LES CORRECTIONS DES SONDAGES AU PLOMB-POISSON

(CORRECTIONS TO SOUNDINGS WITH THE FISH LEAD).

by

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Sounding with the fish lead has been employed by the Hydrographic Expeditions since 1921. It has been described in detail in an article by M. Marti, Ingénieur hydrographe (1).

Here we need only mention the fact that the sounding gear comprises a lead of elongated shape suspended on a metallic wire as fine as possible. The wire unwinds from a reel aboard the surveying boat. Between soundings the lead is lifted one or two metres and is towed parallel to the boat; a small rudder fitted to the lead insures its stability underway.

But the friction of the liquid on the lead and on the wire suspension cable prevents the latter from remaining vertical and at right angles to the course; it bends and assumes a position of curvilinear equilibrium, the curve being concave in the direction in which the boat is moving. This phenomenon produces the following results upon the soundings obtained, viz.—

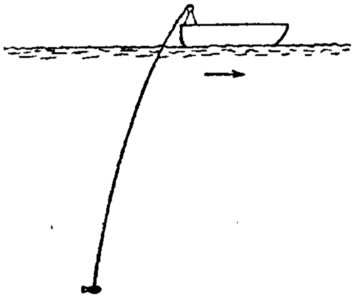

Fig. 1.

In the first place the sounding lead does not remain in the vertical plane containing the point of suspension of the cable in the boat, but at a certain distance δ behind this vertical, such that the position of the lead is not exactly the same as that of the boat at the instant of sounding. It may be deduced by applying this distance δ to the position in the sense opposite to its displacement.

For the rest, the length of the wire between the lead and the surface of the water is somewhat greater than the actual depth of immersion of the lead below the surface; the quantity $k = l - h$ is the subtractive correction to be applied to the length of line paid out in order to deduce the actual depth of the lead.

The corrections δ and k increase with the depth of the sounding and the speed of the boat; further, the same is true for the inclination α of the line to the vertical at the surface of the water.

These corrections, of which the first has not as yet been studied, even though it may become rather large under certain conditions (2), may be calculated as a function of the data

(1) *See* Bibliography N° 9.

(2) *It may, under normal conditions in practice, reach a value of one-fourth of the depth.*

of the sounding gear, of the length l and the speed v; but, this latter quantity, being difficult to measure instantaneously, has been replaced by the inclination α at the surface of the water, which may be measured more readily at the instant of each sounding.

The determination of the expression for k and δ is associated with the more general problem of the shape of the sounding wire. The authors who have treated the question of the equilibrium of a moving wire in a resistant medium have been led to evolve several hypotheses on hydraulic resistance, and have therefore necessarily arrived at different solutions; from each of these one may obtain an expression for the quantities k and δ.

Hydrographic Engineers Courtier and Cathenod (1), and Captain Tonta (2), have worked out the immersion correction by assuming different hypotheses regarding the resistance to horizontal traction of an element of wire inclined to the vertical. The results obtained are in agreement so long as the angle of inclination α at the surface of the water remains moderately small; only when this exceeds about 30 degrees do the divergences begin to be accentuated. Lacking the necessary experiments to determine the most probable hypothesis and for the adoption of the corresponding corrections, one is therefore led in taking soundings to reduce the speed of the surveying boat to a point where the angle α shall remain less than 30 degrees. Under these conditions there is no practical reason for adopting one rather than another of the correction formulae, since owing to the relatively low value of the angle α these are practically independent of the part relating to the horizontal resistance of the wire, which is more or less unknown.

At this speed, however, one can also profit by the fact that this limiting angle is sufficiently small to lend itself to the development of the series, thanks to which it becomes possible to integrate the equations of equilibrium, making only a few preliminary hypotheses regarding the resistance of the water.

This is the method which we shall use in determining the two corrections to be applied to the soundings by fish lead, a method which gives us the means of treating the problem in a more general manner than before because, by assigning limiting numerical values to the coefficients in the formulae which we shall obtain for the immersion correction, we may re-discover the principal terms of the formulae already established. With this procedure we are also enabled to take into consideration the weight of the sounding wire, which the authors have previously had to neglect in order to effect their integrations. Finally this method makes manifest the coefficients of hydraulic resistance which it is important to determine empirically if one wishes to make use of soundings when the suspension wire is inclined to the vertical by angles greater than 30 degrees. It thus allows us to orient the experimenters with regard to the research to be undertaken in order to formulate the laws of hydraulic resistance in the cases under consideration.

First we shall determine the expression for the sounding correction by integrating the equations of equilibrium, after having expanded them in series in accordance with increasing values of the inclination. We shall thereupon study the order of magnitude of the different terms of the formulae which result, a fact which allows us to give to these formulae, in most of the practical cases, a very simple form. In the final chapter we shall discuss the formulae and give the methods for the determination of the constants, and finally we shall establish the limits of practical employment of the sounding equipment in the general case, and in the particular case in which we wish to take soundings without the necessity of making corrections.

I. *GENERAL EXPRESSION FOR THE SOUNDING CORRECTIONS.*

Preliminary Hypotheses : We shall asume in the first place, for reasons of symmetry, that the curve formed by the wire is plane and contained in the vertical plane parallel to the course of the surveying boat. As a matter of fact the symmetry is not absolute because the suspension wire is composed of several helical strands, and this has the tendency to cause the wire to depart from a plane curve. The effect of this lack of symmetry on the depth of immersion is probably very slight and can be neglected without error. Furthermore, this procedure has been followed by all the authors who have treated similar problems. With regard to the position of the sounding the effect is probably somewhat greater, but in any case we shall assume that it may equally be neglected.

(1) *See* Bibliography N° 2.
(2) *See* Bibliography N° 11.

We shall assume also that the liquid is stationary or — which comes to the same thing in this study — that the mass flows as a uniform current so that the movement is everywhere identical for all particles in the medium.

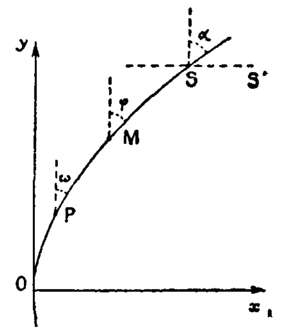

FIG. 2.

Correction formulae : Let us consider the position of equilibrium of the towed wire and let us take on its plane the two axes of coordinates Ox and Oy, the first being horizontal in the direction of the course and the second vertically upright. As origin of coordinates let us take the point on the curve at which the tangent is vertical — this coinciding with the axis Oy. It is readily seen that the point O will not lie on the actual arc PS, comprised between the lead and the surface of the water SS', but somewhat below this arc.

Let M be any point on the curve with coordinates x, y; and let us designate by φ the angle which the curve at this point makes with the axis Oy. We shall adopt a parametric representation of the curve of equilibrium : $x = x(\varphi)$, $y = y(\varphi)$; finally let us call $s(\varphi)$ the length of the arc of the curve OM.

The angle φ varies from a value of ω , inclination of the wire at the point of attachment to the lead, to α , inclination of the wire at the surface of the water.

The length of the line paid out is : $l = s(\alpha) - s(\omega)$.

The depth of immersion of the lead is : $h = y(\alpha) - y(\omega)$.

The correction for the immersion is therefore :

$$k = [s(\alpha) - y(\alpha)] - [(\omega)s - y(\omega)].$$

The correction for position, the horizontal distance from P and S, is :

$$\delta = x(\alpha) - x(\omega).$$

Equations of Equilibrium: Now let $T(\varphi)$ be the tension of the wire at M. Let us call F_n and F_t the projections, on the normal and the tangent to the curve at the point M, of the

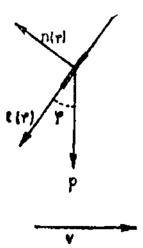

FIG. 3.

resultant of the forces acting together on the unit length of wire at the point M. The equations of equilibrium of the cable are :

$$\frac{dT}{ds} = F_t, \quad T\frac{d\varphi}{ds} = F_n,$$

F_n and F_t depend upon the characteristics of the wire and the lead, on the towing speed and on the position of the point M, therefore on φ .

Let us assume that the angle is sufficiently small so that one can develop these forces as a convergent series with respect to the successive powers of φ and put :

$$F_t = A + B\varphi + C\varphi^2 + \dots$$
$$F_n = D + E\varphi + F\varphi^2 + \dots$$

It would be easy for us from now on to integrate the equations of equilibrium and to obtain an expression for k and for δ ; but it will be preferable for us to examine

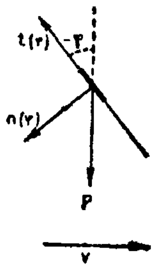

FIG. 4.

first in somewhat more detail the forces F_t and F_n . We may in fact consider that the unit length of the wire is subjected only to its weight (in the water) p and to the resistance to forward movement of which the components are $n(\varphi)$ along the normal and $t(\varphi)$ along the tangent of the curve at M. Consequently :

$$F_t = t(\varphi) + p\cos\varphi,$$
$$F_n = n(\varphi) - p\sin\varphi.$$

If we assume the medium to be homogeneous and the resistance of the water independent of the variation in pressure, it is logical to admit that $t(\varphi)$ cancels out and changes sign with φ , while $n(\varphi)$ does not change when one changes φ to $-\varphi$. Consequently the development of $t(\varphi)$ should only contain the odd-number powers of φ , while that of $n(\varphi)$ only the even-number powers. We shall stop the progression at the terms in φ^2 , given the uncertainty that exists with regard to the coefficient of the term in φ^2 of $n(\varphi)$ and with regard to the term in φ of $t(\varphi)$, as shown in the various treatises on this question. We may then write :

$$F_t = p + B\varphi - \frac{p}{2}\varphi^2 = \frac{dT}{ds},$$

$$F_n = D - p\varphi + F\varphi^2 = T\frac{d\varphi}{ds}.$$

Integration of the Equations of Equilibrium: Taking the first derivatives of these equations we obtain :

$$\frac{dT}{T} = \frac{p + B\varphi - \frac{p}{2}\varphi^2}{D - p\varphi + F\varphi^2} d\varphi = \left[\frac{p}{D} + \left(\frac{B}{D} + \frac{p^2}{D^2}\right)\varphi + \ldots\right] d\varphi.$$

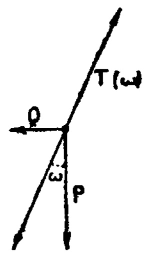

FIG. 5.

from which :

$$\text{Log}\,\frac{T}{T_0} = \frac{p}{D}\varphi + \left(\frac{B}{D} + \frac{p^2}{D^2}\right)\frac{\varphi^2}{2}.$$

$$T = T_0\left[1 + \frac{p}{D}\varphi + \left(\frac{B}{2D} + \frac{p^2}{D^2}\right)\varphi^2\right].$$

The constant of integration T_0 is obtained by noting that the tension $T(\omega)$ of the cable at the point of attachment of the lead is counter-balanced by the weight P of the lead (in the water) and its resistance Q to horizontal traction.

$T(\omega)$ has further the inclination ω, and therefore

$$T(\omega) = \frac{P}{\cos\omega} = P\left(1 + \frac{\omega^2}{2}\right) = T_0\left[1 + \frac{p}{D}\omega + \left(\frac{B}{2D} + \frac{p^2}{D^2}\right)\omega^2\right],$$

and

$$\tan\omega = \frac{Q}{p}, \text{ from which we deduce : } \omega = \frac{Q}{p} - \frac{1}{3}\frac{Q^3}{p^3}.$$

from which :

$$T_0 = P\left[1 - \frac{p}{D}\frac{Q}{P} + \left(1 - \frac{B}{D}\right)\frac{Q^2}{2P^2}\right],$$

D represents the resistance to the forward movement of the unit length of wire held perpendicular to the direction of movement. It is generally agreed that the resistance to the forward movement is sensibly proportional to the square of the speed v. We may therefore write :

$$Q = Q_0 v^2, \qquad D = q_0 v^2.$$

The coefficients Q_0 and q_0 are themselves proportional to the sections of the liquid filaments which act upon the lead and the unit length of wire held vertically. $\frac{Q}{D} = \frac{Q_0}{q_0}$ measures the length C_0 of a wire having the same normal resistance to the forward movement of the lead. Further we shall put $\mu = \frac{p}{P}$ as the ratio between the weight of the lead and the weight of the unit length of wire; $\frac{1}{\mu}$ is a measure of the length of cable which has the same weight as the lead.

We may therefore write :

$$T_0 = P\left[1 - \mu C_0 + \left(1 - \frac{B}{D}\right)\frac{Q^2}{2P^2}\right],$$

T_0 varies very slightly with the speed of the sounding lead, the latter not intervening except through the term in $\frac{Q^2}{p^2}$ and $\frac{Q}{p}$, sensibly equal to ω and being in general a small angle.

The second equation of equilibrium gives : $\frac{ds}{d\varphi} = \frac{T}{D - p\varphi + F\varphi^2}$,

$$\frac{ds}{d\varphi} = \frac{T_0}{D}\left(1 + 2\frac{p}{D}\varphi + 3\frac{p^2}{D^2}\varphi^2 + \frac{B - 2F}{2D}\varphi^2\right).$$

from which, by integrating between o and φ, and by putting $R = \frac{B - 2F}{D}$ in order to simplify the equation, we have:

$$s = \frac{T_0}{D}\varphi\left(1 + \frac{p}{D}\varphi + \frac{p^2}{D^2}\varphi^2 + \frac{R}{6}\varphi^2\right) = T_0\varphi\frac{1 + \frac{R}{6}\varphi^2}{D - p\varphi}.$$

Let us note therefore that we have :

$$s(\omega) = C_0\left[1 + \left(1 - 2\frac{B + F}{D}\right)\frac{\omega^2}{6}\right].$$

in the same manner as T_0, $s(\omega)$ varies very slightly with the speed of the sounding boat and remains close to C_0 .

General Expression for the Corrections : In order to calculate the expressions for x and $s - y$, it suffices to state that :

$$dx = ds\sin\varphi, \qquad dy = ds\cos\varphi, \qquad d(s - y) = ds\,(1 - \cos\varphi).$$

Consequently :

$$\frac{dx}{d\varphi} = \frac{T_0}{D}\varphi\left(1 + 2\frac{p}{D}\varphi + 3\frac{p^2}{D^2}\varphi^2 + \frac{3R - 1}{6}\varphi^2\right),$$

$$\frac{d(s - y)}{d\varphi} = \frac{T_0}{D}\frac{\varphi^2}{2}\left(1 + 2\frac{p}{D}\varphi + 3\frac{p^2}{D^2}\varphi^2 + \frac{6R - 1}{12}\varphi^2\right),$$

and, by integrating between o and φ :

$$x = \frac{T_0}{D}\frac{\varphi^2}{2}\left(1 + \frac{4}{3}\frac{p}{D}\varphi + \frac{3}{2}\frac{p^2}{D^2}\varphi^2 + \frac{3R-1}{12}\varphi^2\right),$$

$$s - y = \frac{T_0}{D}\frac{\varphi^3}{6}\left(1 + \frac{3}{2}\frac{p}{D}\varphi + \frac{9}{5}\frac{p^2}{D^2}\varphi^2 + \frac{6R-1}{20}\varphi^2\right).$$

In order to eliminate from these equations T_0, p and D, we shall first write :

$$x = s\frac{\varphi}{2}\left(1 + \frac{1}{3}\frac{p}{D}\varphi + \frac{1}{6}\frac{p^2}{D^2}\varphi^2 + \frac{R-1}{12}\varphi^2\right),$$

$$s - y = s\frac{\varphi^2}{6}\left(1 + \frac{1}{2}\frac{p}{D}\varphi + \frac{3}{10}\frac{p^2}{D^2}\varphi^2 + \frac{8R-3}{60}\varphi^2\right).$$

But we have, by neglecting the terms of the higher order, $T_0 = P\,(1 - \mu\, C_0)$

$$\frac{\varphi}{D} = \frac{s}{T_0}\left(1 - p\frac{s}{T_0}\right) = \frac{s}{P}\left[1 - \mu\,(s - C_0)\right] \qquad \frac{p}{D}\varphi = \mu s - \mu^2 s\,(s - C_0),$$

and by substituting this expression in x and $s - y$ we obtain :

$$(1)\quad \begin{cases} x = s\dfrac{\varphi}{2}\left[1 + \dfrac{1}{3}\mu s - \dfrac{1}{6}\mu^2 s\,(s - 2\,C_0) + \dfrac{R-1}{12}\varphi^2\right], \\[2ex] s - y = s\dfrac{\varphi^2}{6}\left[1 + \dfrac{1}{2}\mu s - \dfrac{1}{10}\mu^2 s\,(2s - 5\,C_0) + \dfrac{8R-3}{60}\varphi^2\right]. \end{cases}$$

The correction for position δ and the correction for immersion k are respectively the differences in the values of x and $s - y$ for $\varphi = \alpha$ and $\varphi = \omega$, the quantity $s\,(\alpha)$ being taken equal to $l + s\,(\omega)$.

We find again the principal part of the immersion correction given by the various authors by introducing their hypotheses in $(s - y)_\alpha - (s - y)_\omega$ that is, by making $\mu = 0$ and giving to R a running value in accordance with the hydraulic theory adopted.

II. *ORDER OF MAGNITUDE OF THE COEFFICIENTS.*

In the above shape the corrections are not directly applicable; for practical purposes it is necessary to simplify them by neglecting certain terms of little importance. This procedure cannot be accomplished, however, unless we know the order of magnitude of the coefficients μ, C_0 and R, of which the first two depend upon the sounding gear and the third on the hydraulic resistance.

Characteristics of the Sounding Material : We shall study successively the type of cable used, the leads and the sounding gear and the means of securing the lead to the cable. The numerical values which we give have been obtained from various sources; they have been deduced from the results of experiments reported in several papers on the question of the fish-lead and in particular those by M. Courtier.

Cables : Taking the metre as the unit of length, p is the weight in water of 1 metre of the sounding line.

If we consider solely the small metallic cables of steel wire (of 6 to 7 strands in general) we may assume that p is proportional to the square of the diameter of the line, total external diameter (or quotient by π of the circumferential envelope of the strands). By expressing d in millimetres and p in kilogrammes, we find that the order of magnitude for the ordinary cables is about : $p = 0.0037 \, d^2$.

The resistance ρ of these cables to the traction is about 80 kgs per square millimetre of the total section; it is proportional to d^2 and may be taken as equal to $63 \, d^2$.

The coefficient q_0 is proportional to the section of the filaments of water which act normally upon one metre of the cable, and therefore proportional to the diameter d of the cable; if we express the diameter in millimetres and the speed in knots we find approximately $q_0 = 0.012 \, d$ for steel wires. The Hydrographic Engineer HATT had adopted for hemp lines the equation $q_0 = 0.0205 \, d$. (1)

Leads : The coefficient Q_0 is proportional to the largest transverse section of the lead; if we assume the leads to be geometrically similar, Q_0 is proportional to $P^{\frac{2}{3}}$. For those of the type designed by M. MARTI we find $Q_0 = 0.010 \, P^{\frac{2}{3}}$, speeds being expressed in knots. The ratio of Q_0 to the maximum section of the lead (in sq. metres) is about equal to 8.75; it depends primarily on the shape of the lead and the nature of the surface.

Sounding Gear : In general by this term is meant the whole of the equipment comprising the lead and the cable of suspension. In the foregoing we have made use of the expressions $C_0 = \frac{Q_0}{q_0}$ and $\mu = \frac{p}{P}$. From the expressions for q_0 and Q_0 we deduce directly $C_0 = \frac{10 \, P^{\frac{2}{3}}}{12 \, d}$ for steel wires, and $C_0 = \frac{20 \, P^{\frac{2}{3}}}{41 \, d}$ for hemp lines.

M. COURTIER gives the following orders of magnitude for C_0:

For a lead of	70 kgs	(in air) and	fine metal wire	$C_0 = 10$
—	30	—	—	$C_0 = 5$
—	10	—	—	$C_0 = 3$
—	50	—	and hemp line,	C_0 between 0.5 and 1.

By applying these data to the relations between C_0, P, and d we may deduce the following values for the diameter of the wire : 1.3, 1.5 and 1.2 mm. for the steel wire; 8 or 9 mm. for the hemp lines. These values correspond to the dimensions actually in use for the sounding lines, and serve as a check on our formulae.

It is interesting also to study the ratio γ between the resistance of the line to traction and the weight of the lead : $\gamma = \frac{\rho}{P}$; this ratio is in a measure the coefficient of safety of the sounding gear; the greater this radio the less danger of the line breaking under the traction of the lead. γ is necessarily greater than unity, because at that limiting value the weight of the lead is just sufficient to bring about the rupture of the line.

It is easy to establish the following relations :

$$\mu = \frac{\gamma}{17.000}, \qquad C_0 = \frac{6,62}{\gamma^{\frac{1}{2}}} P^{\frac{1}{6}}, \qquad d = 0,126 \, \gamma^{\frac{1}{2}} \, P^{\frac{1}{2}}.$$

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(1) *It should be noted that the graduations of the line, if they are not very carefully carried out, may increase the friction and therefore the resistance to traction; the best is to have the marks painted, but owing to the lack of permanence of such marks they have been substituted by fine metal wire, in preference to the usual graduations of bunting, soft leather and housing line (twine).*

Standard Equipment : In general the sounding line to be used with a certain lead is chosen such that the value of the coefficient γ will be about equal to 6, which necessitates:

$$d = 0{,}31\ P^{\frac{1}{2}}, \quad \mu = \frac{1}{2{,}800}, \qquad C_0 = 2{,}7\ P^{\frac{1}{6}}.$$

For the standard equipment, C_0 will vary very little with the weight of the lead; it reaches 3.95 m. for P = 10 kg. to 5.80 m. for P = 100 kg. Since P is the value in salt water, or approximately 9/10ths of the value in air for these leads, the relation $d = 0.31\ P^{\frac{1}{2}}$ gives us the following results :

P in kilogrammes.	WEIGHT IN AIR in kilogrammes.	DIAMETER OF THE LINE to be adopted in millimetres.
9	10	0.9
16	17.5	1.2
25	27.5	1.5
64	70	2.5

These values are in complete accord with those given by M. Marti who advises adopting :

for leads of 12 to 15 kgs, a cable of 1 mm. dia.
— 25 to 30 — 1.5 —
— 60 to 80 — 2.5 —

Upper Limits : From these orders of magnitudes we deduce the following results, which are of particular interest for us :—

(*a*) γ being greater than unity, C_0 is less than 6.62 $P^{\frac{1}{6}}$ and consequently remains less than 14 metres provided P does not exceed 100 kg. (lead of 110 kg.). In practice, even with very heavy leads, C_0 will not exceed about 10 metres.

(*b*) If we employ lines having twice the diameter of those of the standard type, γ is then multiplied by four, and therefore becomes equal to 24; μ is then equal to 1/700. We assume that this may be considered as the greatest value of μ.

Coeffcients of the Hydraulic Resistance : With regard to the hydraulic resistance, in the absence of direct measurements, we shall content ourselves with reviewing the hypotheses established by the various authors and with the determination of the values which result from them for the coefficients $\frac{B}{D}$ and $\frac{F}{D}$ as well as for the expression $R = \frac{B - 2F}{D}$.

A first group of authors assumes that the tangential component of the resistance will be equal to zero : $B = 0$, from which $R = -\frac{2F}{D}$. Among these M. Cathenod, who obtained a circle for the curve of equilibrium of the wire, has assumed the normal resistance to be constant, which implies that $F = R = 0$. M. Courtier has assumed that it is proportional to $\cos \varphi$, whence $\frac{F}{D} = -\frac{1}{2}$ and $R = 1$. Finally Captain Tonta assumes a catenary by taking the normal resistance proportional to $\cos^2 \varphi$, from which $\frac{F}{D} = -1$ and $R = 2$.

Other authors consider that the resistance is horizontally directed and arrive at the conclusion that $B = D$; $\frac{t(\varphi)}{n(\varphi)} = \tan\varphi$ and $R = 1 - \frac{2F}{D}$. If we grant that it is constant, we find that the wire still takes the shape of the catenary : $F = 0$ and $R = 1$. If, with M. di MARCHI we assume it to be proportional to $\cos\varphi$, we obtain a parabola and $\frac{F}{D} = 1$, $R = 3$.

For the rest, in a study of the equilibrium of the towed wire (1), M. HABERT has assumed from his experiments that $\frac{B}{D}$ equals 0.0075. He considers further that the component normal to the resistance is proportional to $\cos\varphi$: $\frac{F}{D} = -\frac{1}{2}$ whence $R = 1.0075$.

Granted the small value of B with respect to D, his hypothesis is very close to that of M. COURTIER and is in practical agreement with it for the range of inclinations which we are considering.

III. *EXPRESSION FOR CORRECTIONS — SIMPLIFIED FORM.*

Knowing the orders of magnitude for the coefficients of the formulae (1), we shall next study the value of the various terms of these expressions neglecting those which remain below a value too small to have any practical importance :

Terms in ω. Re-write the formulae :

$$(2) \quad \begin{cases} k = [s(\alpha) - y(\alpha)] - [s(\omega) - y(\omega)], \\ \delta = x(\alpha) - x(\omega). \end{cases}$$

It will now be seen that the terms in ω may be neglected in these relations, which means, practically, that we assume the sounding line to be vertical at the line of attachment of the lead — this, however, for the computation of corrections only and not for the drawing up of the correction formulae.

In fact, the principal value of $x(\omega)$ is $\frac{C_0\,\omega}{2}$, that of $s(\omega) - y$ is $\frac{C_0\,\omega^3}{6}$.

These terms may be written, taking into account that $C_0 = \frac{6.62}{\gamma^{\frac{1}{2}}} P^{\frac{1}{6}}$ and that

$Q_0 = 0.010\, P^{\frac{2}{3}}$ and putting $v' = \frac{v}{10}$:

$$\text{for } x(\omega) : \; \frac{3{,}31\; v'^2}{\gamma^{\frac{1}{2}}\, P^{\frac{1}{6}}}, \qquad \text{for } s(\omega) - y(\omega) : \; \frac{1{,}10\; v'^4}{\gamma^{\frac{1}{2}}\, P^{\frac{1}{2}}}.$$

The smaller γ and P, the greater these are; passing then to the inferior limits 1 and 10 of γ and of P, the maxima of these terms are respectively $2.26\, v'^2$ and $0.348\, v'^4$. Before they can attain, the first 1 metre, the second 1 decimetre, sounding velocities must be 6.7 or 7.3 knots, therefore appreciably greater than those in current practice. Consequently the errors introduced by neglecting the terms in ω are less than 1 metre on δ, and 1 decimetre on k, even much less in reality since, in standard material, $\gamma = 6$ and leads are generally more than 10 kg.

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(1) *See* Bibliography N° 5.

For instance, for $\gamma = 6$, $P = 25$ kg. and $v = 5$ knots, we have :

$$C_0 = 4^m,6, \quad \omega = \frac{1}{11,7} = 5^0, \quad x(\omega) = 0^m,20 \quad \text{et} \quad s(\omega) - y(\omega) = 0,006.$$

To sum up, it is quite admissible to adopt :

$$\delta = x(\alpha), \quad k = s(\alpha) - y(\alpha).$$

Influence of weight of line. — This influence is expressed by the terms in μ of δ and of k, which become important only in soundings at great depths. In this case, $s(\omega)$ which is practically equal to C_0 and is of a few metres only, may be neglected before l which may be cancelled out in $s(\alpha)$, so that $\mu s(\alpha)$ practically represents the relation between the weight of the submerged line and the weight of the lead.

$\frac{\mu s(\alpha)}{3}$ and $\frac{\mu s(\alpha)}{2}$ are the errors introduced on δ and on k respectively by neglecting the weight of the line; they are practically proportional to the immersion of the lead. The corresponding absolute errors are $\frac{\mu\alpha}{6} s^2(\alpha)$ and $\frac{\mu\alpha^2}{12} s^2(\alpha)$; roughly, they vary to the same degree as the square of immersion, which explains why they become important at great depths. For a value of 30 degrees which we consider as the maximum of α and remembering that $\mu = \frac{\gamma}{17,000}$, these absolute errors are written $\frac{\gamma}{195,000} s^2(\alpha)$ and $\frac{\gamma}{745,000} s^2(\alpha)$: the immersion error is then nearly one quarter of the position error and remains less than one-hundredth of the length of line paid-out so long as $s(\alpha)$ does not exceed $\frac{7,450}{\gamma}$, i. e. 1,258 metres with standard gear.

With this same gear the errors on δ and k reach 1 metre when $s(\alpha)$ is equal to 179 metres (for δ) or to 350 metres (for k), when the terms in μ^2 are 30 to 40 times smaller and of the order of 3 centimetres.

With lines of diameter n times greater, weight of lead being the same, values of $s(\alpha)$ corresponding to an error of 1 metre on k and δ, must be divided by n; seeing that, in practice, n rarely exceeds 2, it is evident that the influence of the weight of the sounding line is perceptible only in soundings at great depths; in all current soundings it may be assumed that the line has no weight.

Influence of the hydraulic resistance :— The terms containing the unknown coefficient R of the hydraulic resistance are :

$$\text{for } \delta : \quad s(\alpha)\frac{\alpha^3}{24}(R - 1) \qquad \text{and for } k : \quad s(\alpha)\frac{\alpha^4}{360}(8R - 3).$$

Among the hypotheses which we have examined, that of M. di MARCHI gives the greatest value to R ($R = 3$); it is the least favourable from our point of view since it leads up to the highest value of the terms containing R. It is in admitting this hypothesis, therefore, that we shall study these terms which are then written :

$$s(\alpha)\frac{\alpha^3}{12} \qquad \text{and } s(\alpha)\frac{7\alpha^4}{120}.$$

For $\alpha = 30$ degrees, the maximum which we shall consider, they are equal to $\frac{s(\alpha)}{84}$ and $\frac{s(\alpha)}{228}$. If they are neglected, a sounding position error is introduced which is hardly greater than 1 in 100 metres depth and an immersion error of 1 decimetre in about 23 metres depth. These respective errors may very well be accepted; besides, they are maximum errors, the errors usually encountered are less and in practice there is no reason why the terms containing R should not be neglected.

Hydraulic resistance is taken into account in still another term; $s(\alpha)$ is in fact equal to $s(\omega) + l$ and we have seen that :

$$s(\omega) = C_0 \left[1 + \left(1 - 2 \frac{B+F}{D} \right) \frac{\omega^2}{6} \right].$$

Among the various hypotheses on the resistance to horizontal traction, it is that of Captain TONTA which gives the maximum value ($= 3$) to the coefficient $1 - 2 \frac{B+F}{D}$ of the term in ω^2 ; it may therefore be assumed that $s(\omega)$ differs from C_0 by the term ε at most equal to $\frac{C_0 \omega^2}{2}$. Now, we have already established that $\frac{C_0 \omega^2}{6}$, principal value of $s(\omega) - y(\omega)$ is generally much less than 1 decimetre; consequently $s(\omega) = C_0 + l$ to within nearly 0 m. 30 and, if ε is neglected in the expressions of δ and of k, the errors introduced on these quantities are $\frac{\varepsilon \alpha}{2}$, less than 1 decimetre, and $\frac{\varepsilon \alpha^2}{6}$ of the order of 1 centimetre at greatest.

Final expression for corrections :— The superposition of the errors resulting from the various terms neglected is admissible so long as we remain within the limits of inclination adopted and do not reach depths of several hundreds of metres. In these conditions the weight of the sounding line and the influence of the obliquity of the wire on the resistance to horizontal traction are of no practical importance and sounding corrections may be written with sufficient accuracy :

$$(3) \quad \begin{cases} \delta = (C_0 + l) \dfrac{\alpha}{2} \text{ for position,} \\ k = (C_0 + l) \dfrac{\alpha^2}{6} \text{ for immersion.} \end{cases} \qquad \begin{cases} \delta = \dfrac{C_0 + l}{1,15} \dfrac{\alpha}{100}, \\ k = \dfrac{C_0 + l}{1,57} \left(\dfrac{\alpha}{100} \right)^2, \end{cases} \text{ if } \alpha \text{ is expressed in degrees.}$$

With these simple correction formulae it is easy to draw up for any sounding gear tables or graphs rapidly giving, in function of the measured quantities l and α, the corrections to be applied to the soundings.

It may be noted that, when the lead is at the surface of the water, l, h, and δ are zero; now our formulae in their final form give for l-h and δ values different from zero : $\frac{C_0 \alpha}{2}$ and $\frac{C_0 \alpha^2}{6}$; but there is no contradiction for, in this case, α must be assumed equal to ω and we find again terms equal to the terms in ω of the formulae (2), precisely those we have considered negligible.

IV. *RESULTS AND DISCUSSION.*

Magnitude of Corrections. — To give an idea of the magnitude of the corrections, Tables I and II have been drawn up, with values of α every 5 degrees and for values of $C_0 + l$ every 10 metres up to 100 metres; seeing that the corrections δ and k are proportional to $C_0 + l$, it is easy to deduce from the tables, correction values for any value of $C_0 + l$, provided that the simplified correction formulae still hold good. It will further be borne in mind that C_0 has a mean value of from 4-5 metres and only in exceptional cases exceeds 10 metres.

TABLE I

Position Correction δ (in metres).

α / $C_0 + l$ (mètres)	5°	10°	15°	20°	25°	30°
10	0m	1m	1m	2m	2m	3m
20	1	2	3	3	4	5
30	1	3	4	5	7	8
40	2	3	5	7	9	10
50	2	4	7	9	11	13
60	3	5	8	10	13	16
70	3	6	9	12	15	18
80	3	7	10	14	18	21
90	4	8	12	16	20	24
100	4	9	13	17	22	26

TABLE II

Immersion Correction k (in metres).

α / $C_0 + l$ (mètres)	5°	10°	15°	20°	25°	30°
10	0m, 0	0m, 1	0m, 1	0m, 2	0m, 3	0m, 5
20	0 0	0 1	0 2	0 4	0 6	0 9
30	0 0	0 2	0 3	0 6	1 0	1 4
40	0 1	0 2	0 5	0 8	1 3	1 8
50	0 1	0 3	0 6	1 0	1 6	2 3
60	0 1	0 3	0 7	1 2	1 9	2 7
70	0 1	0 4	0 8	1 4	2 2	3 2
80	0 1	0 4	0 9	1 6	2 5	3 7
90	0 1	0 5	1 0	1 8	2 9	4 1
100	0 1	0 5	1 1	2 0	3 2	4 6

Influence of Velocity and Immersion. — As a first approximation we have $\frac{\omega}{\alpha} = \frac{C_0}{C_0 + l}$. But as :

$$C_0 = \frac{Q_0}{q_0} \quad \text{et} \quad \omega = \frac{Q}{P} = \frac{Q_0 v^2}{P} \quad \text{we may write :} \quad \alpha = (C_0 + l)\frac{q_0}{P}v^2.$$

Inserting this expression in formulae (3), we then have :

$$\delta = (C_0 + l)^2 v^2 \frac{q_0}{2P}, \qquad k = (C_0 + l)^3 v^4 \frac{q_0^2}{6P^2},$$

Since $C_0 + l$ does not greatly differ from the immersion depth of the lead h, where l is fairly large in relation to C_0, it may be deduced from these ratios :—

1). — that the position correction, for a known immersion, varies in like proportion with the square of the velocity and, for a known velocity, roughly with the square of immersion;

2). — that the depth correction, for a known immersion, varies in like proportion with the fourth power of the velocity and, for a known velocity, roughly with the cube of the immersion : it therefore increases more rapidly than some authors, who assumed it to be proportional to the square of the depth and to the square of the velocity, have assumed.

Influence of error on inclination α. — The length l of line paid-out is always known with accuracy; but not the inclination of the line on the vertical, which it is difficult to determine more accurately than to within about a degree.

An error $d\alpha$ on the measurement of α introduces in the corrections the errors :

$$d\delta = \frac{C_0 + l}{2} d\alpha \text{ and } dk = \frac{C_0 + l}{3} \alpha d\alpha.$$

If we take $d\alpha = 1$ degree $= 0.0175$, the formulae (3) give δ and k with an approximation equal to $\frac{C_0 + l}{115}$ for position and $\frac{C_0 + l}{172} \alpha$ for immersion; the latter error, for the maximum of 30 degrees assumed for α is itself equal to $\frac{C_0 + l}{329}$. These quantities are somewhat less than the maximum errors introduced by neglecting the term in R of the hydraulic resistance $\left(\frac{C_0 + l}{84} \text{ and } \frac{C_0 + l}{228}\right)$, errors which we have considered quite admissible.

Consequently, the uncertainty of 1 degree in the measurement of the inclination has only a very slight influence on the determination of the sounding, in position and in amount.

Determination of C_0. — In order to determine the value of the constant C_0 of the sounding outfit, the formula $C_0 = \frac{Q_0}{q_0} = \frac{10\, P^{\frac{2}{3}}}{12\, d}$ might be used, and P and d replaced by their measured values, but the numerical coefficients of this expression are mean values which do not correspond exactly to the materiel available; it is preferable to hold this relation in abeyance in order to verify approximately the value of C_0 obtained by another determination.

The best procedure is to take into account the relation $\frac{\omega}{\alpha} = \frac{s(\omega)}{l + s(\omega)}$, (which assumes the weight of the line to be negligible) from which we get $s(\omega) = \frac{l\, \omega}{\alpha - \omega}$.

The determination of ω is made by measuring the inclination of the line when the lead is towed awash; a length l of line is then paid-out and the inclination α read, the towing being carried out at the same speed as exactly as possible. From these measurements the value of $s(\omega)$ is deduced, corresponding to the speed in question. The operation is repeated at different speeds and other values for $s(\omega)$ obtained. We have seen that $s(\omega)$ differed from C_0 by a quantity, function of ω, therefore of velocity, but practically negligible; it is sufficient therefore to adopt the mean of the different values calculated for $s(\omega)$ for the value of C_0.

For the computation of the corrections it is besides unnecessary to know C_0 with any great precision, an error of 1 metre is almost admissible since, for $\alpha = 30°$, it involves errors of only 0 m. 26 on the position correction and of 0 m. 07 on the immersion correction. It is necessary, however, to determine C_0 carefully so that material for the theoretic studies of the soundings may be available if required.

Other determinations :— The measurement of C_0 is the only one necessary for current soundings; one may be led, however, to determine some of the other constants which we have used during these computations.

$\mu = \frac{p}{P}$ is obtained by weighing directly in the salt water the sounding lead and a coil of sounding wire of known length.

The determination of Q_0 is more delicate, for it necessitates a knowledge of the speed of the sounding vessel relative to the water; the relation which enables Q_0 to be obtained is in fact $Q_0 = \frac{P\, \omega}{v^2}$; it is advisable to carry out the measurements at rather varied speeds and to take a mean.

Q_0 being known, it is easy to obtain q_0 by $q_0 = \frac{Q_0}{C_0}$.

$\frac{B}{D}$ may be determined by hanging the sounding line from a dynamometer placed sufficiently near the surface of the water so that the weight of the line between the water and the point of suspension may be neglected. Let us consider the first equilibrium equation; neglecting the term in φ^2 it is written :

$$dT = pds + B\varphi ds.$$

Now ascertain the variation of tension on the line between the lead and the surface of the water :

$$T(\alpha) - T(\omega) = pl + \int_{s(\omega)}^{s(\alpha)} \varphi ds.$$

Now, as a first approximation $\dfrac{ds}{d\varphi} = \dfrac{T_0}{D} = \dfrac{P}{D}$.

Therefore :

$$T(\alpha) - T(\omega) = pl + \frac{P}{2}\frac{B}{D}(\alpha^2 - \omega^2).$$

The variation in tension between the two positions of the lead : at the surface of the water and at immersion corresponding to the length l of line paid-out, may also be given by this formula if it is assumed that the towing speed is the same at the two positions. It is equal to the weight of the line unreeled, increased by a term which is a function of the hydraulic resistance.

If the length l is appreciably greater than C_0, the term in ω^2 is practically negligible, since $\omega = \alpha \dfrac{C_0}{C_0 + l}$ and the tension variation may be written more simply :

$$T(\alpha) - T(\omega) = pl + \frac{B}{D} P \frac{\alpha^2}{2}.$$

Measurement at the dynamometer of the tensions $T(\alpha)$ and $T(\omega)$ should enable the value of $\dfrac{B}{D}$ to be determined not perhaps with great accuracy because of terms of greater magnitude which have been neglected and of uncertainty in the determination of declinations, but this value would doubtless be sufficient to admit of a compromise between the two principal hypotheses emitted on the resistance to horizontal traction which is considered either normal to the sounding line $\left(\dfrac{B}{D} = 0\right)$, or horizontal $\left(\dfrac{B}{D} = 1\right)$; unless one is led to adopt for $\dfrac{B}{D}$, which is possible, a definitely different value from 0 and 1.

This measurement, which in our opinion is essential, does not appear difficult to carry out and does not necessitate a very sensitive dynamometer. Let us examine, for instance, the case of a sounding lead of 74 kilogrammes and a suspension cable weighing 23 grammes to the metre (standard type); let us assume a towing speed of 4 knots; the inclination ω when the lead is at the surface of the water is 2°.3; theoretically the tension on the line is then about 64,051 grammes. When 65 metres of line are paid-out, the inclination attains 30 degrees and tension should increase by 1495 grammes, weight of line unreeled and of $\dfrac{B}{D} \times 8{,}723$ grammes, value of the term, function of the hydraulic resistance; if the hypothesis $\dfrac{B}{D} = 0$ were exact, a tension of about 65,500 grammes would be found; should $\dfrac{B}{D} = 1$ be the sound hypothesis, tension would be about 74,300 grammes. The difference is considerable and a direct measurement, even if rather roughly carried out, would do much towards clearing up the question.

Limits of use of Sounding Gear. The relation $\alpha = (C_0 + l)\frac{q_0}{P} v^2$ shows that, for a known speed and a known immersion, the heavier the weight and the finer the line, the smaller the inclination; the better these conditions are realised, therefore, the more time it will take for the inclination limit to be reached.

On the other hand, given the same gear, the inclination increases, for a known depth, in proportion to the square of the speed; for a known speed, it varies almost proportionally to the depth.

It results from the above that the conditions determining the limit of use of any gear will be given by the relation between the immersion of the lead and the speed of the boat, when the allowed maximum inclination is reached.

Let us discover, therefore, the relation between h, α and v.

We have, approximately :

$$C_0 + h = C_0 + l - k = (C_0 + l)\left(1 - \frac{\alpha^2}{6}\right) = (C_0 + l)\frac{\sin \alpha}{\alpha}$$

But :

$$\frac{C_0 + l}{\alpha} = \frac{P}{q_0 v^2} = \frac{C_0 P}{Q_0 v^2}. \quad \text{whence} \quad h = C_0\left(\frac{P \sin \alpha}{Q_0 v^2} - 1\right).$$

For ordinary leads and cables we have :

$$\frac{P}{Q_0} = 100\, P^{\frac{1}{3}}, \qquad C_0 = \frac{6{,}62\ P^{\frac{1}{6}}}{\gamma^{\frac{1}{2}}},$$

We may therefore write :

$$h = \frac{6{,}62\ P^{\frac{1}{6}}}{\gamma^{\frac{1}{2}}}\left(\frac{100\, P^{\frac{1}{3}} \sin \alpha}{v^2} - 1\right).$$

The depth h which may be reached under the inclination α is, for a known speed, the greater in proportion with the smallness of γ, therefore nearer to 1; at the limit it is equal to $6.62\, P^{\frac{1}{6}}\left(\frac{100\, P^{\frac{1}{3}} \sin \alpha}{v^2} - 1\right)$. For instance, for a lead of 64 kgs, the inclination of 30 degrees is reached at the depth $h = 13.24\left(\frac{200}{v^2} - 1\right)$, i. e. in function of the speed.

v...	0ⁿ,5	1ⁿ	2ⁿ	3ⁿ	4ⁿ	5ⁿ	6ⁿ	7ⁿ	8ⁿ	9ⁿ	10ⁿ
h...	10.579ᵐ	2.635ᵐ	649ᵐ	281ᵐ	152ᵐ	93ᵐ	60ᵐ	41ᵐ	28ᵐ	20ᵐ	13ᵐ

The depths thus obtained are maximum, since the sounding line is just sufficient to support the weight of the lead; these depths could only be exceeded by using special high resistance steel lines. In practice, it has been seen that γ is of the order of 6 (standard gear), the limit values of h are therefore two and a half times less and satisfy the ratio $h = 135\, \frac{P^{\frac{1}{2}}}{v^2} - 2.70\, P^{\frac{1}{6}}$. Table III below gives in function of the weight of the lead and the speed of the boat, the depths at which the inclination of 30° is reached in the case of the standard gear.

TABLE III.

LIMIT DEPTHS ($\alpha = 30°$) REACHED WITH THE STANDARD GEAR ($\gamma = 6$).

P / v in knots.	16 kilogrammes	36 kilogrammes	64 kilogrammes	100 kilogrammes
	metres.	metres.	metres.	metres.
0.5	2,156	3,235	4,135	5,396
1	536	805	1,075	1,344
2	131	198	265	332
3	56	85	115	144
4	29	46	62	79
5	17	27	38	48
6	11	18	25	32
7	7	12	17	22

These numbers show clearly the preponderant rôle of the speed which enters by its square while the weight of the lead is shown at the power of only 1/2 in the principal term.

Sounding with a fish-lead becomes practically impossible at speeds exceeding 4 to 5 knots, the inclination of 30 degrees being attained, at these speeds, as soon as the current depths of from 30 to 50 metres are reached.; if greater inclinations were tolerated, there would be involved the risk of the uncertainty of the hydraulic resistance very considerably falsifying the corrections made to the soundings.

Soundings without correction. One may seek to discover the limits that must be respected if it is desired to carry out soundings without having to make the immersion correction. We will presume that this correction is negligible when it is less than 1/100th of the depth : $k < \frac{h}{100}$ which may be written $101\, k < 1$.. The limit value of α is then given by the inequality :

$$101\ \frac{\alpha^2}{6} < \frac{l}{C_0 + l}\,.$$

If l is superior or equal to C_0, $\frac{l}{C_0 + l}$ will be at least equal to $\frac{1}{2}$ and the inequality will be satisfied so long as α remains less than 10 degrees. If l is less than C_0, the correction k is inferior to $\frac{C_0\,\alpha^2}{3}$, hence to $\frac{C_0}{101}$ if the same inclination limit of 10 degrees is adopted; given the order of magnitude of C_0, this correction will as a rule be less than 1 decimetre.

Consequently, 10 degrees is the greatest inclination limit for soundings without correction. In practice, if l is relatively great in relation to C_0, there may be taken for α a limit between 10 and 15 degrees; the difference between the immersion and the length of line paid-out obviously increases with α, but its value still remains less than 1/100th of the depth. In soundings without immersion correction, the position correction may attain a value close to 1/10th of the depth.

With standard gear, the inclination of 10 degrees is attained, according to the weight of the lead and the speed of the boat, for known depths by tabulation N° IV and computed from the relation :

$$h = 46{,}53\,\frac{P^{\frac{1}{2}}}{v^2} - 2{,}70\,P^{\frac{1}{6}}\,.$$

The figures of this tabulation show that, in order to sound without correction, with ordinary gear, the speed of 2 to 3 knots must not be exceeded.

TABLE IV.

LIMIT DEPTHS ($\alpha = 10^\circ$) ATTAINED WITH STANDARD GEAR ($\gamma = 6$).

P / v in knots.	16 kilogrammes	49 kilogrammes	100 kilogrammes
	metres.	metres.	metres.
0,5	740	1,298	1,855
1	182	331	459
2	42	76	110
3	16	31	46
4	7	15	23

BIBLIOGRAPHY

1 H. CATHENOD ... *Sondage en marche à allure moyenne avec le plomb-poisson* (*Annales Hydrographiques,* 1922).

2. A. COURTIER .... *Sur la courbure de la ligne et la correction d'inclinaison dans les sondages au plomb-poisson* (*Annales Hydrographiques,* 1925-1926).

3. A. GOUGENHEIM.. *Nomogramme donnant la correction d'inclinaison de la ligne dans les sondages au plomb-poisson* (*Annales Hydrographiques,* 1931-1932).

4. A. GOUGENHEIM.. *Détermination nomographique de la correction d'inclinaison dans les sondages au plomb-poisson* (*Revue Hydrographique,* 1932. IX-2).

5. J. HABERT .... *Equilibre d'un fil traîné dans l'eau* (*Annales Hydrographiques,* 1933).

6. P. HATT .... *Influence des courants sur les mesures de profondeur prises au moyen des lignes de sonde* (*Annales Hydrographiques,* 1877).

7. H. LAROSE ...... *Note sur le mouvement uniforme d'un fil dans un milieu résistant,* 1917.

8. L. DI MARCHI ... *Teoria degli scandagli d'alto mare,* 1913.

9. P. MARTI .... *La sonde au plomb-poisson* (*Annales Hydrographiques,* 1925-1926).

10. P. MARTI .... *Matériel de sondage au moyen du plomb-poisson* (avec enregistreur) (*Revue Hydrographique,* 1933, X-1).

11. L. TONTA .... *Sur la courbure de la ligne de sonde et la correction d'inclinaison* (*Revue Hydrographique,* 1927, IV-2).

12. L. TONTA .... *Sur la courbure de la ligne de sonde et la correction d'inclinaison* (*Revue Hydrographique,* 1930, VII-2).