A CALENDAR OF THE FUNDAMENTAL HARMONIC TIDES AND DIAGRAMS FOR THEIR GRAPHICAL PREDICTION

by

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The height of the tide in a place at any moment T is given in the harmonic notation by the well-known formula :

$$y = A_0 + \Sigma \, R \cos \varphi \qquad (1)$$

in which : $\varphi = V_0 + \mathrm{n}T - \mathrm{g}$ with $R = \mathrm{f} \times H$

A_0 is the height of the mean sea level above chart datum; R is the half range of a constituent; V_0 is the equilibrium argument at the initial epoch from which the time T is reckoned; n is the speed of the constituent; nT is the increment of the equilibrium argument from the time 0^h; lastly, g is the particular phase lag of the constituent for the locality under consideration referred to the meridian of Greenwich.

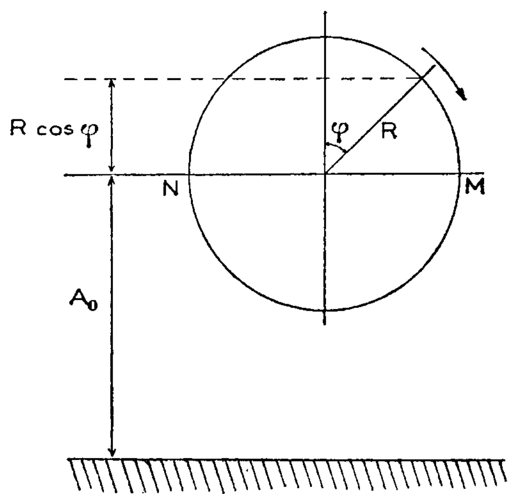


FIG. 1.

In figure (1) it will be seen that High Water of the constituent occurs when $\varphi = 0°$ and for consecutive integral multiples of $360°$.

HEURES DES POTENTIELS MAXIMA À GREENWICH

JANVIER 1940

TABLE T_m

JOUR DU MOIS f=	M_2 1.04	S_2 1	N_2 1.04	K_2 0.77	K_1 0.89	O_1 0.83	P_1 1	Q_1 0.83	M_4 1.07	MS_4 1.04
1	04.54	12.00	05.21	05.11	23.08	11.11	00.36	12.21	04.55	02.24
2	05.44	12.00	06.40	05.07	23.04	13.00	00.40	15.13	05.46	02.49
3	06.35	12.00	07.59	05.03	23.00	14.50	00.44	18.05	00.23	03.14
4	07.25	12.00	09.18	04.59	22.56	16.39	00.48	20.57	01.14	03.39
5	08.16	12.00	10.37	04.55	22.52	18.28	00.52	23.49	02.04	04.04
6	09.06	12.00	11.56	04.51	22.49	20.17	00.56	26.41	02.55	04.28
7	09.57	12.00	00.35	04.47	22.45	22.07	01.00	02.41	03.45	04.53
8	10.47	12.00	01.54	04.44	22.41	23.56	01.04	05.33	04.36	05.18
9	11.38	12.00	03.13	04.40	22.37	25.46	01.08	08.25	05.26	05.43
10	00.03	12.00	04.32	04.36	22.33	01.46	01.12	11.17	00.04	00.01
11	00.54	12.00	05.51	04.32	22.29	03.35	01.16	14.10	00.55	00.26
12	01.44	12.00	07.10	04.28	22.25	05.24	01.20	17.02	01.45	00.51
13	02.35	12.00	08.29	04.24	22.21	07.13	01.24	19.54	02.36	01.16
14	03.25	12.00	09.48	04.20	22.17	09.02	01.27	22.46	03.26	01.41
15	04.15	12.00	11.07	04.16	22.13	10.51	01.31	25.38	04.16	02.05
16	05.06	12.00	12.26	04.12	22.09	12.41	01.35	01.38	03.06	02.30
17	05.56	12.00	01.05	04.08	22.05	14.30	01.39	04.30	05.57	02.55
18	06.47	12.00	02.24	04.04	22.01	16.19	01.43	07.22	00.35	03.20
19	07.37	12.00	03.43	04.00	21.57	18.08	01.47	10.15	01.26	03.45
20	08.28	12.00	05.01	03.56	21.54	19.57	01.51	13.07	02.16	04.09
21	09.18	12.00	06.21	03.53	21.50	21.46	01.55	15.59	03.07	04.34
22	10.09	12.00	07.40	03.49	21.46	23.36	01.59	18.51	03.57	04.59
23	10.59	12.00	08.59	03.45	21.42	25.24	02.03	21.43	04.48	05.24
24	11.50	12.00	10.18	03.41	21.38	01.24	02.07	24.35	05.38	05.49
25	00.15	12.00	11.37	03.37	21.34	03.14	02.11	00.35	00.16	06.13
26	01.06	12.00	00.17	03.33	21.30	05.03	02.15	03.28	01.07	00.32
27	01.56	12.00	01.36	03.29	21.26	06.52	02.19	06.20	01.57	00.57
28	02.47	12.00	02.55	03.25	21.22	08.41	02.23	09.12	02.47	01.22
29	03.37	12.00	04.14	03.21	21.18	10.30	02.27	12.04	03.38	01.46
30	04.28	12.00	05.33	03.17	21.14	12.20	02.31	14.57	04.28	02.11
31	05.18	12.00	06.52	03.13	21.10	14.09	02.35	17.48	05.19	02.36

TABLE DES ÉTABLISSEMENTS

g (HEURES)

g	M_2	S_2	N_2	K_2	K_1	O_1	P_1	Q_1	M_4	MS_4	Sa	Ssa
°	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m		
001	00.02	00.02	00.02	00.02	00.04	00.04	00.04	00.05	00.01	00.01	22 MARS	22 MARS - SEPT.
002	00.04	00.04	00.04	00.04	00.08	00.09	00.08	00.09	00.02	00.02	23 "	22 " "
003	00.06	00.06	00.06	00.06	00.12	00.13	00.12	00.13	00.03	00.03	24 "	23 " "
004	00.08	00.08	00.08	00.08	00.16	00.17	00.16	00.18	00.04	00.04	25 "	23 " "
005	00.10	00.10	00.11	00.10	00.20	00.22	00.20	00.22	00.05	00.05	26 MARS	24 MARS - SEPT.
006	00.12	00.12	00.13	00.12	00.24	00.26	00.24	00.27	00.06	00.06	27 "	24 " "
007	00.14	00.14	00.15	00.14	00.28	00.30	00.28	00.31	00.07	00.07	28 "	25 " "
008	00.17	00.16	00.17	00.16	00.32	00.34	00.32	00.36	00.08	00.08	29 "	25 " "
009	00.19	00.18	00.19	00.18	00.36	00.39	00.36	00.40	00.09	00.09	30 "	26 " "
010	00.21	00.20	00.21	00.20	00.40	00.43	00.40	00.45	00.10	00.10	31 MARS	26 MARS - SEPT
011	00.23	00.22	00.23	00.22	00.44	00.47	00.44	00.49	00.11	00.11	1er AVR.	27 MARS - SEPT.
012	00.25	00.24	00.25	00.24	00.48	00.52	00.48	00.54	00.12	00.12	2 "	27 " "
013	00.27	00.26	00.27	00.26	00.52	00.56	00.52	00.58	00.14	00.13	3 "	28 " "
014	00.29	00.28	00.30	00.28	00.56	01.00	00.56	01.03	00.15	00.14	4 "	28 " "
015	00.31	00.30	00.32	00.30	01.00	01.05	01.00	01.07	00.16	00.15	5 AVR.	29 MARS - SEPT.
016	00.33	00.32	00.34	00.32	01.04	01.09	01.04	01.12	00.17	00.16	6 "	29 " "
017	00.35	00.34	00.36	00.34	01.08	01.13	01.08	01.16	00.18	00.17	7 "	30 " "
018	00.37	00.36	00.38	00.36	01.12	01.18	01.12	01.21	00.19	00.18	8 "	30 MARS - SEPT
019	00.39	00.38	00.40	00.38	01.16	01.22	01.16	01.25	00.20	00.19	9 "	31 MARS - 1er OCT
020	00.41	00.40	00.42	00.40	01.20	01.26	01.20	01.30	00.21	00.20	10 AVR.	31 MARS - 1er OCT
021	00.43	00.42	00.44	00.42	01.24	01.30	01.24	01.34	00.22	00.21	11 AVR.	1er AVR - 1er OCT
022	00.46	00.44	00.46	00.44	01.28	01.35	01.28	01.39	00.23	00.22	12 "	1er AVR - 2 OCT
023	00.48	00.46	00.49	00.46	01.32	01.39	01.32	01.43	00.24	00.23	14 "	2 AVR - 2 OCT
024	00.50	00.48	00.51	00.48	01.36	01.43	01.36	01.48	00.25	00.24	15 "	2 AVR - 2 OCT
025	00.52	00.50	00.53	00.50	01.40	01.48	01.40	01.52	00.26	00.25	16 AVR	3 AVR - OCT
026	00.54	00.52	00.55	00.52	01.44	01.52	01.44	01.56	00.27	00.26	17 "	3 " "
027	00.56	00.54	00.57	00.54	01.48	01.56	01.48	02.01	00.28	00.28	18 "	4 AVR. - OCT
028	00.58	00.56	00.59	00.56	01.52	02.01	01.52	02.05	00.29	00.29	19 "	4 " "
029	01.00	00.58	01.01	00.58	01.56	02.05	01.56	02.10	00.30	00.30	20 "	5 " "
030	01.02	01.00	01.03	01.00	02.00	02.09	02.00	02.14	00.31	00.31	21 AVR.	5 AVR. - OCT.

Let us choose for instance as initial epoch 0^h of Greenwich on 1st January 1940. The first H. W. of the constituent will take place at the time

$$T = \frac{360 - V_0}{n} + \frac{g}{n} \qquad (2)$$

the succeeding High Waters of this same constituent will follow this instant at the consecutive integral multiples of $\frac{360}{n}$

In formula (2) the term $\frac{g}{n}$ is a purely local constant of the considered constituent which, to simplify matters, we might call its " establishment on Greenwich ".

The term $T_0 = \frac{360 - V_0}{n}$ is independent of the locality and is computed astronomically for the initial instant and the prime meridian of Greenwich. In condensed form, by a sort of fiction, this term may be designated as the " time of maximum potential of the constituent at Greenwich ", or else, " Time of transit at Greenwich of the Fictitious Body ".

The Calendar, a page of which is reproduced overleaf, gives for each day of the year 1940 the time of maximum potential at Greenwich, or Transit at Greenwich,

$$T_m = T_0 + \text{multiple } \frac{360}{n} \qquad (3)$$

of each of the fundamental harmonic constituents M_2 S_2 N_2 K_2 K_1 O_1 P_1 Q_1 M_4 MS_4 of the tide.

To obtain the time of the actual H. W. of each of these fundamental constituent tides for a given place expressed in the local time of that place, it suffices simply to add to the time T_m taken out of the Calendar for the given day and for the constituent considered, the establishment $\frac{g}{n}$ peculiar to the place.

(This establishment $\frac{g}{n}$ hours is easily deduced from the harmonic constant g° by the aid of a permanent conversion table, a section of which is reproduced overleaf).

The tabulation given below shows how the Calendar, which is a species of Annual of the fundamental harmonic tides, may be quickly compiled.

The equilibrium argument has been recomputed for the first day of each month. For the subsequent days certain multiples of $\frac{360}{n}$, corresponding to the daily increment of this argument, shown in the last column of the tabulation, have been added. Thanks to a small calculating machine, these additions are effected very quickly and the compilation of the Calendar for the whole year requires but very little time.

CONSTITUENT	n	1/n	PERIOD	DAILY INCREMENT
M_2	28°.984	$2^m.070$	$12^h.421 = 12^h.25^m.24 \times 2 = 24^h.50^m.48$	$+ 50^m.48$
S_2	30°.000	$2^m.000$	$12^h.000 = 12^h.00^m.00 = 24^h.00^m.00$	$0^m.00$
N_2	28°.440	$2^m.110$	$12^h.658 = 12^h.39^m.49 = 25^h.18^m.99$	$+ 1^h.18^m.99$
K_2	30°.082	$1^m.995$	$11.^h967 = 11^h.58^m.04 = 23^h.56^m.07$	$- 3^m.93$
K_1	15°.041	$3^m.989$	$23^h.935 =$ .... $= 23^h.56^m.07$	$- 3^m.93$
O_1	13°.943	$4^m.303$	$25^h.819 =$ .... $= 25^h.49^m.16$	$+ 1^h.49^m.16$
P_1	14°.959	$4^m.011$	$24^h.066 =$ .... $= 24^h.03^m.95$	$+ 3^m.95$
Q_1	13°.399	$4^m.478$	$26^h.868 =$ .... $= 26^h.52^m.05$	$+ 2^h.52^m.05$
M_4	57°.968	$1^m.035$	$6^h.210 = 6^h.12^m.61 \times 4 = 24^h.50^m.48$	$+ 50^m.48$
MS_4	58°.984	$1^m.017$	$6^h.103 = 6^h.06^m.20 = 24^h.24^m.80$	$+ 24^m.80$

	daily	
S_a	0°.99	Max. 21st March.
S_{sa}	1°.97	

In the calendar, for each day only one of the hours of maximum potential relating to the semi-diurnal or quarter-diurnal High Waters is given. The other, or the others, may as far as required be deduced therefrom by simple addition, from the periods of the constituents indicated in the above tabulation.

Should it be desired to use the harmonic constants directly for the prediction of tides by computation, the data supplied by a small Calendar of this kind would in a sense do away with all the tedious calculation of the initial phase of each constituent. Further, by the graphical method set forth below, which dispenses with any costly or complicated gear and necessitates but insignificant materiel easily constructed by one's self during a hydrographic expedition, the calculation of the hourly ordinates is confined to a few additions.

Having determined by means of the harmonic Calendar, as stated above, the times of the various High Waters of the local harmonic constituents for a given day, a series of " sinusoidal reglettes " (rules) made of transparent celluloid, each corresponding to one of the constituent tides M_2 S_2 N_2 K_2 K_1 O_1 P_1 Q_1 M_4 et MS_4, are laid successively on the diurnal diagram (Diagram A - Fig. 2) graduated from 0^h to 24^h. For this purpose it suffices to place the index engraved on each reglette (rule) on the time of H. W. as read off the diagram, while bringing the horizontal middle axes of the reglette and of the diagram into coincidence.

Figure 3 represents, for example, the sinusoidal reglette for tide M_2.

Figure 4 shows this reglette laid on diagram A for a H. W. of M_2 occurring at 1 h. 25 m.

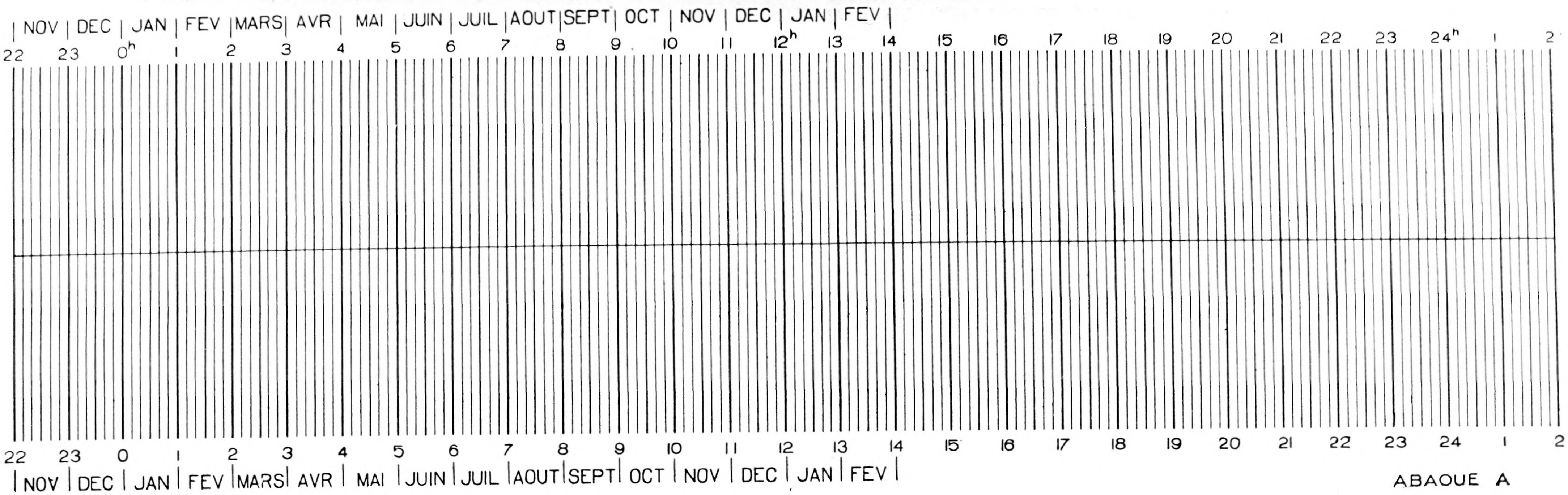


Fig. 2.

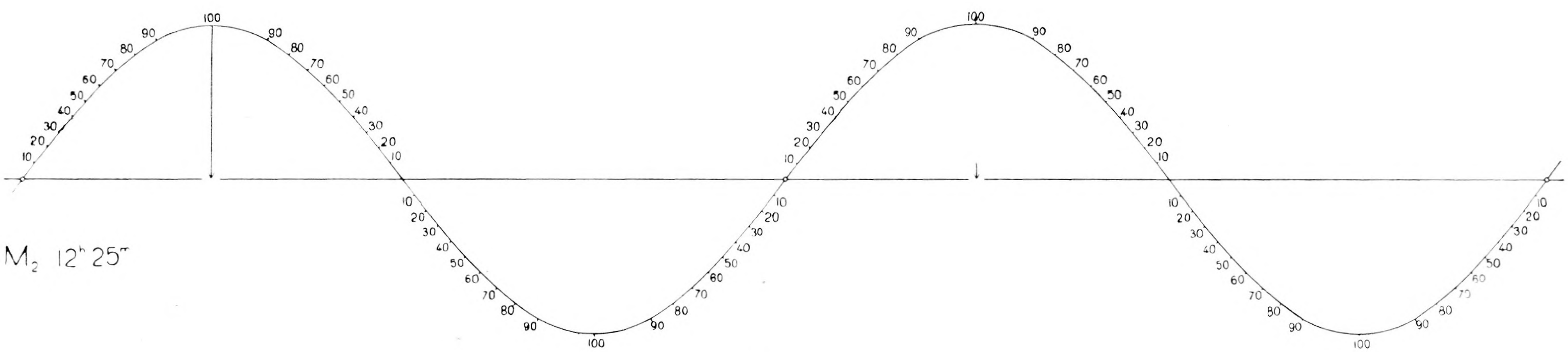


Fig. 3.

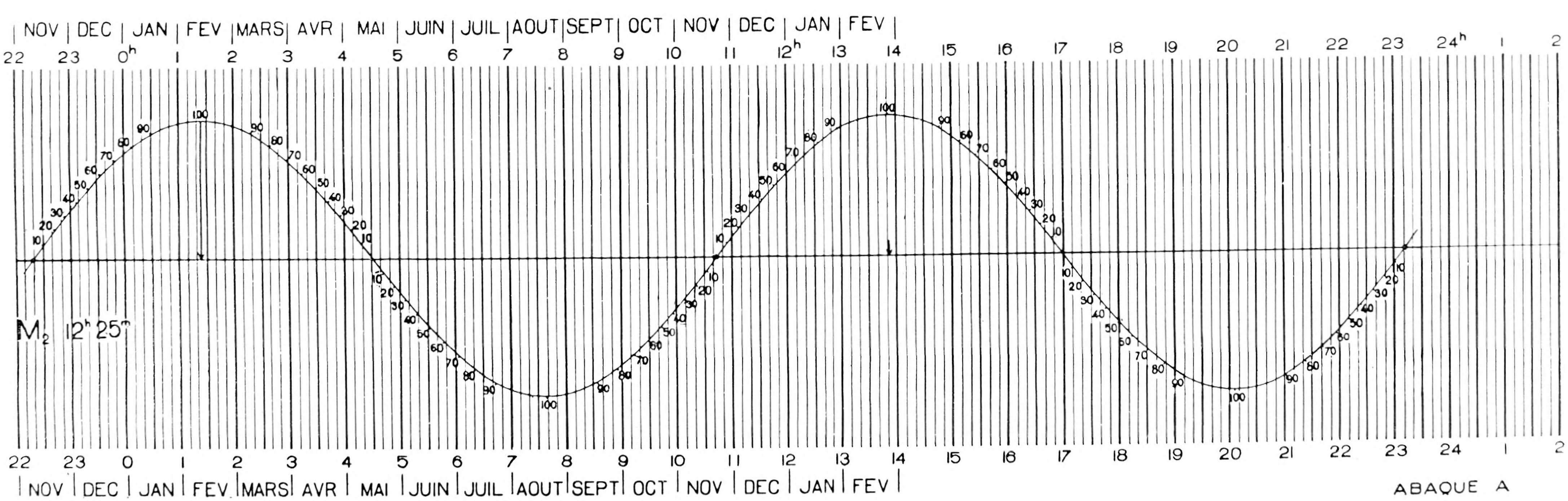


Fig. 4.

The ordinates due to each of the constituent tides are then read off the diurnal diagram with each of the graduated sinusoidal reglettes and for each even hour.

To this end the sinusoidal reglettes carry a graduation in hundredths of the semi-range of each tide, above and below the line of mean sea level, and interpolation is easily effected by inspection between the divisions of the " reglette ".

Thus it suffices for each constituent to multiply its half-range by the number of hundredths of the rise or fall read for each even hour on the sinusoidal rule. This multiplication is readily done by inspection by superposing on diagram B (Fig. 5) (auxiliary diagram for interpolating hundredths of range) the various scales graduated in centimetres (or in feet) corresponding to the desired range of the semi-amplitude of the constituent considered. (Fig. 6).

Fig. 6 shows two of these scales printed on a celluloid batten. A set of six of these reglettes suffices to cover the practical range of all the semi-amplitudes likely to occur.

Fig. 7 illustrates one of the scales laid down on diagram B for a half range of 17 centimetres or feet.

In this manner both diagrams A and B and their respective reglettes will be used simultaneously for reading by inspection, without computation, the hourly ordinates due to each constituent tide, by means of which a tabulation of the form C may be filled in (Fig. 8).

By algebraic summation of the corresponding ordinates of all the constituents there will be deduced the hourly ordinates of the total tide of the place, for the whole day or the period considered, referred in plus or minus to mean sea level.

These summations may be readily made provided some form of systematic calculation is available, for instance like that of the appended model (Form C) (Fig. 8).

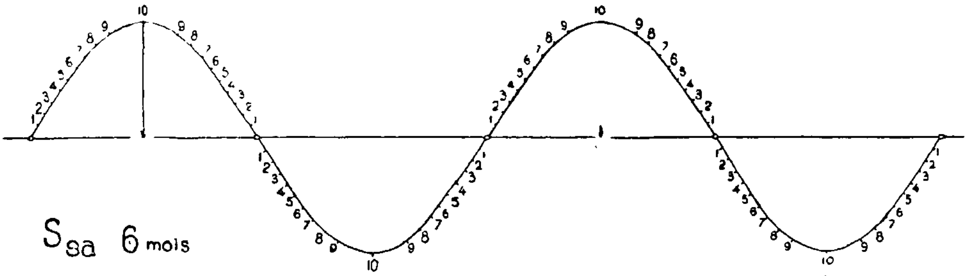
In order to refer the hourly ordinates thus determined to chart datum, it suffices to add the height A_0 of mean sea level above chart datum indicated as a rule by the chart or by the tabulations of harmonic constants.

These ordinates will enable the graphical curve of the tide for the particular day or period considered to be traced on cross section paper. From this graph the High Waters and Low Waters are then deduced and, generally speaking, the curve enables all the tidal problems to be solved by inspection, without assuming new hypotheses concerning the law of vertical movement of the water, for instance the so-called Rule of Twelfths, and without tedious interpolation calculations.

REMARKS : 1. — It is obvious, from the above, that for the semi-diurnal and quarter-diurnal constituents it suffices to know the time of one of the daily High Waters only, since the sinusoidal reglette automatically furnishes the other, or the others, on the diagram A in face of the consecutive maxima of the sinusoid.

2. — The semi-ranges, as also the phase lag g of the various constituents of the tide, are given by the Lists of Harmonic Constants published by certain Hydrographic Offices as well as by the International Hydrographic Bureau in its Special Publication N° 26. These constants are now-a-days known for more than 1950 ports distributed all over the world.

If the phase lag is not referred to the meridian of Greenwich, but to a local meridian (Kappa number) it will be necessary for its utilization as above, to reduce it first to the meridian of Greenwich, by means of the formulae and tables which are given, together with detailed explanations, in the Preface of December 1930 of the International Hydrographic Bureau's Special Publication N° 26.



3. — In the Table for converting the establishments are given the dates on which the annual and semi-annual constituents S_a and S_{sa} reach their local maxima in terms of their respective phase lags, so that by using the two reglettes (red) peculiar to these constituents (Fig. 9) conjointly with the monthly portion of diagram A (Fig .2), it is possible to deduce from this diagram for each day of the year the corresponding plus or minus correction to be applied to mean sea level; this correction is then entered on Form C (Fig. 8).

As an example a tabulation is given below of some of the regional harmonic constants H and $\frac{g}{n}$ of the tide from Saigon to Hong-Kong.

An harmonic tide calculator comprising both diagrams A and B described above, a set of 12 sinusoidal reglettes and 6 metric scales, together with a supply of calculation forms, C, is exhibited in the Library of the International Hydrographic Bureau. The whole is contained in a case the outer dimensions of which are 27 × 72 × 3 centimetres (7 3/4 × 18 × 3/4 ins.), and weighs 1.6 kilograms.

SUMMARY : The compilation of a very simple Calendar of the Fundamental Harmonic Tides would facilitate the direct use of the harmonic constants already known for a great many ports.

Easily traced diagrams make it possible to reduce the calculations for prediction to a few additions at most.

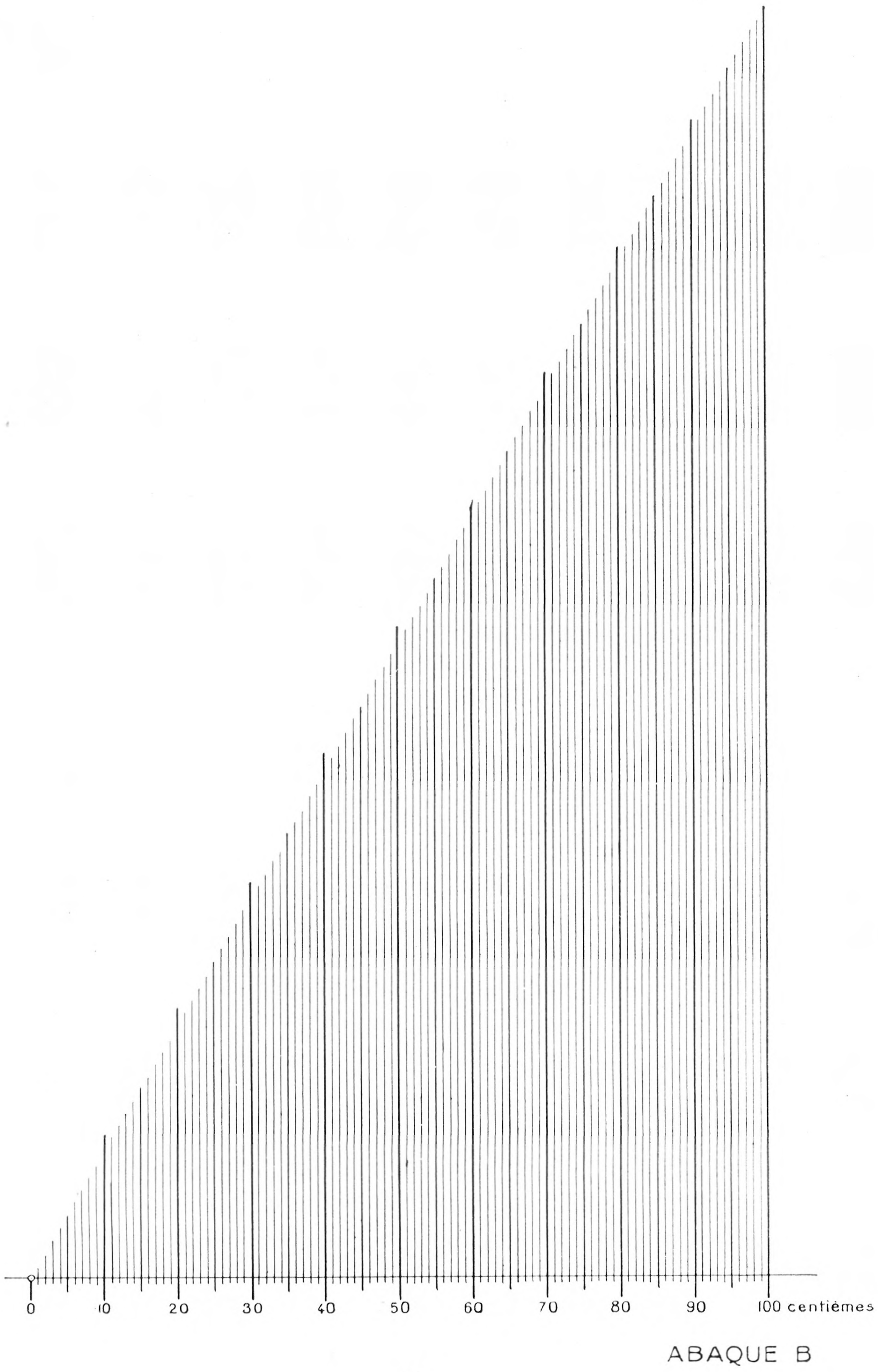


Fig. 5.

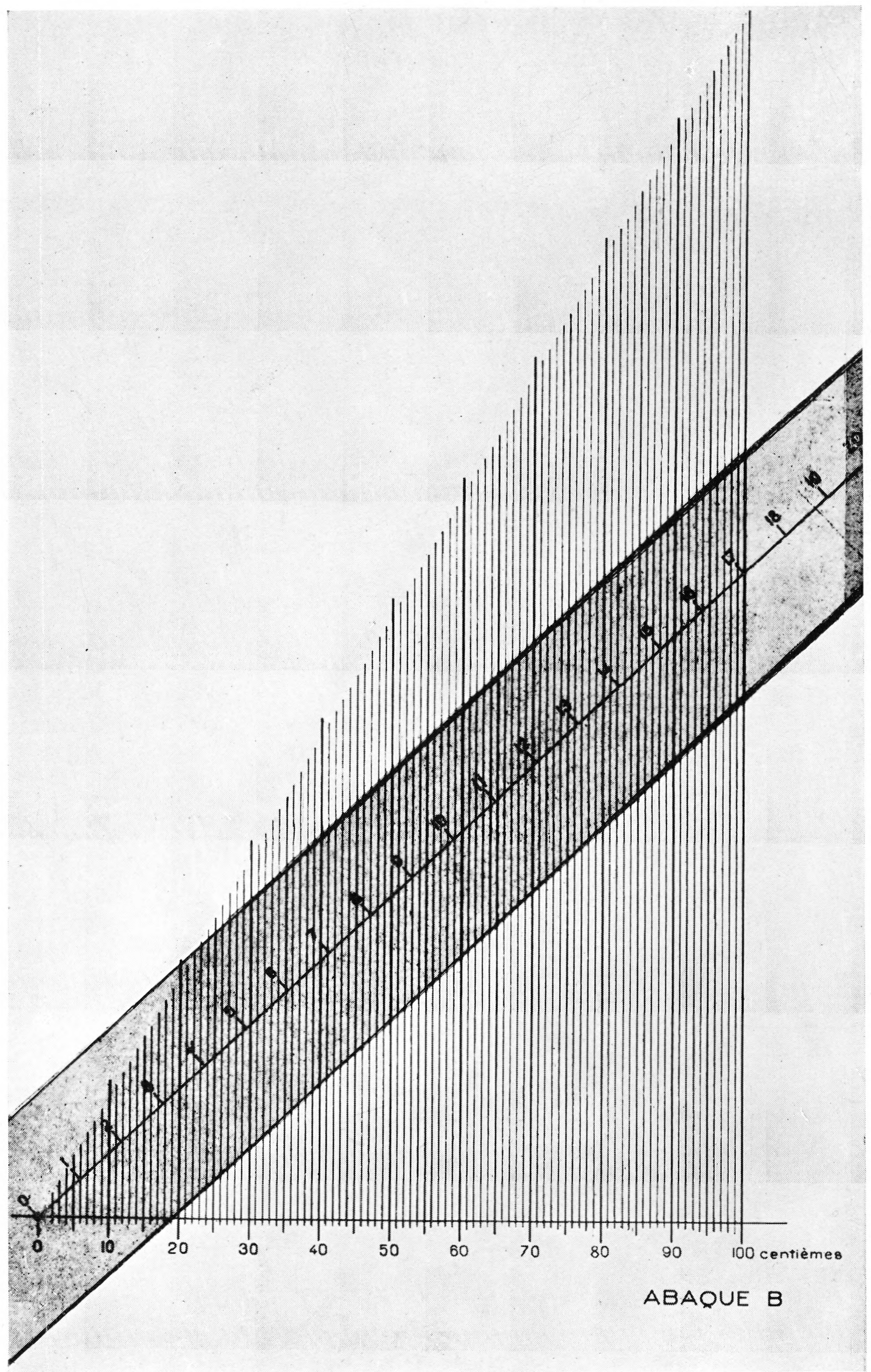


Fig. 7.

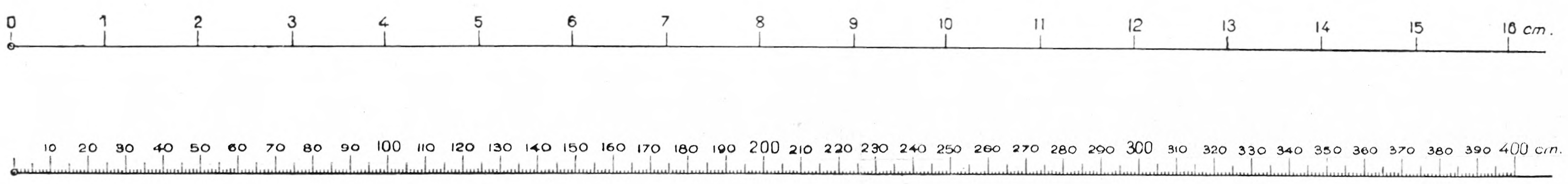


Fig. 6.

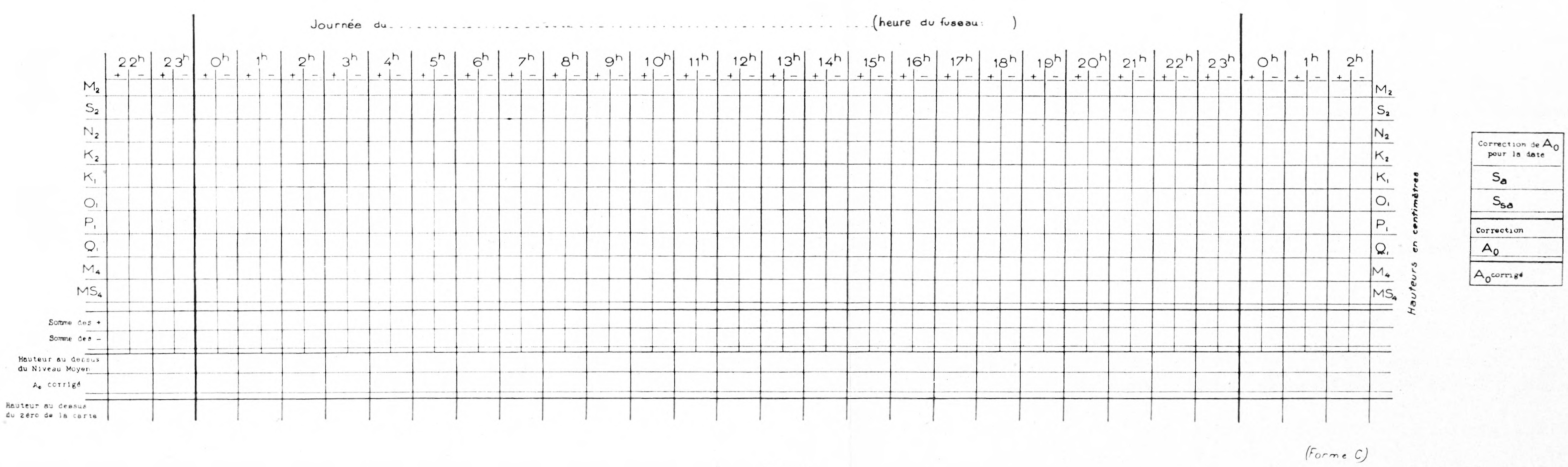


Fig. 8.

Tableau des Constantes Harmoniques régionales de la Marée de Saïgon à Hong-Kong.

Lieu	Fuseau	M_2	S_2	N_2	K_2	K_1	O_1	P_1	Q_1	M_4	MS_4	A_0	S_a	S_{sa}
Po. Condore	-7	80 01.41	28 03.44	15 01.41	8 03.43	64 21.09	45 19.39	21 21.12	9 20.09			228		
C. S. Jacques	-7	79 01.15	31 02.42	15 00.25	9 03.49	60 20.45	45 18.47	19 20.51	9 17.55	1 04.18	3 06.04	305	23 5.I	8 13.V
Cangiou	-7	80 01.29	32 03.02		11 03.04	70 20.25	44 19.00	23 20.31				260		
Ph. Kéga	-7	36 12.13	16 01.16	6 12.35	4 01.18	45 19.05	38 18.04	15 19.11	10 18.48			179		
Pt. Lagan	-7	22 10.54	10 00.06	4 11.00	3 00.06	40 19.09	33 17.52	13 19.15	6 18.17			106		
Po. Cecir M.	-7	19 10.19	7 11.34		1 11.34	32 19.09	28 17.26	10 19.15				161		
Cam Ranh	-7	20 10.21	9 11.30		2 11.28	35 19.25	29 18.04	12 19.27				124		
Nhatrang	-7	17 10.04	7 10.42	3 10.08	2 10.42	33 19.25	30 17.34	11 19.27	6 17.59			106		
Qui-Nhon	-7	19 09.54	6 11.14	4 09.59	2 11.12	32 18.45	30 18.00	11 18.47	6 18.31			95		
Tourane	-7	17 10.23	6 11.20	4 09.40	2 00.02	20 19.17	13 17.30	6 19.15	2 17.28	1 04.27	1 04.27	96	18 16.XI	3 24.IV
Hon-Nieu	-7	30 00.04	10 02.46	6 12.35	3 02.48	50 05.47	59 03.05	17 05.49	11 02.55			182		
Hon Ne	-7	18 12.07	7 03.54	4 12.12	2 03.53	68 16.19	69 01.13	23 06.17	13 01.03			223		
Hondau	-7	5 01.37	4 03.32	1 02.47	3 02.50	73 06.03	74 02.01	20 05.25	14 26.43	1 04.31	0 05.29	186	10 16.X	6 5.V
Haiphong	-7	4 02.25	5 04.40	0 02.34	2 05.03	64 07.15	73 03.05	18 06.41	15 00.31	1 05.11	1 00.28	202	13 17.IX	4 25.V
Apowan	-7	2 03.25	1 04.26	0 03.20	0 04.25	70 05.03	81 02.01	24 05.05	16 01.52			304		
P. Hydres	-7	5 05.02	2 04.48	1 05.00	1 04.47	69 05.11	75 01.35	23 05.13	14 01.25			210		
Hongay	-7	7 03.58	4 03.54	1 03.54	1 03.53	85 05.51	90 02.22	28 05.53	17 02.14			256		
Cam Pha	-7	14 05.39	5 07.28	3 05.38	1 07.29	76 04.55	68 01.39	25 04.53	13 01.25			215		
Tsieng Mun	-7	19 05.11	7 06.40		2 06.39	72 05.23	81 01.09	24 05.21				204		
Lo Shu Shan	-7	27 05.00	7 06.12	5 04.58	2 06.13	80 04.31	79 01.18	27 04.33	15 01.03			235		
Pak Hoi	-7	38 05.39	12 06.42	7 05.38	3 06.41	103 04.47	113 02.39	34 04.49	22 02.32			297		
Nau Tsheou	-7	79 09.52	32 11.08	15 09.55	9 11.06	41 20.17	37 18.52	14 20.19	7 19.20			234		
P.^b Beaumont	-7	95 10.29	35 00.16		10 00.18	39 20.29	37 19.56	13 20.31				234		
Macao	-8	47 10.33	18 11.24	10 10.12	6 11.22	36 21.09	29 19.09	12 20.59	6 18.08	3 01.33	2 02.09	202	9 7.X	2 2.IV
Whampoa	-8	67 01.17	20 02.34	12 00.36	9 02.36	33 00.04	25 22.05	11 01.24	4 23.53	5 05.34	4 00.17	174	15 12.IX	4 8.V
Hong Kong	-8	44 09.19	17 10.02	9 09.00	5 09.46	37 19.57	29 17.43	12 19.51	5 17.19	2 05.51	2 00.13	130	11 8.XI	6 20.IV

demi amplitudes en centimètres, établissements en heures et minutes, et rapportés au Méridien de Greenwich.