VISIBILITY OF LIGHTS.

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(Translation from the Dutch text).

INTRODUCTION. — The range of visibility of a light depends not only on its power but, even more, on atmospheric conditions which latter are essentially variable.

However, it is of the highest importance to the seaman to be able to estimate, as nearly as possible, the distance at which a light will be seen and the knowledge of its geographical range or of its intensity does not suffice for this purpose.

As far back as in 1889 the Washington Conference had recommended the insertion, in Lists of Lights, of a uniform method of indicating visibility; the International Hydrographic Conference, London, 1919 adopted the following resolution on this subject: —

« As at present no formula is known which is sufficiently elastic to satisfy the varying conditions of the atmosphere between the source of light and the point of observation, the observation method adopted by many nations appears to hold the field; in order to arrive at an unanimous solution this matter should be referred to the International Hydrographic Bureau. In the meantime each nation may retain its own method, and it is considered desirable that observations should be made by each nation with a view to accumulating data for determining constants and developing relations to serve as a basis for an acceptable formula, and these observations should be sent to the International Hydrographic Bureau. »

The following information as to the experience gained in working out the necessary statistics of the results of observations for the empirical determination of the visibility of the lights on the coast of the Netherlands and as to the extent of agreement between the values calculated by means of the Blondel-Rey law combined with the Allard formula and the observed values, might assist in reaching the object aimed at in the above mentioned resolution.

2. *The Allard formula**. — ALLARD in 1876 laid down that the relation between the intensity of a light and its range of visibility is: —

$$\lambda = \frac{L\, a^x}{x^2}$$

where λ is the minimum of light perceptible to the eye,
L the intensity of the light,
a the coefficient of atmospheric transparency, indicating the available residue of light after the unit of distance has been traversed,
and *x* the range of visibility, *i.e.* the maximum distance at which the light could be observed when the atmosphere has a transparency corresponding to the coefficient *a*.

In accordance with the results of his research ALLARD fixed the value of λ *in vacuo* as 0.1 decimal candle-power per kilometre.

The observations of French lights from those in their neighbourhood, which have been made regularly since 1863, have enabled the minimum values of *a*, which may be depended upon during particular portions of the year, to be established by means of this formula and these may be divided into areas in which the mean transparency is quite clearly differentiated. The Allard formula is applicable only when used in conjuction with reliable local observations; if it is applied where the cœfficient of atmospheric transparency adopted for the area under consideration is unknown its results are meaningless and may lead to considerable error.

After the introduction of the Bourdelle flashing apparatus and the substitution of sources of light more powerful than oil lamps, the luminous ranges of lights obtained by observation were found to be considerably shorter than those given by the Allard formula.

3. *The purely empirical method.* — After a close examination of the observations available, RIBIÈRE** was convinced that all formulae should be dropped and that it would be preferable, in the future, to determine luminous range entirely by means of direct observations, made during a great number of years, of all the lights of different type within each area.

The method of making these observations which is generally known and was applied by RIBIÈRE is the following : —

* Intensité et portée des phares, by *E. Allard*, Imprimerie Nationale, Paris, 1876.

** Phares et Signaux Maritimes, by *A. Ribière*, Encyclopédie Scientifique, Octave Doin, Paris.

Each light-keeper notes on a form, at certain fixed hours three times each night, which of the neighbouring lights are visible and which are not. The annual percentage of visibility of each light is calculated from the entries on the forms for each of the different distances at which it was observed. By means of a system of coordinates, in which the ordinates are the percentages and the abcissae are the corresponding distances, it is possible to trace a curve of visibility for each light from which may be obtained the probable minimum range of visibility percent of sighting, *e.g.* in 90 and in 50 cases in each 100.

4. *Disadvantages of the purely empirical method.* — Similar observations have been made, since 1907 of the lights on the Netherlands coast and the results obtained are not, in most cases, as positive as might be expected. Cases occur where a light is observed from one station only or from two stations at about equal distances though perhaps in different directions from the light ; under these circumstances the data obtained are obviously insufficient to enable the curve of visibility to be drawn with any certainty.

Likewise, it happens fairly frequently, even when a sufficient number of observations, made by stations at unequal distances, are available, that some stations observe a light under conditions which are very inferior to those of a light-vessel or the other stations.

The transparency of the atmosphere is sensibly different in various directions, in general it may be said to increase with altitude and at a given moderate height it is generally greater over the sea than over the land. Consequently observations made more or less along a coast give results which are frequently very much less favourable than those made from seaward or from a station from which the line of vision passes over sea over a great part of its length. When the line of vision passes over low-lying and swampy ground the transparency of the atmosphere is sensibly influenced unfavourably by the emanations therefrom and, usually, mouths and estuaries of rivers reduce the transparency.

Besides, the determination of the curves of visibility of the different types of lights on the coast of the Netherlands has been seriously interfered with by the facts that since the observations were begun the greater number of lights has undergone change once and some few of them several times even, that during the five years of the war several lights were not exhibited and finally that all the light vessels, from which the most reliable observations are made, were not replaced in their proper stations until 1921.

5. *Closer examination of the Allard formula.* — The need being admitted for a formula which will admit of the calculation of missing data by means of observations which are known to be exact or of checking results on which not too much reliance is placed, the following question inevitably arises : — Why does the Allard formula give much greater ranges for lights and particularly for those of high intensity, than does direct observation ?

The Allard formula sets out the problem quite correctly except that it assumes that the coefficient of atmospheric transparency is the same throughout the line of vision from the observer to the point observed, which is a condition that probably never occurs in reality.

The formula remains useful for the object aimed at, so long as the variation of transparency occurs to the same amount throughout a determined area. The atmosphere with an invariable transparency, which is assumed by the formula, becomes simply a hypothetical atmosphere which transmits, throughout the distance under consideration, a quantity of light equal to that transmitted by the real atmosphere of variable transparency.

Only those cases where the local conditions are extremely favourable or unfavourable cannot be met by the formula. Nevertheless its use cannot be rejected *a priori* for supplementary calculations, for such cases cannot be met by direct observations either, unless exceptionally favourably situated observing stations are available.

The reasons for the erroneous results given by the Allard formula were, *first*, that the luminous intensity ascribed to modern powerful flashing lights was exaggerated and, *second*, that the value of λ was too low.

6. *Definition of Luminous Value.* — It is well known that the impression received by the retina does not depend on the intensity of the source of light only but also on the duration of the light phenomenon.

In Allard's time, it was unnecessary to consider this for the flashing lights of his day gave such long flashes that, with reference to range of visibility, they could be taken as fixed lights.

Since then, however, the duration of the flashes of modern lights has diminished to such an extent that account must indubitably be taken thereof.

When flashing lights were first introduced it was thought that the minimum duration of flash which would permit or full perception was one tenth of a second*, *i.e.* that, for the purposes of observation, any flash which exceeded 0.1 sec. in duration had the same value as a fixed light of the same intensity. Shortly afterwards it was demonstrated, in practice, that the above opinion was false and that lights with flashes of very much longer duration than the limit then laid down could not be treated as equivalent to fixed lights of equal power.

Since then the researches of BLONDEL and REY** have provided more precise data; according to the theory put forward in their publications the relation between the intensity I, of a flash of duration t, and the intensity I_c of a fixed light which is equivalent thereto from the point of view of observation, is expressed by the equation : —

$$I_c = I \frac{t}{a + t}$$

where a is a constant which, according to their experiments, has a value of 0.21 seconds.

This formula presupposes that the luminous intensity during t is constant; where this intensity varies (as in revolving lights) the relation is expressed by : —

$$I_c = \frac{\int_{t_1}^{t_2} I\,dt}{a + (t_2 - t_1)}$$

in which t_1 is the moment of commencement of the impression on the retina and t_2 that of its end, both at the limit of visibility.

The formula may be expressed : —

$$\int_{t_1}^{t_2} I\,dt - I_c\,(t_2 - t_1) = I_c\,a$$

In Plate I, fig. 1, let the curve represent the successive intensities of the pencil of light, I_c the value of the equivalent fixed light and $a = 0.21$ secs. Then the area of the rectangle $I_c\,a$ will be equal to the shaded portion of the curve. Therefore, if the data include a curve of intensities, the value of I_c is determined without difficulty.

From the above it is obvious that the photometric intensities of lights with flashes of different durations are quantities of which use cannot be made in order to express their powers which give the amount of the luminous range. For this purpose the intensity of equivalent lights of equal duration of flash must be taken as the basis.

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* On flash lights and physiological perception of instantaneous flashes. ***A. Blondel***, International Maritime Congress, London, 1893.

** Sur la perception des lumières brèves à la limite de leur portée, par ***A. Blondel*** et ***Jean Rey***. Journal de Physique. - July & August 1911.

As BLONDEL and REY have stated, it is not admissible to take the intensity of equivalent fixed lights as the basis because the time which is necessary to discover the position in space where a point of light occurs which is just capable of producing, when fixed, a glimmer of sensation, must be so great that, in practice, the eye in searching for this point will never remain fixed on it long enough to find it.

It would be very difficult, if not impossible, to find out how long the eye remains fixed on a certain spot at night or what is the minimum duration of flash which would be necessary to make the flashing light equivalent to a fixed light of the same intensity; possibly the observations made in the Netherlands give an indication from which more or less information on this subject may be gathered.

For some years Goeree (Westhoofd) light and Westschouwen light, which are nine sea-miles apart, have been observed each from the other and each has recorded exactly the same number of sightings of the other, from which it may be concluded that these lights have equal luminous values.

The maximum photometric intensity (I_m) of Westhoofd light is 310,000 Hefner* candles, the duration of the flashes is 0.46 secs; and that of maximum intensity is 0.25 secs; the figures for Westschouwen are 235,000, 3 and 1.5 respectively. The respective intensities of equivalent fixed lights (I_c), deduced from the curves of intensity as mentioned earlier, are 180,000 and 210,000 Hefner candles.

The curves of intensity, after having been determined photometrically in camera obscura, must be multiplied by a factor, which has been determined for all lights, in order to make practical use of them. The factor was determined as follows: — Let it be granted that, in regular practice, the intensity is diminished 30 % as a maximum, or 15 % on the average for various causes, *e.g.* gas burner not in perfect condition, (or the mantle if incandescent light is used, either gas or oil vapour), low voltage or carbonisation of bulb, if incandescent electric lights are used. Likewise let it be granted that on an average 10 % of light is occulted by the upright and cross-bars of the lantern, and a further 10% absorbed by the lantern glazing, then the practical intensity of the equivalent fixed light (I_{cp}) will be $0.75 \times 0.9\, I_c = 0.675\, I_c$.

From these premisses the values obtained for I_{cp} are: —

Goeree (Westhoofd) — 121,000 Hefner candles.

Westschouwen — 142,000 Hefner candles.

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* 1 Hefner candle = 0.88 British candle-power.

As practice has shown that in reality, these two lights are equivalent it is obvious, as was stated above, comparable *luminous values* should not be based on the intensity of equivalent fixed lights but on that of equivalent flashing lights with flashes of a relatively short and equal duration. Undoubtedly this would have to be less than 1.5 secs. for, according to the law of BLONDEL and REY, the intensity of a fixed light equivalent to Goeree (Westhoofd), whose flashes are 1.5 secs. in duration would be $\frac{0.21 + 1.5}{1.5} \times 121{,}000 = 138{,}000$ Hefner candles only and this is less than the intensity found for the fixed equivalent of Westschouwen.

Since the duration of the maximum intensity of the beam of Westschouwen during each flash is 1.5 secs. this duration must be considered in calculating the *luminous value* of the light; therefore, this value L of the two lights will be $0.675 \times 235{,}000 = 157{,}000$.

The duration (t) of the flash of the equivalent light at Goeree (Westhoofd) is obtained by the equation : —

$$I_{cp} = L \frac{t}{0.21 + t}$$

As L is 157,000 and I_{cp} is 121,000 then t must be 0.7 sec. and, in order to ascertain the *luminous value* of lights whose flashes are of shorter duration than 0.7 sec., the calculated value of I_{cp} must be multiplied by the factor

$$\frac{0.21 + 0.7}{0.7} = 1.3$$

The following Table 1 gives the intensities and the luminous values, calculated in accordance with the Blondel-Rey law, for various types of lights on the Netherlands coast. Comparison of these figures shows that the Allard formula, if the first values were used, would give absolutely different results to those obtained when the second are taken as the basis of calculation.

7. *Determination of* λ. — ALLARD obtained the value of λ from observation at comparatively short distances of sources of light under varying atmospheric conditions. In clear weather the sources used were of very low power (sometimes a single candle, even) and their intensities were reduced by a known amount by means of varying numbers of glass screens; in foggy or misty weather more powerful sources were employed.

TABLE 1.

LIGHT.	Character.	OPTICAL DATA.			Duration of flash at the limit of visibility in secs.	Luminous intensity $0.675 \times I_{max}$.	Luminous value $1.3 \times 0.675 \times I_c$ for flashes shorter than 0.7 secs.; $0.675 \times I_{max}$. for longer flashes.
		Focal length in metres.	Number of panels.	Time in which one revolution is made in secs.			
Terschelling	1 flash	0.30	2 × 4	20	0.22	4,000,000	2,200,000
Scheveningen. . . .	2 flashes	0.50	4	20	0.16	4,530,000	2,200,000
Kykduin	2 flashes	0.92	4	20	0.22	525,000	330,000
Goeree (Westhoofd).	3 flashes	0.70	6	30	0.33	210,000	157,000
Westschouwen . . .	2 flashes	0.92	12	180	1.7	157,000	157,000
Ameland.	3 flashes	0.92	15	150	1.4	127,000	127,000
Schiermonnikoog . .	4 flashes	0.50	4	20	0.3	117,000	86,000
Eierland.	occult.	0.92	fixed light 360°	—	4	30,000	30,000
Hoek van Holland (N. Hoofd) . . .	flashing	0.1875	2	10	0.15	12,000	5,000

The luminous ranges of the sources of light of known intensities were determined, for varying degrees of atmospheric transparency, by withdrawing the observers to greater distances from each light until the limit of visibility was reached.

Taking the Allard formula (*see* paragraph 2, above) and transposing : —

$$\frac{L}{x^2} = \frac{\lambda}{a^x} \text{ or } \log\left(\frac{L}{x^2}\right) = \log \lambda - x \log a$$

$$\text{now let } y = \log\left(\frac{L}{x^2}\right)$$

$$\text{then } y = \log \lambda - x \log a$$

which equation corresponds to a straight line which cuts the axis of the abcissae at an angle the tangent of which is — *log a* and which cuts the axis of the ordinates at a point $y = \log \lambda$. If each observation be represented by a point the abcissa of which is the value found for x and the ordinate that of $\log \frac{L}{x^2}$ which corresponds thereto, then all the points thus fixed for one set of observations, *i.e.* taken under the same atmospheric conditions, should be situated in a straight line.

Each set of observations taken under different atmospheric conditions will give a set of points lying in another straight line which will cut the axis of the abcissae at a new angle, but all such lines will meet at a point in the axis of the ordinates the distance of which from the point of origin is log λ.

It is obvious that the results obtained by ALLARD were not very certain on account of the difference in vision of the observers, the difficulty in determining the exact limit of visibility of a source of light and the variations in the transparency of the atmosphere which, of course, cannot be controlled while a set of observations is being taken. It could not be expected, therefore, that the points resulting from the same set of observations would lie in a perfectly straight line and thus a point on the axis of the ordinates had to be chosen from which could be drawn the most probable straight lines for each set of points obtained by observation.

It was in this way that the value of λ was determined to be 0.1 decimal candle* at a distance of 1 kilometre, *i.e.* the eye is illuminated 10^{-7} Lux.

The researches of BLONDEL and REY have shown much smaller values of λ for certain persons ; in some cases as low as 0.06 decimal candle at 1 kilometre. Nevertheless, in observing a light under prac-

* One decimal candle = 0.99 British Candle power.

tical conditions, it appears that these figures cannot be adopted as the bases, *first* because such observations are made in all weathers which, of course, prejudices the results considerably and *second*, because the conditions differ greatly when the observer is gradually withdrawn from the light, which he keeps in sight until the limit of visibility is reached and when a seaman or lightkeeper must search in the darkness for a light the position of which is uncertain. It is obvious, therefore, that the value of λ must be determined under the same conditions as those which obtain when the light is observed in practice, *i.e.* it should be deduced from well established luminous ranges of light of various intensities within the same area.

Researches have been made with a view to establishing the degree of accuracy with which it is possible to determine the value of λ from the results of observations made on the coast of the Netherlands. The following are the results obtained.

Plate II gives, for lights of different types and of various luminous powers, the percentages of sighting deduced from the series of observations shown in Table 2, which observations extended over at least six years.

The positions of these lights and their reciprocal distances are shown on Plate III. An examination of these positions gives clear proof of the disadvantages, mentioned in paragraph 4, which arise in tracing the most probable curve of visibilities from the results of observations.

Terschelling Light is the only one which has a sufficient number of observing stations situated at varying distances and which give a clearly defined curve though, even in this special case, these stations are not equally favourably situated. For instance, the visual ray between Vlieland and Terschelling passes over the estuary and over the Noordvaarder, the vast sandy beach which lies in front of the dunes of the island. The Terschellingerbank Light-ship is the sole station where, for visibility, the conditions are analogous to those of a vessel to seaward ; those of Ameland, Eierland and Kykduin are worse.

Therefore the curve must be drawn near the point obtained from Terschellingerbank and pass above the points obtained from the other observing stations ; the curve thus drawn gives the following probabilities : —

Visibility in 90 cases out of 100 .... 9.5 naut. miles
» » 50 » » » 100 .... 27.7* » »

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* The article on "Visibility of Lights" in the previous number of the *Review* gives 27.2 nautical miles. The figure given by the Engineer-in-Chief of the Light House Service should be accepted, of course.

TABLE 2.

Nº	Observing station	Distance in nautical miles	Percentage of sighting
Terschelling.			
1	Vlieland	6.9	91.9
2	Tersch. Bank	13.9	84.5
3	Ameland	15.8	75.6
4	Eierland	16.7	75.4
5	Kykduin	30	42
Kykduin.			
1	Eierland	14.4	73.6
2	Haaks	15.2	75
3	Vlieland	23.7	45
4	Terschelling	30	26.1
Goeree (Westhoofd).			
1	Westschouwen	9	83
2	Maas	13	74.6
3	Hoek van Holland Semaphore	13.6	63.6
4	Schouwenbank	15.3 •	68.6
5	Scheveningen	23	35
Ameland.			
1	Terschelling	15.8	62.25
2	Schiermonnikoog	19	58.3
3	Vlieland	22.5	48
Schiermonnikoog.			
1	Ameland Kustwacht	19	56.7
2	Borkum	19.7	46.6
Eierland.			
1	Vlieland	10	77.1
2	Kykduin	14.4	60.1
3	Tersch. Bank	16.1	55
4	Terschelling	16.7	52.6
Hoek v. Holland (Noorderhoofd).			
1	Semaphore	1.1	97
2	Maas	7.3	83.5
3	Scheveningen	9.9	58

Light-buoys, 200 Hefner candle-power.

Nº	Light-buoy	Observing station	Distance in naut. miles	Percentage of sighting
1	Oostgat Nº 1	Westcapelle	1.25	96.2
2	Plaat van Breskens	Nieuwe Sluis	1.6	96.1
3	N. R. W.	Hoek van Holland	1.8	93
4	Sardynguel Nº 3	Nieuwe Sluis	2.6	92.6
5	N. R. W.	Hoek van Holland	2.8	83
6	Oostgat Kaloo	Westkapelle	3.2	85.2
7	id. Nº 4	Nieuwe Sluis	4.5	78
8	Wielingen Nº 2	id.	5.5	68.2

The Haaks Light-ship forms an excellent observing station for *Kykduin*. With this and Terschelling a probable luminous range, for 50 %, of 23 miles is obtained. The curve cannot be drawn with sufficient accuracy to obtain the 90 % range.

The only stations which observe Goeree (Westhoofd) and on which reliance can be placed, are the Maas and Schouwenbank Light-ships; Scheveningen and the Hook of Holland are so placed that they observe along the coast and over estuaries. The observations made from seaward give, for 50 %, a probable luminous range of 20 miles; that for 90 % cannot be satisfactorily determined as Westschouwen observing station is likewise badly placed.

A probable luminous range, for 50 %, of 21.8 miles may be deduced with sufficient accuracy from the data available for drawing the curve of *Ameland Light*, but none whatsoever are available for 90 %.

The two approximately equidistant observing stations for *Schiermonnikoog Light* give a probable visibility at 19 miles for 50 %. The available data suffice for the drawing of a curve for *Eierland Light* from which may be deduced luminous ranges of 6.2 and 17 (approximately) miles for 90 and 50 % respectively.

In the case of the *Hook of Holland* (Noorderhoofd) the observations from Maas Light-ship are the only ones on which reliance can be placed. These give a probable luminous range of 5 miles for 90 %. The visual ray between this light and Scheveningen follows the coastline throughout and, therefore, it is best not to take it into account.

The probable visibility of the *Light-buoys*, *i.e.* 2.8 and 6.5 miles for 90 and 50 % respectively, are the means of the visibilities deduced from observations made by different observing stations at various distances.

The probable luminous ranges are grouped in Table 3 for the purpose of determining the value of λ by means of Fig. 2 and Plate 1.

From this the value of λ can be fixed, with considerable accuracy, at 1.14 Hefner candles at a distance of 1 nautical mile, which corresponds to 0.3 decimal candle at 1 kilometre or 3×10^{-7} Lux. This is no less than three times the value found by ALLARD and corresponds with that deduced by REY* from various luminous ranges of lights on the Mediterranean coast which were determined by the *Direction des Phares*.

* Notice sur un nouveau système de phares à réflecteurs métalliques. *Jean Rey*; Paris, 1923.

TABLE 8.

Light	Power in Hefner candles	x_1 Luminous range 90°/o	x_2 Luminous range 50°/o	$\log\left(\frac{L}{x_1^2}\right)$	$\log\left(\frac{L}{x_2^2}\right)$
Terschelling . . .	2,200,000	9.5	27.7	4.387	3.458
Kykduin	330,000	—	23	—	2.795
Goeree (Westhoofd)	157,000	—	20	—	2.593
Ameland	127,000	—	21.8	—	2.428
Schiermonnikoog . .	86,000	—	19	—	2.377
Eierland	30,000	6.2	17	2.892	2.017
Hoek van Holland (Noorderhoofd). .	5,000	5	—	2.301	—
Light-buoy	200	2.8	6.5	1.406	0.675

8. *Calculation of curves of visibility.* — The calculation of luminous ranges by the use of the Allard formula is most easily and quickly done by transposing the equation of ranges by means of a logarithmic abacus. Allard described a method for this purpose, but that of Rey* is the simplest and most practical.

Take the Allard formula, paragraph 1 : —

$$\frac{\lambda}{L} = \frac{a^x}{x^2}$$

$$\text{let } \log \frac{1}{x^2} = y,$$

$$\text{then } \log \frac{\lambda}{L} = \log \frac{1}{x^2} + x \log a \qquad (1)$$

$$\text{then } y = (-\log a)\, x + \log \frac{\lambda}{L} \qquad (2)$$

which equation represents a straight line which cuts the axis of the abcissae at an angle whose tangent is — log a and which passes through a point on the axis of the ordinates at a distance of $\log \frac{\lambda}{L}$ from the zero point.

The point where this straight line cuts the logarithmic curve $y = \log \frac{1}{x^2}$ satisfies equation (1).

Therefore if, in a rectangular system of coordinates (*see* Plate IV), the logarithmic curve $y = \log \frac{1}{x^2}$ is constructed and the logarithmic values $\log \frac{\lambda}{L}$, calculated for the various *luminous values*, are set off on the axis of the ordinates and if the straight lines are drawn between the point on the axis of the ordinates which corresponds to the *luminous value* of a given light and the points on the logarithmic curve the abcissae of which represent the luminous ranges of this light under varying atmospheric conditions, then the angles which each such straight line makes with the axis of the abcissae will give the angular coefficient of the corresponding atmospheric condition.

Conversely, if the angular coefficient of an atmospheric condition has been determined in this manner then, for this atmospheric condition, the range of any other light will be represented by the abcissa of the point of intersection of the logarithmic curve and the straight line cutting the axis of the ordinates at a point representing the *luminous value* of the light, and the angle between which and the axis of the abcissae is equal to the angular coefficient of the atmospheric condition under consideration.

As is shown by Fig. 2, Plate I, the luminous range, *i.e.* 23 miles for 50 % of Kykduin, the *luminous value* of which is 330,000 Hefner candles, corresponds exactly with the value determined for λ and the direction of the straight line for 50 % in Plate IV is obtained by joining the point representing 330,000 on the axis of the ordinates and placed $\log \frac{1.14}{330{,}000} = \bar{5}.461$ from zero, with the point on the curve $\log \frac{1}{x^2}$ the abcissa and the ordinate of which are 23 and $\log \frac{1}{23^2} = \bar{2}.723$.

In the same way the range of 9.5 miles of Terschelling was used for the direction of the line of 90 % and that of 15.2 miles (distance of Haaks Light-ship) of Kykduin for the direction of the 75 % line.

The diagrams on Plate V were drawn by this graphic method. Table 4 compares the ranges deduced from the observations used for calculating the value of λ (*see* paragraph 7) and the results obtained by the graphic method of calculation

Finally Plate VI shows the probable curves of visibility of these lights, as obtained by means of the graphic method, for 90, 75 and 50 %. The points from Plate II, which were obtained from observations, are inserted.

The discrepancies of these points with reference to the new curves are, generally speaking, less than those which might reasonably be

TABLE 4.

LIGHT.	LUMINOUS RANGES.			
	Observed.		Calculated.	
	90 %	50 %	90 %	50 %
TERSCHELLING.	9.5	27.7	9.5	28.35
KYKDUIN.	—	23	8	23
GOEREE (WESTHOOFD) .	—	20	7.45	20.21
AMELAND.	—	21.8	7.25	20.45
SCHIERMONNIKOOG . . .	—	19	7	19.4
EIERLAND.	6.2	17	6.2	16.6
HOEK VAN HOLLAND (NOORDERHOOFD) . . .	5	—	4.95	12.35
LIGHT-BUOY.	2.8	6.5	2.9	5.95

expected in dealing with a matter which is subject to so many different influences and in which precision is difficult to obtain. In the few cases in which these discrepancies are fairly considerable they may be accounted for by the unfavourable situation of the observing station in question.

The facts that the points referring to the observations of Ameland are above the calculated curve and that those of Goeree (Westhoofd) are below it, are probably due to different atmospheric conditions in the neighbourhood of these observing stations in addition to their more or less favourable situations.

No discrepancies *towards one side only* appear between the luminous ranges of high and low power lights respectively as obtained by graphic calculation. It was such discrepancies which were the cause of discredit of Allard's formula.

The agreement between the results obtained by calculation and those deduced directly from observations gives remarkable proof of of the accuracy of the Blondel-Rey law.

9. *Conclusion.* — The curves obtained by graphic calculation give for any area, in addition to checks of the observed values of the luminous ranges, the ranges of lights which it is not possible to obtain

directly from observations and they can be extremely useful, likewise, for the calculation of the *luminous value* which would be suitable for lights which it is intended to establish, in order to obtain any required luminous range.

It is highly probable that, in other areas, the Allard formula could be of great service in obtaining luminous ranges also, for which the necessary reliable observations are lacking, provided that three different definitely known probabilities of sighting of the same light or of other lights whose intensities are known, are available. Under these circumstances the graphic calculations must be based on the *luminous value* and the new value of λ. Nevertheless, it is obvious that the reliability of the results obtained will be in proportion to the number of data deduced from direct observation.

Since the luminous range of a light does not depend on its intensity but on its *luminous value*, it would be preferable if this latter were shown in Light Lists instead of the former. A uniform method of determining this value should be adopted for this purpose and, where a diagram of photometric intensity is not available, as happens frequently, a method of approximation should be employed.

This diagram may be taken as an isoceles trapezium of which the height is the maximum intensity, the long base is the total divergence of the pencil of light and the short side the divergence of the maximum part of the intensity of the pencil.

The maximum intensity may be represented, in practice, with sufficient accuracy by $a \times O \times e$, if O is the area of the projection of a lens panel on a plane perpendicular to the optical axis, e the mean intrinsic brilliance of the source of light and a a coefficient which, according to researches made, should be valued at 0.5.

The maximum divergence of the luminous pencil is the angle subtended by the source of light in a horizontal plane at a distance equal to the focal length. The divergence of the central part of the pencil of maximum intensity is the smal lest angle subtended by the source of light at the furthest points of the lens panels.

In this way satisfactory results will be obtained, when the constants of the lenses and of the source of light are known with exactitude, provided that the same coefficient of reduction, *i.e.* 0.675, is applied. However, the luminous value of flashing lights depending on the electric arc cannot be determined by this method because the intensity of these lights is very much influenced by the displacement of, and the more or less variable inclination taken up by, the planes of the craters as the carbons are burnt away.

Every such displacement and change of inclination is a reason for the application of a much greater coefficient of reduction than that mentioned above and for this reason, likewise, it would be extremely difficult to determine the coefficient.

Perhaps some conclusions may be drawn from the fact that the change from arc lamps of 60 Amps and 45 Volts, twin apparatus, with 2×4 lens-panels of 30 cms. focal length, to electric incandescent lamps giving a mean intrinsic flash of 1000 Hefner candle-power per square centimetre, which was made at Terschelling, has not sensibly influenced the luminous range of this light.

It would seem, from the consistency of the *luminous value* before and after the change, ascertained by this summary method, and the relation between the intrinsic brilliance and the dimensions of the two sources of light, that in practice the *luminous value* of arc-lights should be taken as being but a quarter of that calculated by the method cited above.

As the durations of the flashes of a light diminish as the limit of the luminous range is approached, it would be advisable to strive to reach uniformity in the notation of duration by adopting, for all flashing lights, the duration *at the luminous range.*

Finally it appears necessary to consider what data, deduced from a curve of visibility of a coastal area, should appear in Lists of Lights.

It being obvious that the curve for each light cannot be shown, it is necessary to ascertain what peculiarities are of greatest interest from the seaman's point of view.

In some Light Lists the geographical range only of the lights is given, others show the luminous ranges for 50 and 90 %. It is indisputable that the latter give the more useful information to the seaman, but the fact that the range for 50 % may lead to error in many cases, for it greatly exceeds the geographical range of high power lights, must be taken into account. The luminous range given has usually been determined by observers situated at such an altitude that they can see the light itself (not its reflection or glare) at a much greater distance than could the seaman; and though the reflection of high-power lights is visible, in many cases, to a much greater distance than the geographical range, this does not occur as often as might be expected, to judge from the distances mentioned.

It might be possible to avoid all confusion by inserting, in addition to the geographical range for a height of eye of 5 metres*, the probable visibility of the light at that range. When dealing with low-power lights which are visible at their geographical range less than 50 times in 100, it would suffice to show the 50 % range only.

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* It is a fact that, in large vessels, the height of eye is considerably over 5 metres. However it is best to retain this height, which is established by international agreements, for the true height of eye varies considerably in the various types of vessels. The seamen can easily add the increment, due to a height above 5 metres, to the ranges given in the Lists of Lights. The approximate increment for H metres may be found by the formula Increment $= \sqrt{4.08\ H} - 4.5$ nautical miles.