THE NEW 400-GRADE DIVISION

by

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(Translated from the German).

As a result of the ministerial decision of the *Reichs-und Preuss. Ministers des Innern* of 18th October 1937, the 400-grade division of the circle will hereafter be applied to all surveying work in Germany (1). This decision, which has been taken in the interest of the standardisation of the German topographical system, is very welcome, since previously existing conditions (Northern Germany keeping the former division; Southern Germany using the new division) were most undesirable.

In the following are given a retrospective outline of the inception of this new division for the circle, and a statement as to its advantages and inconveniences when compared with the former sexagesimal division:–

HISTORICAL ACCOUNT.

When in 1617 Henry BRIGGS (1556-1630) calculated and published the logarithms of the natural numbers having 10 as a base (2), he intended also to extend the logarithmic calculation to trigonometry. This led him to introduce a more convenient division of the circle than had been used up to that time. BRIGGS retained the division in 360° but, for parts of degrees, used the decimal division. However, it was not permitted to BRIGGS to complete the projected work. After his death, Henri GELLIBRAND continued the work and in 1633 published the "Trigonometria britannica", which gives the values of the sines to 15 decimal places and the values of the tangents and secants to 10 decimal places for each hundredth of a degree. This publication is in use even to-day and in more recent years it formed the basis of the works of PETERS to 7 and 10 decimal places (3).

On its publication, however, the BRIGGS-GELLIBRAND table was not accorded the recognition it deserved, the reason perhaps being the almost simultaneous publication by Adrian VLACQ of a table of logarithms to 10 decimal places of the values of the sines, cosines, tangents and co-tangents for each 10". With regard to the subdivision, the latter table was in fact more complete than that of BRIGGS-GELLIBRAND, and soon supplanted it. On the other hand, as the VLACQ table was calculated according to the former division of the angle ($1^\circ = 60'$; $1' = 60''$), and as, doubtless, measuring instruments with the BRIGGS decimal division of the degree did not exist, the reform undertaken by BRIGGS for the division of the angle fell into oblivion.

Towards the end of the XVIIIth century, the question of the more appropriate division of the angle was again taken up; a German was the first to attempt it. Oberbaurat Joh. Carl SCHULZE of Berlin had just calculated, according to the BRIGGS-GELLIBRAND scheme, tables for the earlier division with decimal subdivisions, when the proposal was made to him to divide also the quadrant into tenths (4). SCHULZE immediately investigated the question and prepared, first, instructions for the calculation of these tables by means of the EULER formulae. However, it seems that SCHULZE was prevented by other occupations from continuing the work which he had commenced; but the importance which he attributed to the decimal division of the quadrant appears in the following sentence written at the end of his statement:– "he had the fervent desire to find assiduous co-operation in so useful an undertaking for the application of mathematics". SCHULZE died in 1790 without completing his projected work. The mathematician

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(1) For geographical co-ordinates and for the lines of the network, as well as in astronomy the 360° division has been retained.

(2) Known as the BRIGGS or common logarithms, or decimal logarithms.

(3) PETERS: "7stellige Werte der trigonometrischen Funktionen von 0° bis 90° für jedes Tausendstel des Grades", Berlin, 1918, and PETERS: "10stellige Logarithmen der trig. Funktionen von 0° bis 90° für jedes Tausendstel des Grades", Berlin, 1919.

(4) This proposal, made about 1780, came from LAGRANGE, then in the Mathematical Chair of the Berlin Academy of Sciences.

HOBERT and the astronomer IDELER devoted themselves to the work and in 1799 published a logarithmic table to 7 decimals of the terms, sin, cos, tan, and cot., calculated with extreme accuracy; this was the first complete table dividing also the quadrant according to the decimal system.

Towards the end of the XVIIIth century, a lively interest was also manifested in France for the decimal division of the quadrant. From 1792 DELAMBRE used repeating circles with the decimal division of the quadrant. This fact is the more surprising because DELAMBRE, for want of decimal division tables, must have had to reduce his observed results to the earlier division.

In Germany, the first divided circle with decimal division of the quadrant was made in 1799 by the engineer WAGNER of Berlin. In Bavaria, for the principal triangulation begun in 1801, circles divided into hundredths were also used — at first procured in Paris but soon afterwards from the excellent engineers at Munich. REICHENBACH-ERTEL theodolites with the decimal division of the quadrant were introduced in Baden also in 1823.

If, at the commencement of the XIXth century, the decimal division of the quadrant attracted so much attention everywhere, it is rather astonishing that the 360° division was not finally completely supplanted. There are various reasons for this: counter currents existed in science also; many could not, or would not, see any reason for a change in the division of the angle from that which had been hitherto uniformly adopted. The extension of the new division was also made very difficult by the lack of a sufficient number of tables and corresponding instruments, so that observers and calculators very often could not truly appreciate the advantages of the new system of division.

The question of the most appropriate division of the angle was, however, only slumbering. In fact, at a congress of the principal Physics Group of the German Society of Naturalists and Physicists held in 1889 at Munich, this question was thoroughly discussed without the members being able to arrive at complete unanimity (5).

The *Zeitschrift für Vermessungswesen* has repeatedly advocated the decimal division of the angle. From the first year of its publication, VORLÄNDER (6) emphasised the advantages of the 400-grade subdivision of the circle. Later, P. SCHADE (7) advocated the decimal division for the quadrant as well as for the day. Finally Professor C. MÜLLER (8) turned against the decimal division proposed for the earlier degree, and recommended the much more practical decimal division known as the new "grade".

If, by the above-mentioned ministerial decision, the decimal division of the quadrant is now definitely introduced into the German topographic system, it is because, from the point of view of instruments and the available tables, the present situation is much more favourable than it was 100 years ago. As, for a considerable time, makers have been producing theodolite circles with the former division as well as with the new, no alteration in the construction of the instruments is necessary: it is only a matter of a very desirable simplification (9). Trigonometrical tables according to the new division are already available to 4, 5 and 6 decimal places. A table to 8 decimal places is now being calculated, and there also exist conversion tables from the former division into the new, and conversely.

ADVANTAGES AND INCONVENIENCES OF THE NEW SYSTEM OF DIVISION.

The advantages of the new system of division must be sought less in the measurements than in the calculations; but full advantage is obtainable only when measurements and calculations are both made in the new system of division.

As the new division fits perfectly into the decimal system, we note for each trigonometrical or polygonometrical calculation, in addition to increased security, a saving of time which, in view of the developed calculations involved in secondary geodesy, is of the highest importance. An advantage of the decimal division of the angle is also found in the fact that it is possible to terminate at will the sequence of the numerals. For instance, when calculating a chain of polygones, it is not necessary to show, as with the older division, the meaningless exact seconds (for instance 42°16'27") for the simple reason that the minute itself (42°16') might, as the final unit, be too large. With the new division the angles of the polygon and the bearing angles are written with an accuracy

(5) *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Vol. 8 (1899), p. 138.
(6) *Zeitschr. f. Verm. - Wesen* 1872, p. 101.
(7) *Zeitschr. f. Verm. - Wesen*, 1910, pp. 254 & 305.
(8) *Zeitschr. f. Verm. - Wesen*, 1918, p. 154.
(9) The introduction of the 360° division with decimal subdivision, which had formerly been recommended from various quarters, (see among others theReport mentioned in note 5) would have put the makers in face of a new problem, because circles with this system of division had never before been used.

of about 10cc, for instance: 54.368 g ; *i. e.* with ample precision, one decimal place is economised. Also in calculations for circles the new division shows a decided superiority in that the conversion factor ρ = 63.6620 g = 6366.20 c = 636620 cc, contains the same numerals for all units concerned, only the position of the decimal point is varied (10). Consequently, with the new division, a table of arcs may be dispensed with.

A very remarkable relation between the angular measurement of the new division and the arc on the Earth's surface results from the establishment of the metre as the 10,000,000th part of the quarter of the Earth's meridian. To a central angle of the Earth of 1c there corresponds an arc of 1 km on the Earth's surface (11). Consequently parallel circles traced on a map at intervals of 1 c give a highly convenient kilometric division (12). Finally let it be said that with the older division decimal writing cannot always be avoided. When reading the compass, for instance, 163°.2 is obtained, *i. e.* within a tenth part of the former degree. In higher geodesy it is also current to write 76°48'25".39 (13), or, for geographical co-ordinates 49°23'46".4832. In such a case the former sexagesimal division has been associated with the decimal division ; here, however, the new division which uses only 10ths is the more logical.

To the above advantages of the new division there is opposed on the part of astronomers the inconvenience of transforming the measurements of time to angle measurements which is less simple with the new division (1h = 15°) (14). Another reason which certainly plays an important rôle in the reticent attitude of astronomers towards the 400-grade division is the breaking away from tradition. Introducing the 400-grade division into astronomy would give rise to an immense work of conversion, for the definite results of astronomical calculations, mostly expressed in angular co-ordinates, would have to be converted into the new system of division (15). However great the importance of that may be, the permanent advantages to be derived by astronomy from the use of the new division would weigh more heavily in the balance than any difficulties of conversion, which would in any case be merely temporary. What has just been said applies to geography also and, finally, to the whole technique. It would at any rate be most desirable to reach conditions in which all scientific .data and technique would employ a unified standard division of the angle, based on the decimal system.

NOTE BY THE I. H. B.

For geodetical instruments and calculations, the division of the quadrant into 400 grades, having themselves decimal subdivisions, is already used in several countries which appreciate the above advantages.

It is more difficult to apply the system to astronomy for the reasons given in this article, and for a few others.

It seems very difficult to use it in astronomy, particularly for seamen, as the change on all charts, instruments and nautical documents could be accomplished only after a lapse of many years, during which seamen would be exposed to dangerous mis-apprehensions which the absolutely standard use of the sexagesimal division by seamen of *every* nationality without exception helps in the meantime to avoid.

This standardisation is very rare in the domain of the other units, and it is so valuable that anything which might lead to a departure from it must be considered as very serious, whatever convenience might otherwise result from the point of view of calculations.

P. V.

(10) It is also interesting to draw attention to the similarity of the number r = 6366 km. (Earth radius).

(11) A similar relation exists between the minute of the older division and the nautical mile (= 1852 m.). In this order of idea the km. might be called the metrical nautical mile.

(12) This division is obtained for instance on French maps because on these the geographical lines of the network are plotted in the 400—grade division.

(13) The former method of writing in thirds (1''' = 1/60") is now never used.

(14) For a decimal division of the day there would also exist a simple reduction figure between time and angle measurements.

(15) For geodesy the matter is appreciably simpler because in this case final results — exception made of geographical co-ordinates — are most frequently expressed in lengths.