INTERFERENCE OF TIDE WAVES

by

VAN ROON.

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(Translated from the Dutch)

In *De Zee* of 1936, Mr. NOORDRAVEN spoke about the tidal streams in the North Sea and the English Channel in the articles entitled "The Tidal Streams", which, at the instigation of *De Zee*, appeared later as reprints.

Appended to these articles is a chartlet showing the lines of simultaneous H.W. (cotidal lines). These lines have been obtained by observations of the times of H.W. at various places. On examining this diagram one is struck by the peculiar phenomenon that in the North Sea these lines of simultaneous H.W. originate from two points. Probably at these points no vertical tidal movement occurs.

Superficially considered, this suggests the idea of a tide wave which does not travel in a straight line. Yet such a phenomenon can also be the result of interference of crossing waves. This phenomenon is not explained in more elementary books on tidal movements and we therefore propose to give here a simple form of explanation.

Tide waves are differentiated from wave movements on the surface in that with the tide waves the whole body of water is involved, so that in practice the water particles in the same vertical plane, at right angles to the direction of travel of the wave movement, have the same horizontal velocity.

When a wave movement of this kind travels in a channel of constant section and undetermined length, the water particles describe ellipses about a determined point. The vertical axes of these ellipses are small compared with the horizontal. Each water particle is in the movement in its elliptic trajectory slow on the particles situated in front in the direction of the wave movement on the same level.

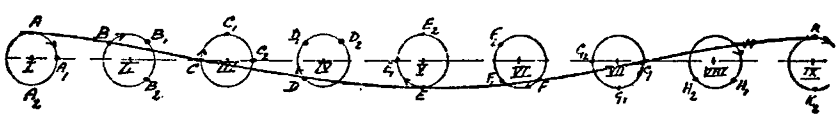

Fig. 1.

Fig. 1 gives a schematic representation of this movement, whereby the trajectories, for greater simplicity, are drawn as circles, so that the vertical movement is exaggerated as compared with the horizontal. If particle I is at A, particle II is then at B, III at C, IV at D, V at E, etc. At A and K (particles I and IX) the highest position is attained and the horizontal component of the velocity of the movement the greatest. In C the water particle is at the mean level and there remains only a vertical upward movement; in G the water particle is likewise at the mean level and there is only a vertical downward movement. The line passing through the points A, B, C and K gives the shape of the water surface.

After 1/4 period I is at A_1, III at C_1, V at E_1, VII at G_1, IX at K_1. The crest of the wave has moved to C_1, the trough to G_1.

After 1/2 period the crest of the wave lies in E_2, the trough in $K2$, and after a full period the wave crest lies again in A and K.

The crest of the wave has moved within one period from A to K. The distance between the points A and K is called the wave length. The velocity with which the crest of the wave travels over the distance equal to the wave length is called the wave velocity. The velocity of the wave is thus not the horizontal component of the velocity which the water particles themselves possess in their trajectories.

For the sake of completeness this well-known theory is re-stated here.

The wave movement described is called a progressive wave. The rate of travel of the progressive wave depends of the depth on the channel. If h is the depth of the channel, g the acceleration of gravity, then the rate of travel is $c = \sqrt{gh}$.

If the channel is closed at one end, the wave is reflected. For convenience it will be admitted that this reflection takes place against a vertical wall in the channel of constant cross section.

If it be granted that during the progress of the wave and the reflection no loss through friction or other causes has occurred, the wave then travels back by the reflection with the same wave height and the same displacement of the water particles in their trajectories.

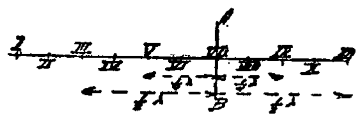

FIG. 2

If (*see* Fig. 2) I, II, III, IV, V, etc., are points at same mutual distances, whereby I-V = 1/2 wave length and VII is the point where reflection has taken place against the plane *AB*, then point VI acquires through the reflection a movement which is the reflected image with respect to *AB* of the movement which a point VIII of the progressive wave would have, and which in the absence of the plane *AB* would continue. In like manner V acquires a movement which is the reflected image of the movement of IX with respect to *AB*, IV one which is the reflected image of X, and so on.

Point V at one fourth wave-length from *AB*, and point IX, likewise at one fourth wave-length from *AB*, which thus lie one half wave-length from one another, have movements such as represented in Fig. 3, whereby the numbers are given for the same moment.

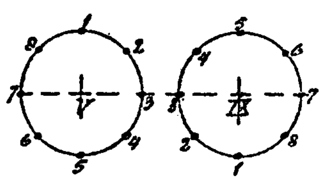

FIG. 3

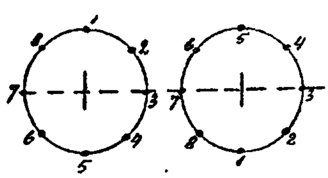

FIG. 4

Through the joint action of the original progressive wave and of the reflected wave the movement IV is obtained composed of the movements of Fig. 4.

The figure shows that the vertical movements of both waves cancel each other: if one of them gives a rise above mean level, the other gives a fall of equal magnitude. The horizontal movements reinforce each other: in both movements there occur simultaneously shiftings towards the right and towards the left. If we consider III, at a half wave length from VII, and XI likewise at a half wave-length from VII, whereby III and XI lie mutually at a distance of a full wave length, then these points have exactly the same movement. Through the joint action of the original wave and

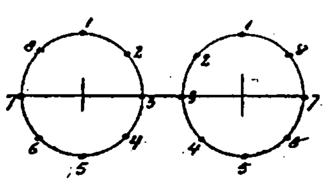

FIG. 5

the reflected wave the movement of III is thus composed of the movements of Fig. 5. Here the vertical movements reinforce each other and the horizontal cancel out.

We thus find a wave movement whereby in *Q* and *S* (Fig. 6) there exists no verti-al movement but simply a horizontal one. On *P*, *R* and *T* we have a vertical movement alone and no horizontal. In places situated between these points there is vertical movement with smaller amplitude than in P, R and T, together with a horizontal movement which is directed towards the point where the water rises highest. Thus if P and T have a rising movement, then between P and R the stream is towards left, the strongest in Q; between T and R, towards right and the strongest in S. If the water is in the highest position no current flows anywhere. On the falling of the water all the points return in like manner to the mean position. At this moment the horizontal movement is the strongest.

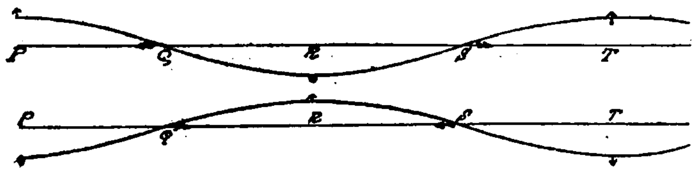

FIG. 6

The joint action of interference of the original progressive wave and of the reflected wave gives a *stationary wave*, in which the points with the strongest vertical movement are called the loops, the points where no vertical movement occurs, the nodes. The nodes are situated at a mutual distance equal to the half wave-length, the loops lie half-way between two nodes. The plane where reflection occurs always comprises a loop.

Thus from two progressive waves a stationary wave may result. Conversely, a stationary wave may be considered as resulting from the interference of two progressive waves.

It is further possible to consider a progressive wave as resulting from the interference of two stationary waves. In order to illustrate this point, let us first examine how the height and the velocity of the horizontal movement depend on time.

The height of the water in relation to the mean water level for a given moment can be determined in the case of a progressive wave as follows:

Let us conceive a circle with a diameter equal to the height of the wave (i.e. equal to the double amplitude of the wave). Along this circle let us conceive a point which within a time interval equal to the wave period makes one revolution. The angle which the radius towards the moving point makes with the vertical diameter is called the phase of the movement. In the highest position the phase is $= 0°$, in the lowest position equal to $180°$, for the mean position and falling water, $90°$, for the mean position and rising water, $270°$. The position at any arbitrary moment may thus be found by projecting the auxiliary point on the vertical diameter. If therefore y is the height above the mean position, then $y =$ ampl. cos phase.

The rate of the horizontal movement (the velocity of the current) with a progressive wave is found in the same manner. The radius of the circle is thus the maximum speed v_0 and the phase is counted from the horizontal diameter. The projection of the radius of the auxiliary point in movement in the circle on the horizontal diameter gives then the velocity v of the given phase. We have thus $v = v_0$ cos phase.

With a progressive wave, for all places along which the wave passes the amplitude and the maximum speed are the same. For points which lie in the direction in which the wave moves, the phase lags behind that of points which lie in the opposite direction. If points I, II, III, IV are spaced in such a way in the channel that the intervals equal $1/4$ wave-length, and if the wave moves in the direction I to IV, the phase of II is then $90°$ behind that of I, the phase of III $180°$ behind that of I, that of IV $270°$ behind that of I. At each point the maximum current coincides with the greatest rising or falling and there in no current when the water is in the mean position.

With a stationary wave it is possible to deduce the vertical movement in the same way as for the progressive wave by means of the movement of an auxiliary point. Here also $y =$ amplitude cos phase. *The amplitude thus changes from place to place.* For points I, II, III, IV, V which also lie $1/4$ wave length from each other, the successive amplitudes are: at I zero, at II max., at III zero, at IV max., at V zero. Between I and III the phase is the same for all points, for points between III and V the phase varies by $180°$ with that of the points between I and III. All points between I and III have H.W. when those between II and V have L.W., and conversely.

For the velocity $v = v_0$ cos phase is again valid, whereby v_0 acquires from point to point another value. In the above-mentioned points we have therefore: at

I v_0 max., at II $v_0 = 0$, at III $v_0 =$ max., at IV $v_0 = 0$, at V v_0 max. The direction of the stream is the same at all points between I and II, between II and IV opposed thereto, between IV and V the same as between I and II. In each point the phase of the current differs by 90° from that of the vertical movement.

Considering now a progressive wave of wave-length λ, we see that the phase difference of two points is proportional to their mutual distance and is 360° when the points lie at a mutual distance equal to the wave-length. Thus, with the phase difference θ,

$$\theta : 360^\circ = x : \lambda.$$

If the phase at the first point is $= \varphi$, at the second point it is $\varphi - \theta$.

For this point the rising of the water is then $y = A \cos(\varphi - \theta)$ and the velocity of the current $v = v_0 \cos(\varphi - \theta)$.

We have then

$$y = A \cos\varphi \cos\theta + A \sin\varphi \sin\theta.$$
$$v = v_0 \cos\varphi \cos\theta + vo \sin\varphi \sin\theta.$$

If we now consider the movements which are determined by $y = A \cos\theta \cos\varphi$ and $v = v_0 \sin\theta \sin\varphi = v_0 \sin\theta \cos(\varphi - 90^\circ)$, this apparently represents a stationary wave movement. The vertical movement $y = A \cos\theta \cos\varphi$ is a movement with amplitude $A \cos\theta$. This amplitude differs from place to place. For $x = 0$, $\theta = 0$, thus the amplitude $= A$. For $x = 1/4\ \lambda$, $\theta = 90^\circ$ and the amplitude 0. For $x = 1/2\ \lambda$, $\theta = 180^\circ$, the amplitude thus $-A$, which in effect represents a movement which is in 180° phase difference with that for $x = 0$. For $x = 3/4\ \lambda$, $\theta = 270^\circ$, the amplitude is again 0; for $x = \lambda$, $\theta = 360^\circ$ the amplitude is $= A$.

The horizontal velocity lags 90° behind y. The maximum velocity varies from point to point. This maximum velocity is $v_0 \sin\theta$ and is zero for $x = 0$, v_0 for $x = 1/4\ \lambda$, zero for $x = 1/2\ \lambda$, $-v_0$ (i.e. oppositely directed to the velocity in $x = 1/4\ \lambda$) when $x = 3/4\ \lambda$ and again zero when $x = \lambda$.

We thus find all the typical phenomena of a stationary wave. $y = A \cos\theta \cos\varphi$ with $v = v_0 \sin\theta \sin\varphi$ represents thus a stationary wave.

In like manner $y = A \sin\theta \sin\varphi = A \sin\theta \cos(\varphi - 90^\circ)$ with $v = v_0 \cos\theta \cos\varphi$ gives a stationary wave. In this wave the vertical movement lags 90° behind that of the first and the horizontal velocity is 90° ahead of the first.

At the same time the loops of the first wave coincide with the nodes of the second and conversely.

A progressive wave can thus be considered as the result of two stationary waves.

Therefore where different waves exist (progressive and stationary) one can either replace each progressive wave by two stationary, or each stationary by two progressive waves. The resulting movement may be considered as composed entirely of stationary or entirely of progressive waves.

Whenever two progressive waves with the same wave period, but with different amplitudes, move one against the other, peculiar phenomena occur which, supercifially examined, may often lead to erroneous conclusions.

Let us suppose that for a given point the movements in both waves have the same phase. For a point which lies in the direction of travel of the first wave, the phase for the movement of the first wave then lags, by an amount θ behind that in the first-mentioned point. The phase of the movement in the first wave is therefore leading by θ.

The resulting vertical movement is therefore:

$y = A \cos(\varphi - \theta) + B \cos(\varphi + \theta)$, in which A and B are the amplitudes of the waves and φ the phase for the point where both movements have the same phase.

If v and w are the maximum current velocities of the two waves, the resulting velocity is then $u = v \cos(\varphi - \theta) - w \cos(\varphi + \theta)$.

By developing the terms appearing in these expressions we have

$$y = (A + B) \cos\theta \cos\varphi + (A - B) \sin\theta \sin\varphi.$$
$$u = (v - w) \cos\theta \cos\varphi + (v + w) \sin\theta \sin\varphi.$$

In order to consider the vertical movement more closely, let us take the fixed numerical values $A = 1$ m., $B = 0.20$ m. If the wave period is 12^h and the time of H.W. for the point where both movements are in the same phase 0^h, then at t^h $\varphi = t \times 30^\circ$.

We have on the other hand $y = 1.2 \cos \theta \cos \varphi + 0.8 \sin \theta \sin \varphi$. With $P = 12.\cos \theta$, $Q = 0.8 \sin \theta$, y is then $= P \cos \varphi + Q \sin \varphi$. The highest level is attained when $\tan \varphi = \frac{Q}{P}$ and gives a rise above mean level $= \sqrt{(P^2 - Q^2)} = \sqrt{(A^2 + B^2 + 2AB \cos 2\theta)}$.

The value of θ depends on the locality; at the point where for both wave movements the phase is the same, $\theta = 0$, one quarter of wave-length farther in the direction of the movement of the first wave $\theta = 90^\circ$, at a distance equal to the half wave-length $\theta = 180^\circ$. If the distance from which the point where the phases are equal be computed, then for a given value of θ the distance equals $\frac{\theta}{360} \times \lambda$.

We can now calculate for which places the time of H.W. is 0^h, 1^h, 2^h, 3^h, and so on, by starting from

$$\tan \varphi = \frac{Q}{P} = \frac{0.8 \sin \theta}{1.2 \cos \theta}$$

There follows therefrom $\tan \theta = 1.5 \tan \varphi$, which then gives :

H. W. at	φ	θ	x	*Ampl.*
0^h	0°	0	0	1.20
$0^h\ 30^m$	15°	21.9°	$0.061\ \lambda$	1.15
1^h	30°	40.9°	$0.114\ \lambda$	1.04
$1^h\ 30^m$	45°	56.3°	$0.156\ \lambda$	0.94
2^h	60°	69.0°	$0.192\ \lambda$	0.86
$2^h\ 30^m$	75°	79.9°	$0.222\ \lambda$	0.82
3^h	90°	90°	$0.25\ \lambda$	0.80

The interference of the two waves produces a wave which travels in the direction of the strongest wave. Not all localities have hereby equally strong vertical movements, as with a purely progressive wave. The amplitude varies from 1.20 to 0.80. Yet a stationary wave is also not the resulting movement, for there exist no places without vertical movement. The movement should be considered as a progressive wave of variable shape.

If the moment of H.W. be noted from place to place, and the lines of simultaneous H.W. be drawn, then these lines for equal time differences appear to lie at unequal distances. For places with large tidal range these lines lie further spaced, for places with small tidal range, closer together. From the position of the lines of simultaneous H.W. one must infer a progressive wave, the velocity of travel of which is variable and must be attributed to a variable depth of water.

In Fig. 7 is given, for a distance of a 1/4 wave-length, the shape of the water surface at different instants, but the vertical distances are strongly exaggerated in comparison with the horizontal. In the figure the places H indicate where at this instant H.W. occurs (i.e. where the rising movement of the water is transformed into a falling one). These are not the places in the wave where water level at this instant is the highest. These places are given by M. In the points M the water has a falling movement.

We can proceed in like manner with the current. It is then found that at a given moment the strongest currents occur at the places where it is exactly H.W. and not at the places where the highest level of the surface of the wave is attained. Concurrently the maximum current velocity is the greatest where the range is the smallest, and the maximum current velocity the smallest where the range is the greatest. If for instance, the range is $A + B$, then the current is $v - w$, whereas for a range $A - B$ the current is $v + w$.

With the simultaneous occurrence of stationary waves analogous phenomena are obtained. The above-mentioned case is in actual fact also interference of two stationary waves :

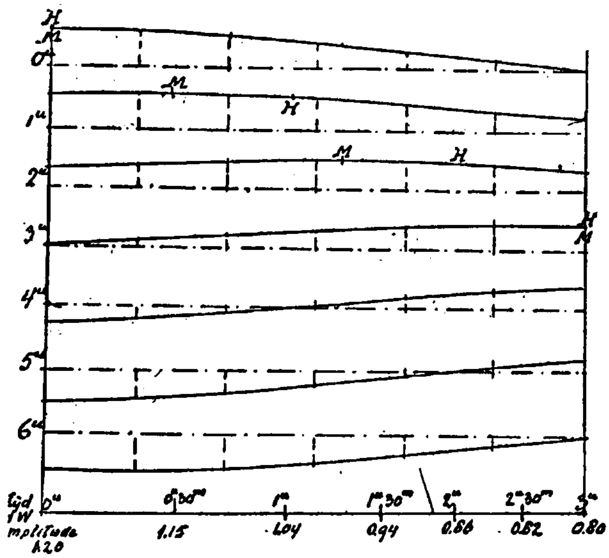

FIG. 7

one of them is determined by $y = (A + B) \cos \theta \cos \varphi$ and $u = (v + w) \sin \theta \sin \varphi$, the other by $y = (A - B) \sin \theta \sin \varphi$ and $u = (v - w) \cos \theta \cos \varphi$.

If we investigate the points of the wave where the water is at mean level ($y = 0$) and the current nil ($u = 0$), it is found that these are not the same instants as in a progressive wave. The phenomenon which thus often occurs: maximum current not at *H* or L.W., reversal of the current not at the mean level, and which is attributed to change of depth of the water, whereby the movements of the water particles no longer occur in ellipses with their long axes horizontal, may thus also be explained by interference of wave movements.

In the above considerations we have always started from two waves which travel in the same direction or in opposed directions: waves in a channel having no velocities perpendicular to the axis of the channel.

In a vast basin in which waves can enter from various sides and are reflected against shoals and coasts which are not perpendicular to the direction of the movement of the wave, waves can occur with different directions of travel and in which the wave

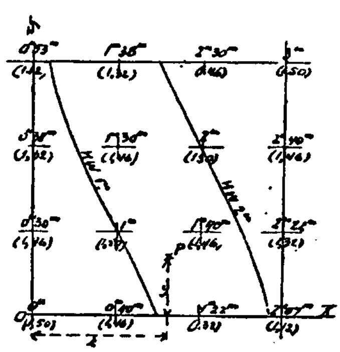

FIG. 8

crests intersect each other at various angles. Here several particularities occur which we shall explain in the light of a few simple cases.

In the first instance let us consider the case where two progressive waves intersect at right angles.

The first wave travels in the direction *OX* (West-East) and the second in the direction *OY* (South-North) (Fig. 8). We have assumed that at the point *O* the phase for both wave movements is equal (phase at a given moment $= \varphi$).

At a point P lying at a distance x from OY the phase of the W.-E. travelling wave is $\varphi - \frac{x}{\lambda} \times 360^o = \varphi - \theta_1$. If the point P lies at a distance y from OX, the phase of the S-N travelling wave at that point is then $\varphi - \frac{y}{\lambda} \times 360^o = \varphi - \theta_2$.

If the amplitudes of these waves are A and B, the water level in P is $y = A \cos (\varphi - \theta_1) + B \cos (\varphi - \theta_2)$.

It follows therefrom $y = (A \cos \theta_1 + B \cos \theta_2) \cos \varphi + (A \sin \theta_1 + B \sin \theta_2) \sin \varphi$.

H.W. occurs there when

$$\tan \varphi = \frac{A \sin \theta_1 + B \sin \theta_2}{A \cos \theta_1 + B \cos \theta_2}$$

If at 0^h $M.T.$ $\varphi = 0$ and the wave period 12^h, at t^h $M.T.$ the value of $\varphi = t \times 30^o$.

It is thus possible with the aid of the given formula to calculate how late H.W. occurs at various points. In the figure there are given, for various points, the times of H.W. and (in parentheses) the values of the amplitudes for these points for the case $A = 1.00$ and $B = 0.50$. At the same time are given the lines passing through the places where H.W. occurs at 1^h and 2^h.

The lines of simultaneous H.W. are displaced parallel to themselves in the direction N-E. On the S-W, N-E travelling lines the amplitude is the same.

With regard to the direction of the stream at a given point the latter is the resultant of a current along the meridian and along the parallel. Both components are variable and do not in general varv proportionally. The resultant varies thus as a rule both in magnitude and in direction. The North component is $v \cos (\varphi - \theta_2)$, the East component $w \cos (\varphi - \theta_2)$.

The azimuth of the direction of the stream is determined by

$$\tan \alpha = \frac{w \cos (\varphi - \theta 1)}{v \cos (\varphi - \theta_2)}$$

For the points for which $\theta_1 = \theta_2$, i.e. on the line running through O in the N-E direction, $\tan \alpha = w:v$. Here the direction of the stream does not turn, but changes by reversal 180^o in direction.

For other points the stream direction is variable and the strength is never zero at any moment. On one side of the above mentioned line the stream turns with the sun, on the other side against the sun.

Interference of two progressive waves thus gives rise to a uniform shifting of the lines of simultaneous H.W. with revolving streams. At the same time the amplitude (range) is not the same at all places.

In the interference of two stationarv waves whose wave crests intersect, other phenomena can again occur.

In order to avoid involved considerations, let us assume two stationary waves whose crests are at right angles to each other. For one of the waves let us suppose that (Fig. 9) a node occurs along OY and a loop along AB. For the second wave a node falls along OX and a loop along BC. The greatest amplitude for the vertical movement of the first wave is $= A$, for the second $= B$.

If θ_1 and θ_2 are angles which depend in the same manner on the location of the point considered as the phase difference by the progressive waves of the preceding example, at a point P the vertical movement is then determined by

$$y = A \sin \theta_1 \cos \varphi + B \sin \theta_2 \cos (\varphi - \alpha).$$

Therein φ is the phase of the vertical movement in the first wave, $\varphi - \alpha$ that of the vertical movement in the second wave.

To obtain an idea of the development of the times of H. W., let us take $A = 1.00$ $B = 0.80$, $\alpha = 90^o$, for which $y = 1.00 \cos \theta_1 \cos \varphi + 0.80 \sin \theta_2 \sin \varphi$.

H.W. in a given point occurs then when

$$\tan \varphi = \frac{0.80 \sin \theta_2}{1.00 \sin \theta_1}$$

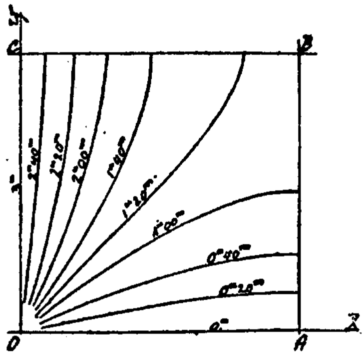

FIG. 9

For various combinations of θ_1 and θ_2 it is now possible to calculate the times of H.W. and deduce therefrom the run of the lines of simultaneous H.W.

The lines of simultaneous H.W. are curved lines all of which pass through a point O where the vertical movement for both waves is nil.

The amplitude varies from place to place: at the point A the amplitude (semi-range) is 1.00, at C, 0.80, at B, 1.28.

In Fig. 9 the direction of the cotidal lines alone is drawn for the area above the line OX and right of the line OY. If the reflected image of the figure with respect to OY be formed, one then obtains the lines of H.W. at $3^h\ 20^m$, $3^h\ 40^m$, and so on till 6^h. By taking the reflected image with respect to OX of the figure thus formed we find the lines for H.W. $6^h\ 20^m$, $6^h\ 40^m$, and so on.

When the wave crests are not at right angles to each other and the phase difference of the waves is not exactly 90^o, the run of the cotidal lines is less symmetrical, but they nervertheless pass through the point of intersection of the lines where both waves have a node.

The stream at any arbitrary point has again, in general, a component parallel to OX and one parallel to OY. In B the stream is always nil, in O it is the strongest, at 0^h and 12^h it is directed along OY, at 6^h along OX. As a rule revolving streams are produced; alone along AB and CB the stream changes its direction by 180^o at its reversal.

From the above one sees that through interference various peculiarities of the tidal waves in the propagation of the tidal movement can be explained.

Thus when a tide wave enters a sea area bounded by coast lines of varied direction and in which the depth changes, it is reflected by the coast and banks. Moreover transverse streams can occur which are produced by the difference in velocity of travel through change in the depth in a direction perpendicular to that of the wave movement.

Through all these possibilities different waves, progressive and stationary, of diverging directions, interfere at a given point.

In each case the vertical movement of the water at any arbitrary point can be given by a formula of the form $y = A \cos (\varphi - \alpha)$, in which A is the amplitude for the locality, φ the phase of the tidal movement for a chosen determined point (basic station) and α the amount by which the phase in the point considered lags behind that of the basic station.

If we put $A \cos \alpha = P$, $A \sin \alpha = Q$, then $y = P \cos \varphi + Q \sin \varphi$, in which P and Q for each place are constant values which vary in general from place to place. With a tide period of 12^h, on the assumption that $\varphi = 0$ at 0^h, we have:

For all places where $Q = 0$, H.W. occurs at 0^h. or 6^h, for places where $P = 0$, H.W. occurs at 3^h or 9^h.

All points where $Q = 0$ lie on one or more curved lines which are the lines of simultaneous H.W. at 0^h or 6^h; whereas all points where $P = 0$ lie on one or more curved lines, the lines of simultaneous H.W. at 3^h or 9^h.

If the lines thus found for 0^h and 3^h intersect, we have a point through which the cotidal lines also pass at some other hour. The lines then form an amphidrome. At that point both P and $Q = 0$; the entire vertical water movement is there nil.

If the lines do not intersect for $P = 0$ and $Q = 0$, no amphidromes are produced.

With regard to the image of the stream: with intersecting waves, either progressive or stationary, rotating tidal streams should be the rule.

Wave movements of the character of tide waves occur also when for one cause or another a disturbance of equilibrium has taken place.

Thus if in a channel closed at one end, of constant cross section and depth, a progressive wave is formed, the same reflects against the closed extremity and a stationary wave is produced. At the closed end a loop of the stationary wave is formed. If at the other extremity the channel opens out into a deep extensive sea and at this open end there is a node of the stationary wave, then at the mouth of the channel there occurs solely horizontal and no vertical movement. The further one penetrates into the channel the stronger the vertical movement becomes and the weaker the horizontal.

When such is the case the original wave must have a wave-length of λ which is equal to $4 \times$ the length L of the channel.

If T is the wave period, c the velocity of propagation of the progressive wave, then $cT = \lambda = 4\,L$, whereby $c = \sqrt{gh}$ (h being the depth of the channel).

There results therefrom $T = \dfrac{4\,L}{\sqrt{gh}}$

This is called the period of oscillation of the open channel.

For a channel or sea basin of arbitrary, variable, cross section such a period is difficult to find by calculation and can only eventually be determined by approximation. In any case the water body in such an area has a natural period of oscillation.

For a channel closed at both ends a stationary wave may be produced on both ends by reflection; these stationary waves have a loop at the extremities and at half length of the channel a node. This can only occur when the length L of the channel is equal to the half wave-length. If such is the case, the wave period must be $T = \dfrac{2\,L}{\sqrt{gh}}$. This value gives then the period of oscillation of the channel closed at both ends.

With a variable cross section and depth the body of water acquires also a fixed period of oscillation.

In such case one can also obtain oscillations of shorter duration, whereby not one but several nodes occur.

These phenomena were first studied on the occasion of periodical movements in Lake Geneva, which movements are called "seiches". Later on such oscillations were also observed and investigated in other lakes and sea basins, and characterized broadly by the name "seiches".

Now if the period of oscillation of a channel, a sea basin or a bay is equal or nearly equal to the period of a tide generating force, the tide movements become very strong by resonance. On the basis hereof, the strong tidal movements such, for example, as occur in the Bay of Fundy (Nova Scotia), are to be ascribed to resonance of proper oscillation and tidal forces and not, as mostly alleged, to "a tide wave, which coming from the ocean, by penetrating the bay, attains a greater height through the diminishing width and depth".

The considerations developed above touch of course upon the problem of tide waves in a very superficial manner. They are also merely intended to give a rough idea of the possibilities with interference and resonance phenomena. For those who wish to go further into the subject, reference may for instance be made to Parts XIII and XIV of the "Probleme der Kosmischen Physik" (THORADE — *Probleme der Wasserwellen*). For more elementary considerations reference may be made to "The Tide" by H.A. MARMER.