LIGHT-STAR SIGHTS

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The expedient of utilizing a light to create a point in the horizon for observing an altitude of a star at night is employed quite commonly by navigators. The objections to measuring an altitude in this manner lie (1) in the maneuvering that may be required to cause the light and star to appear in a vertical plane of the observer, and (2) in the uncertainty of attaining the desired degree of verticality when the larger altitudes are observed.

It will be shown in this discussion that a line of position will result in observing the angle between a horizon light and a star high in altitude (regardless of whether the angle lies or does not lie in a vertical plane of the observer) provided that a near-horizontal angle between the light and a star low in altitude is also observed at the same time. The star high in altitude should be one selected preferably within 10° of the zenith; and the star low in altitude, one preferably within 25° of the horizon and within the limits of 60° and 120° in angular distance from the light.

In connection with the solution of this sight, a computation is made in order to obtain the true angular distance and direction between the two stars. Once made for a pair of stars, these computed elements, remaining constant, may be used in connection with sights repeated on successive nights. The angle at the near-zenith star between the light and the near-horizon star is also used in the solution and is computed from the three known sides of the spherical triangle formed by the two stars and the light projected on the celestial sphere. If the near-zenith star is within about 6° of the zenith, the remainder of the solution may be made graphically, except for some minor calculations.

The light used for creating a point in the horizon may be one showing on a distant vessel, or it may be a self-igniting water light or lighted float dropped from the observer's vessel when the occasion and conditions arise for taking a sight. The vessel, of course, steams several miles away from the water light before making the observations. The distance between the observer and the light should be preferably not less than 4 miles. The distance, obtained by patent log or range finder, and the difference in elevation between the light and observer are required for determining the dip correction.

The following is a summary of the elements required to establish the line of position:

1. The sextant angle observed between the light and near-zenith star.
2. The sextant angle observed between the light and near-horizon star.
3. The time of observing the two angles simultaneously.
4. The distance between the light and observer.
5. The difference in elevation between the light and observer.

In Fig. 1, the light L, the observer Z, and the observer's horizon are shown projected on the celestial sphere. Other designations are as follows:

S, the near-zenith star.
H, the near-horizon star.

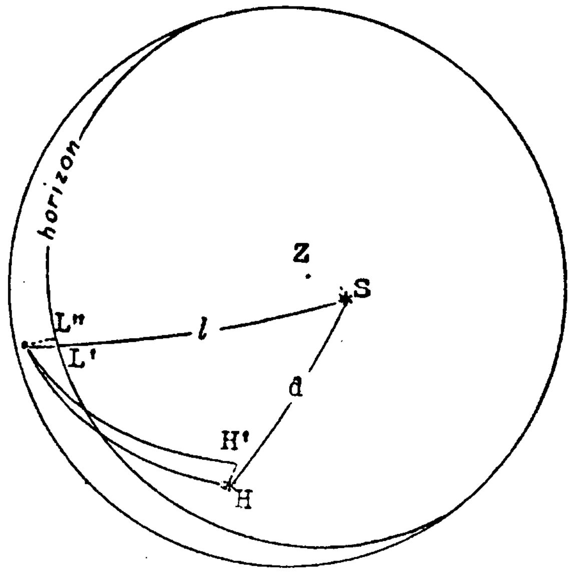


Fig. 1.

H', the displaced position of H due to refraction.

LL'' the dip of L below the horizon.

LH', the measure of the near-horizontal angle observed between L and H'

LS, the measure of the near-vertical angle observed between L and S.

(The displacement of the star S due to refraction being less than 0.17' may be neglected).

d, the computed angular distance between S and H.

l, the measure of the part of the arc LS above the horizon, namely between L' (point in the horizontal) and S.

As previously stated the star S is one selected within 10° of the zenith. This makes it practicable to consider LL' equal to the dip LL''. If the height of eye is not more than 70 feet and the distance to the light not less than 4 miles, the maximum error resulting in considering LL' equal to the dip would be 0.15' Therefore, within the limitations specified :

$$l = LS - \text{dip}. \qquad (1)$$

In connection with formula (1), it is interesting to note that the maximum error (0.17'), resulting in neglecting the refracted displacement of the star S, will occur when the star lies in the vertical circle ZL, at which occurrence no error is occasioned in making LL' equal to LL''; and vice versa, no error is occasioned in neglecting the refracted displacement of S when the maximum error results in making LL' equal to the dip. Therefore, we may conclude that in deriving l in formula (1) the combined error resulting from the two sources will never exceed 0.17'. When l is greater than 90° the two errors tend to negate each other.

The true angular distance LH is obtained from the observed angle LH' by applying a correction for refraction. This correction is the component of refraction along the arc LH corresponding to the vertical refraction HH'. Corrections are tabulated below for various values of LH', when the altitude of H is between 5° and 25°. It can be shown that for a given value of LH' the correction is constant for the range of low altitudes specified.

LH'	Corrn	LH'	Corrn	LH'	Corrn	LH'	Corrn	LH'	Corrn	LH'	Corrn
50°	− 0.8'	65°	− 0.4'	80°	− 0.2'	90°	− 0.0'	105°	+ 0.3'	120°	+ 0.6'
55°	− 0.7'	70°	− 0.3'	85°	− 0.1'	95°	+ 0.1'	110°	+ 0.3'	125°	+ 0.7'
60°	− 0.5'	75°	− 0.3'	90°	− 0.0'	100°	+ 0.2'	115°	+ 0.4'	130°	+ 0.8'

In the spherical triangle LSH,

$$\csc \tfrac{1}{2}\, LSH = \sqrt{\frac{\csc (s - LS) \csc (s - d)}{\csc LS \csc d}} \qquad (2)$$

in which $s = \tfrac{1}{2}(LS + d + LH)$.

The azimuth at S to H (azSH) is derived from the celestial coordinates of the stars S and H.

The azimuth at S to L' (azSL') is found from the equation :

$$\text{az}SL' = \text{az}SH \pm LSH. \qquad (3)$$

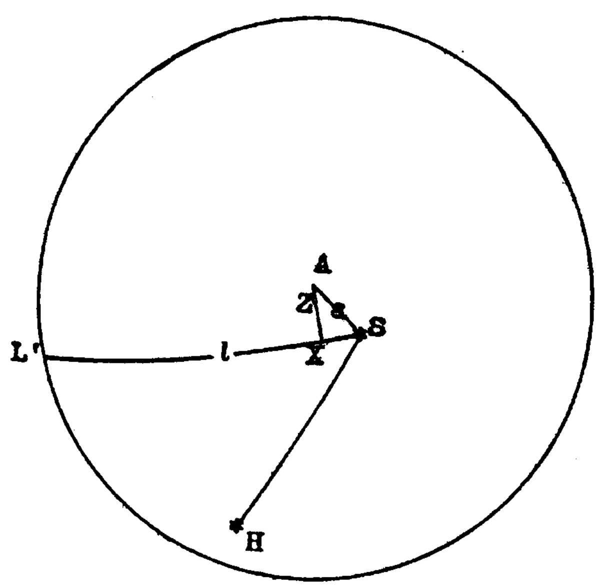


Fig. 2

In Fig. 2, the star S and the point in the horizon L' created by the light are shown projected on the plane of the horizon. Other designations are as follows :

l, the arc $L'S$ (see formula 1).

ZX, the great circle having its pole at L' ; it passes through the zenith Z and is therefore a line of position; it is perpendicular to the arc $L'S$. Arc $XS = l - 90^{\circ}$.

a, the approximate zenith distance of S obtained by scaling the distance between the plotted position of S and the plotted dead reckoning position.

A, the point in the line of position ZX, distant a from S.

The latitude and longitude of the point A and the azimuth at A to X (azAZ) when found will establish the line of position.

In the right spherical triangle AXS,

$$\sec ASX = \frac{\sec l \sec a}{\csc l \csc a} \qquad (4)$$

$$\csc XAS = \frac{\sec l}{\csc a} \qquad (5)$$

The azimuth at S to A (azSA) is found from the equation :

$$\text{az}SA = \text{az}SL' \pm ASX. \qquad (6)$$

The latitude and longitude of A and the azimuth A to S (azAS) are found from the terrestrial coordinates of the Star S, the approximate zenith distance a and the azimuth azSA (formula 6). See solution of problem.

$$\text{az}AZ = \text{az}AS \pm XAS. \qquad (7)$$

When the dead reckoning position is within about 4° of X, the line of position may be established as follows : Compute the latitude and longitude of X and the azimuth azXS from the terrestrial coordinates of the star S, the angular distance co l (SX), and the azimuth azSL' (formula 3).

$$\text{az}XZ = \text{az}XS \pm 90^{\circ}.$$

Plotting the position X and the azimuth azXZ, the line of position XZ is established. On a MERCATOR projection, a MERCATOR course corresponding to the great circle course azXZ may be used if the distance along the course is long enough to require its use.

When the zenith distance of S is less than 6°, the line of position may be established graphically as follows : Plot the position of X using the distance co l and azimuth azSL' (formula 3) from the plotted position of S. Draw XZ perpendicular to the course XS to establish the line of position. If a MERCATOR projection is used, MERCATOR courses along SX and XZ may be used. A refinement of the graphical solution can then be made by computing the position of X and a position in the line of position, using the principle of MERCATOR sailing.

PROBLEM. — A floating pole supporting a light was dropped from a vessel, and after steaming away from it 4.44 miles 96° true, the star Vega (near the zenith) was observed 93° 25.2' in angular distance from the light and the star Antares (near the horizon) was observed 70° 51.1' in angular distance from the light. The time of observation was 23 h. 50 m. 02 s. G.C.T., August 13, 1935. The height of the observer was 35 feet; the light, 3 feet. The dead reckoning position was 42° 54' N. Lat., and 44° 00' W. Long. There was no index correction.

It will be assumed that the distance and azimuth between the two stars were computed on a previous occasion; these values are:

$$d = 71° - 42.3, \quad \text{and} \quad \text{az}SH = 210° \; 19.7'.$$

SOLUTION OF PROBLEM.

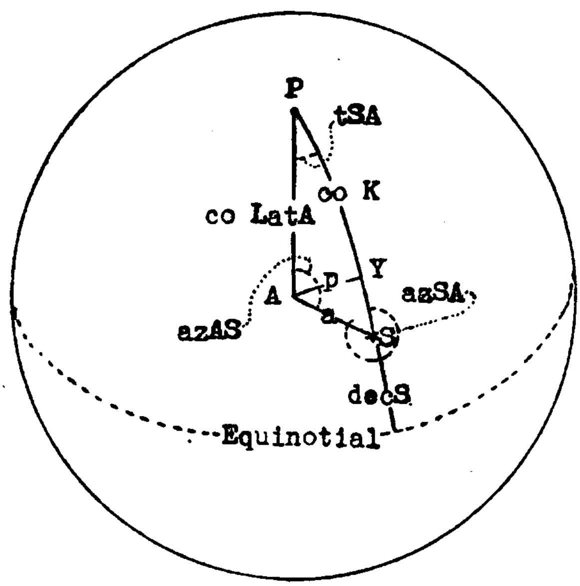


Fig. 3

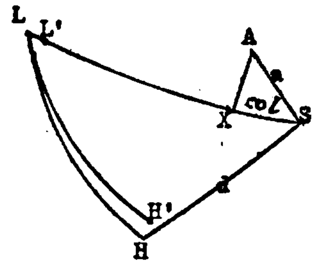
Derivation of formulae for computing latitude and longitude of the point A. In fig. 3 the star S, and the point A are shown projected on the plane of the horizon in relation to the pole P. The arc p (AY) is drawn perpendicular to the meridian of S. The arc K is the angular distance from the equinoctial to Y. The angle tAS is the difference in longitude between A and S (Long $A \pm$ Long S).

In the right spherical triangle AYS

$$\csc p = \csc a \csc \text{az}SA$$

$$\sec (K \pm \text{dec}S) = \frac{\sec a}{\sec p}$$

Solution of Problem



LH' 70° 51.1'
refr. − 0.3
LH 70 50.8
LS 93 25.2 .... A = 77
d 71 42.3 .... + A 2253
2s 235 58.3
s 117 59.1
(s−LS) 24 33.9 A .38114
(s−d) 46 06.8 A .14102
(52216)−(2330)
2) 49886
1/2 LSH 34° 16.2 ..... A 24943
LSH 68 32.4
azSH 210 19.7
azSX 278 52.1

GCT $23^h\ 50^m\ 02^s$
Sid T 0^hGCT 21 22 00.8
3 54.9
GST 21 15 57.7
R.A.S 18 34 42.3
LongS (time) 2 41 10.4
LongS 40° 17.6'

LS 93 25.2
dip − 6.0
l' 93 19.2 B 123722 B 123722 A .73
a 5 00 (−) A 105970 + B 166 + A 105970 A 105970 B 165.6 (a)
XAS 41 38.5 A .17752 (123888)−(106043)
ASX 48 28 .... B(17845) + (−)
azSX 278 52.1
azSA 327 20.1 .... A 26783
A 132753 B .... 48 A 132753 B 48.0
K−decS 4 12.8 ←........ ....B 117.6
decS 38 43.5 ....(−) + ...B 10282 (decS)
K 42 56.3 ....A 16672 +
LatA $\boxed{42\ 52.7}$ ←.... B .13503 A .16720
tSA 3 40.8 ←.... A 119250 .... A 119250 (tSA)
LongS 40 17.6 (−)(130032)
LongA $\boxed{43\ 58.4}$
a ..... A 105970
azAS 144° 55.6 A 24062
XAS 41 38.5
azAX $\boxed{186\ 33.1}$

In the right spherical triangle *PYA*

$$\csc LatA = \sec p \csc K$$

$$\csc tAS = \frac{\sec p}{\sec LatA}$$

In the spherical triangle *PSA*

$$\csc azAS = \frac{\sec decS \csc tAS}{\csc a}$$