METHOD OF MEASURING ANGLES BY « Serial Repetitions » BY MEANS OF A REPEATING INSTRUMENT (1)

by

LT.-COL. CH. SCHMERBER, COMMANDER P. TARDI AND CAPTAIN P. CAILLOL.

attached to the *Service Géographique de l'Armée Française.*

(Extract from the BULLETIN GÉODESIQUE n° 56, Paris,

October-November-December 1937, p. 290.)

(Translated from the French.)

Two classical methods exist for the measurement of angles in the field : the repetition, and the swing (*réitération*) methods.

The first was invented by BORDA and was used by French Engineer-Geographers up till the middle of last century. It was abandoned as a result of the great difficulty experienced in constructing an apparatus which would permit the method to be rigorously applied :— an apparatus in which no *play* should be possible between the alidade, the limb and the axis at the moment when these different parts are brought together as a solid body, nor any *drag* be possible when it is desired to move such parts with respect to each other.

By the use of this method applied to the measurement of an angle AB, the systematic errors of the graduations from the first of the pointings on A to the last of the pointings on B (2), at the end of n repetitions are actually divided by n. Similarly the errors of microscope readings (due to micrometric screws, to the uncertainty of their calibration, to readings effected, etc.) are also divided by n ; the sum of the errors of the sights with the telescope being itself divided by $\sqrt{n}$.

....

Since 1900 it has become more and more usual for geodesists throughout the whole world to measure the angles of triangulations of the first order individually and proceeding by *couples,* a procedure which possesses the considerable advantage of eliminating to a great extent the influence of the twisting of the support : such as the SCHREIBER method, or the method known as that of the couples on the control signal (*couples sur référence*), successfully introduced in France towards 1925 and at present very widespread. (*)

From the above, there is no theoretical reason to prevent an effort to return to the *repetition* methods for the individual measurement of the angles of a first order triangulation, more especially when the sighting telescope is fitted with a micrometer with movable hair.

The *repetition* method is in fact incontestably superior to the swing (*réitération*) method which has now-a-days come into almost exclusive use.

In the swing (*réitération*) method, as is known, independent pointings are carried out, distributed over the whole circumference of the limb ; if the error of graduation is of a simple periodical form, the algebraic sum of these errors should be very close to zero. However, this is by no means always the case, as demonstrated by J. BAILLAUD (3) with reference to calibration measurements of several graduated circles used in astronomical transit instruments. Moreover, as the mean of the n microscopic readings is taken, the corresponding error only varies in the ratio $\frac{1}{\sqrt{n}}$.

Now, it is very easy to use *with a modern repeating instrument* a method (which we have called *serial repetition* ("*par répétitions fractionnées*") which has the benefit of nearly all the theoretical advantages possessed by the so-called *repetition* method proper.

Here also n *independent* measurements of the angle AB are made, each including pointings with the eyepiece movable hair and the microscope readings, both on signal A and on

---

(*) Note by I. H. B. See : P. TARDI : *Traité de Géodésie* — Fascicule I —Gauthier Villars, Paris, 1934, p. 152.

signal B ; measurement number p consists necessarily of a total reading for A (i.e. reading of the limb plus supplementary microscope reading) very close to that already obtained for the signal B on measurement number (p—1). Averaging the *n* measurements, all of the systematic part of the error relating to intermediary graduations on the limb used and the microscopic readings on such graduations, is eliminated.

In this method, the independence of the individual measurements sometimes required by the precarious or fugitive conditions of visibility, is preserved. The possibility of proceeding by couples is also preserved, that is, by "to and fro" measurements — the only method which allows the elimination of the twist of the support of the instrument, which is frequently quite appreciable on elevated geodetic signals.

These two advantages counterbalance the inconveniences which the method presents in comparison with the so-called *repetition* method proper, namely, a considerably greater number of microscope readings, due to the operation being divided into series, which corresponds to a final increase of errors due to those readings and to a certain loss of time.

If the sum $\Sigma p/$ is obtained from *p* consecutive measurements, the most probable value of the angle is $\Sigma p/p$. There is thus obtained for the angle sought, as the observations succeed each other, values which approach closer and closer that which one will be led to adopt definitely when *p* will have attained a value *n* previously fixed. This grouping of progressively improving results enables the observer to note the moment when supplementary observations no longer change the final result except by negligible quantities. Moreover, if by some unfortunate chance the n^{th} and last repetition terminates on a graduation line showing an exceptionally large error, an abrupt variation in the value of $\Sigma p/p$ is produced. The observer, warned by this abrupt variation, may either stop at the $(n-1)^{th}$ repetition, if the sequence $\Sigma p/p$ is sufficiently regular, or make a $(n+1)^{th}$ repetition so that the error of the faulty division line may be eliminated.

....

(1) See on the same subject : Paper submitted to the *Académie des Sciences* on 20th September 1937. (C. R. t. 205, 1937, p. 514).

(2) If A is a reference common to the different directions of the swing, only the second of these errors will stand out, divided by *n*, in the final result of the observations, recorded as a swing.

(3) Jules BAILLAUD : *Sur les erreurs de graduation des cercles employés dans les opérations géodésiques et sur les méthodes qui permettent d'en diminuer l'influence.* BULLETIN GEODESIQUE n° 42, 1934, p. 38-47.