GRAVITY REDUCTIONS AND THE FIGURE OF THE EARTH.

by

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Summary : The accuracy of the various formulae for deducing the rise of the natural geoid with respect to its reference spheroid from the gravity anomalies is considered. The merits and demerits of the various reductions for reducing observed gravity from ground level to geoidal level for the above purpose are discussed.

INTRODUCTION.

In recent years, gravity observations have been carried out apace, and a considerable amount of literature has been published about the interpretation of the gravity anomalies, and the deduction of the figure of the earth from them. The question of the reduction of the observed gravity from ground level to sea-level started as early as the time of Stokes (1849), but is still a subject of discussion, and Dr. Vening Meinesz (1) in his Lisbon report to the International Union of Geodesy and Geophysics has stressed the need for future research in this matter. In the present communication, the question of the proper reduction for the determination of the undulations of the natural geoid will be considered.

1. Our ultimate object is to find the form of the natural geoid of the Earth, or in other words, its deviations from a reference spheroid. This may be done in two ways:—

a) By a suitable hypothesis, all the masses external to the geoid may be removed. The level surface of the new mass system may be called the corrected geoid. The distance between the natural and corrected geoids is easily calculable from the known mass-transfers. Our problem then reduces to finding the form of a level surface having no attracting masses external to it.

b) This method consists in leaving the actual topography as it is, and finding the undulations N of the natural geoid from Δ g's the gravity anomalies, on it.

We will consider each of these in turn.

2. There are several ways of idealizing the Earth, but for the sake of definiteness, we shall suppose that the values of gravity on the earth have been reduced to the level of the geoid on the hypothesis of Hayford's Isostatic compensation. The level surface of the new masses is now the Compensated geoid, and we want to determine its form. A solution of this problem is embodied in the two famous equations of Stokes, which have been proved in a multiplicity of ways. Taking the geoid as

$$r = a\ (1 + \Sigma\, u_n) \tag{1}$$

and the reference surface as

$$r = a\ (1 - \varepsilon \sin^2 \theta) \tag{2}$$

Stokes proved that

$$\Delta g = G \sum_2^{\infty} (n - 1)\, u_n, \quad N = a \sum_2^{\infty} u_n. \tag{3}$$

He connected the two quantities N & Δ G by a quadrature formula

$$N = \frac{a}{4 \pi G} \iint \Delta g\, F(\psi)\, d\omega \tag{4}$$

the integration being on a unit sphere.

He took the reference surface as a spheroid with small meridional ellipticity, and assumed that the level surface whose form was to be determined differed from this spheroid, and from a sphere of equal volume by quantities of the order $a \varepsilon^2$. It must be noted, that Stokes's reference surface $r = a\,(1 - \varepsilon \sin^2 \Theta)$ is not an exact spheroid, but differs from it by $\frac{3}{2} a \varepsilon^2 \sin^2 \Theta \cos^2 \Theta$, which can amount to 300 ft. in latitude 45°. There is no objection to taking 2) as a reference surface. Stokes's method of deducing his equations (3) and (4) is however open to two objections. One is, that he uses an expression for potential

$$V = \Sigma \frac{Y_n}{r^{n+1}},$$

in terms of spherical harmonics, the convergence of which has been doubted in the region near the boundary of the geoid. The second is, that g is taken $= -\delta V/\delta r$, where r is the radius vector at the point considered. Actually $-\delta V/\delta r = g \cos X$, where X is the angle between the normal and the radius vector at the point considered. We can easily see the approximation involved in this for the case of a spheroid. The error is $g\,X^2/2 = 0\,(g\,\varepsilon^2) = 0$ (10 mgals.), which is considerable. Later work has shown that under certain conditions, equations (3) and (4) hold not only when the reference surface is an exact spheroid, but also for the more general case, when the reference surface is given by

$$r = [1 - \frac{2}{3} \varepsilon P_2 + \sum_2^{\infty} y_n].$$

Thus, if

(5) $$r = a\,[1 - \frac{2}{3} \varepsilon P_2 + \sum_2^{\infty} (y_n + z_n)]$$

is the equation to the geoid, and if

(6) $$r_s = a\,[1 - \frac{2}{3} \varepsilon P_2 + \sum_2^{\infty} y_n)$$

be taken as its reference surface, then provided the reference surface is such that the potential on it has the same value as that on the geoid, we have

(7) $$\Delta g = G \,\Sigma\, (n-1)\, z_n, \qquad N = a\, \Sigma\, z_n.$$

It is important to assess the order of accuracy of these equations. For this purpose, it is more convenient to employ the method outlined by Pizetti (2), which leads to the quadrature formula (4) without the help of the intermediary equation (3).

Let the geoid be

(8) $$r_g = a\,[1 - \varepsilon\, \Sigma\, y_n - \varepsilon_1\, \Sigma\, z_n]$$

and its reference surface

(9) $$r_s = a\,(1 - \varepsilon\, \Sigma\, y_n)$$

(10) $$N = - a\, \varepsilon_1\, \Sigma\, z_n.$$

So far as our present experience goes, it has been found that the geoid, and its reference surface can differ by 200 or 300 feet, and not very much more. Hence, by (10), $a\,\varepsilon_1$ can at the most amount to 300 feet, or $\varepsilon_1 = 1/6 \times 10^4 = 0\,(\varepsilon^2)$. The geoidal meridional ellipticity therefore differs only by quantities of the second order from the ellipticity of the reference surface. If the reference surface is an exact spheroid, then

$$\varepsilon\, \Sigma\, y_n = - \frac{2}{3} \varepsilon \left(1 + \frac{23}{42} \varepsilon\right) P_2 + \frac{12}{35} \varepsilon_2\, P_4.$$

Let the potential on the reference surface be $U = W_0$, and on the geoid $W = U + T = W_0$. T is the potential due to the coating between the two surfaces

$$g = -\left(\frac{\delta W}{\delta n'}\right)_{geoid} = -\left(\frac{\delta U}{\delta n'} + \frac{\delta T}{\delta n'}\right)_{geoid}$$

$$-\left(\frac{\delta U}{\delta n}\right)_{geoid} = -\left(\frac{\delta U}{\delta n}\right)_{spheroid} - N\left(\frac{\delta^2 U}{\delta n^2}\right)_{spheroid} = \gamma_0 + N\left(\frac{\delta \gamma}{\delta n}\right)_0.$$

Taking the elements of normal of the geoid and spheroid to be the same, we get

(11) $$g - \gamma_0 = N\left(\frac{\delta \gamma}{\delta n}\right)_0 - \frac{\delta T}{\delta n} = -\frac{2 N \gamma}{r} - \frac{\delta T}{\delta n} = -\frac{2T}{r} - \frac{\delta T}{\delta n}.$$

If we assume $T = \Sigma\, a^{n+1}\, y_n / r^{n+1}$, we see that $\Delta g = \Sigma\, (n-1)\, y_n / a$ and $N = \Sigma\, y_n / G$ which are identical with equations (3). The following approximations are involved in the proof of equation (11) :

(i) $$g = -\frac{\delta W}{\delta n'} = -\frac{\delta W}{\delta n},$$

where dn', dn denote elements of normal of the geoid and spheroid respectively. If x is the angle between these normals, the error is

$$g\,(1 - \cos x) = g\, x^2/2.$$

Now it can be shown that the angle between the radius vector and the normal at a point of surface (8) is given by

(12) $$\tan\mu \doteq \mu = \frac{\sqrt{\sin^2\theta\left[\varepsilon\left(\frac{\delta}{\delta\theta}\Sigma\, y_n\right)+\varepsilon_1\left(\frac{\delta}{\delta\theta}\Sigma\, z_n\right)\right]^2+\left[\varepsilon\left(\frac{\delta}{\delta\Phi}\Sigma\, y_n\right)+\varepsilon_1\left(\frac{\delta}{\delta\Phi}\Sigma\, z_n\right)\right]^2}}{\sin\theta\,[1+\varepsilon\,\Sigma\, y_n+\varepsilon_1\,\Sigma\, z_n]}$$

Hence $X = \mu - \mu_0$, where μ_0 is obtained from μ by putting $\varepsilon_1 = 0$. On simplifying, we find that x is of the order $\varepsilon_1\,\varepsilon_2$ and therefore the error is of $0\,(g\,\varepsilon_1^2\,\varepsilon_2^2) = 0\,(g\,\varepsilon_1^6)$

(ii) $$\left(\frac{\delta T}{\delta n}\right)_{\text{spheroid}} = \left(\frac{\delta T}{\delta r}\right)_{\text{sphere}}.$$

Error is of

$$0\left(\frac{\delta T}{\delta r}\right)\mu^2 = 0\left(\varepsilon^2.\frac{\delta T}{\delta r}\right) = 0\,(G\,\varepsilon^4).$$

(iii) $$N\frac{\delta\gamma}{\delta n} = -\frac{2\,N\,\gamma}{a}$$

$$N\frac{\delta\gamma}{\delta n} = N\frac{\delta\gamma}{\delta r} + 0\left(N\,\varepsilon^2\frac{\delta\gamma}{\delta r}\right)$$

$$= -\frac{2\,g\,N}{a} + 0\left(\frac{2\,g\,N\,\varepsilon}{a}\right) + 0\left(N\,\varepsilon^2\frac{\delta\gamma}{\delta r}\right)$$

$$= -\frac{2\,g\,N}{a} + 0\left(\frac{2\,g\,a\,\varepsilon^3}{a}\right) + 0\left(a\,\varepsilon^3\frac{\delta\gamma}{\delta r}\right)$$

$$= -\frac{2\,g\,N}{a} + 0\,(g\,\varepsilon^3).$$

(iv) Finally, in deducing this equation, the rotation term has been omitted. For a rotating earth,

$$N\frac{d\gamma}{dr} = -\frac{2\,g\,N}{a} - 2\,\omega^2\,N$$

$$N\,\omega^2 = 200^{ft} \times (7 \times 10^{-5})\ \text{sec}^{-2} = 3 \times 10^{-5}\ \text{cm/sec}^2.$$

This is of the same order of magnitude as $G\,\varepsilon^3$

It can easily be seen that each of the terms of the equation (11) is of $0\,(G\,\varepsilon^2)$, for on the geoid, gravity is

$$g = G\,[1 + A\,P_2 + B\,P_4 + (n-1)\,\Sigma\, u_n],$$

and on the spheroid

$$\gamma_0 = G\,[1 + A\,P_2 + B\,P_4 - \frac{23}{63}\,\varepsilon^2\,P_2 + \frac{36}{35}\,\varepsilon^2\,P_4].$$

Hence

$$g - \gamma_0 = 0\,(G\,\varepsilon^2) = 0\ (10\ \text{mgals}).$$

Hence, in equation (11), only terms of the $0\,(G\,\varepsilon^3)$ have been neglected. The quadrature formula (4) can be deduced directly from (11), and from what has been said above, the formula is well adaptable for the calculation of local geoidal rise.

N and Δg can also be connected together by means of an integral equation.

(13) $$N - \frac{3}{4\pi a}\iint\frac{N\,dS}{r} = \iint\frac{\Delta g\,dS}{G\,r}.$$

With the help of Green's functions, a solution of this is

$$N = \frac{a}{4\pi G}\iint \Delta g\,F(\varphi)\,d\omega$$

which is the same as equation (4). It must be borne in mind, that the above equation cannot be applied to the natural geoid of the earth on account of the protruding attracting masses above the geoid, and also due to the fact, that the values of gravity are not observed on the geoid.

3. Next, let us consider method (b). In this, the mass-distribution of the earth is not interfered with, and the problem is to get a value of the potential at a point inside the attracting masses. This case has been considered by Malikin (3), who deduces the following integral formula for N,

$$(14) \qquad g N - \frac{1}{2\pi}\int\int g N\left(\frac{2}{R} + \frac{\omega^2}{g} - \frac{\cos\theta}{r}\right)\frac{d\omega}{r} = 2 U_e + \frac{1}{2\pi}\int\int \frac{\Delta g_P}{r} d\omega$$

where Ue is potential of external masses A on the geoid, and R is the radius of curvature of the geoid at the point considered.

If Δg_P is known, the formula (14) is remarkably accurate. In deriving this formula, Malikin has used terms of $O(\varepsilon N) \doteqdot 1$ ft. We thus see that theoretically we are equipped with very precise formulae for the determination of N.

4. We are now in a position to answer the question, as to which is the best reduction for reducing gravity from ground level to geoidal level from the point of view of determining the undulations of the natural geoid. Stokes, although he established equation (4), did not make any practical use of it, as the masses outside the natural geoid defeated him. There is no doubt however that he thought that free air anomalies would give a good idea of the geoidal rise. Jeffreys (4) has shown in an elegant way, that although there are masses outside the geoid, still equation (4) is valid to the first order in height of the earth above the geoid, if we use values of gravity reduced to the natural geoid by free-air. He concludes, that these are the only anomalies, which should be used for determining the form of the geoid, and "any attempt to allow for the mass of the upper layers or Isostasy merely introduces irrelevant considerations and inaccuracy." This is a result of fundamental importance and needs amplifying. Jeffreys' treatment does not answer the question, "Why is free-air reduction suitable for Stokes's formula ?" Not by chance surely. The reason is as follows :

Imagine all the topography above the natural geoid to be condensed on the geoid, and let the level surface of the new mass distribution be called the condensed geoid. It can easily be shown that for all practical purposes, so far as N is concerned, the natural and condensed geoids may be considered as identical. By Lambert's tables, the geoidal rise due to a cap 3 km. thick, of 100 km. radius, and density$=$2.8 is 34.8 metres. If the mass of this cap be condensed as a coating, the rise due to it is

$$N = \frac{V}{G} = \frac{3}{2a} \cdot \frac{c h \rho}{\rho_m}\left(1 - \frac{h}{c} + \frac{h^2}{2c^2}\right) = 34 \cdot 2 \text{ metres.}$$

This shows how closely identical, the effects of the actual and condensed topography are, even for the unfavourable case that has been considered. This is due to the fact that N depends more on the actual amount of the attracting mass rather than its configuration. If then we can get Δg's on the condensed geoid, these can be used in equation (4) for getting the rise of the natural geoid, since there are no masses external to the condensed geoid.

Let E be a point on the earth, and A the corresponding point on the geoid. Let the masses inside the geoid be designated by M , and the masses between the geoid and the earth's surface by m.

Before condensation,

g_E =attraction of masses M at E+attraction of masses m at E.

After condensation,

g_A =attraction of masses M at A+attraction of condensed masses m at A.

The condensation reduction is

$g_A - g_E = \frac{2gh}{a} +$ (attraction of condensed masses m at A—attraction of masses m at E)

$$(15) \qquad = \frac{2gh}{a} + \left(\int\int\frac{\sigma\, dS}{r} - \int\int\int\frac{dm}{r}\right)$$

where σ denotes the skin density, and the volume integral extends througout the mass m.

The term in brackets on the right hand side of equation (15) can be evaluated rigorously with the help of Hayford's reduction tables, but for our purpose, we may neglect the curvature of the earth. If the masses m between E and A be regarded as an infinite plateau, this term vanishes, and we have $g_A - g_E + 2gh/a$ which is nothing more than Jeffrey's result, that Δg_F's need only be used. In mountainous areas however, we mav regard the topography above A as an infinite plateau plus undulations. After condensation, the effect of the infinite plateau cancels out and we are left with the so-called "Gelände-Reduction" Δg_R.

Hence a more correct expression for g_A is

$$g_A = g_E + 2 g h/a + \Delta g_R \tag{16}$$

Δg_R is always positive. Its values for some of the typical mountain stations in India are as follows:—

Station	Altitude Feet	(in gals)
Domel	2239	+.015
Hayan	6084	+.028
Sonamarg	9050	+.021
Churawan	8151	+.024
Minmarg	9351	+.023
Wozul Hadur	13921	+.019

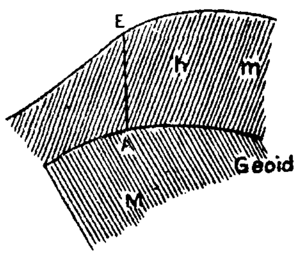

Fig. 1.

Of course, there are some mountain stations for which it is less, but +.020 gals seems a fair average value to take for uneven topography. If then we neglect Δg_R, and obtain N from Δg_F's we are making a systematic error of about 20 mgals in all the mountainous regions. A casual error of this amount in say every degree square will not have much effect on the resulting value of N, but it is not desirable to have such a large systematic error for all the mountainous regions of the globe. These remarks only hold for determining N. If the objective is to determine the ellipticity of the level surface, free-air gravity anomalies can be used without objection.

Yet another way of seeing the propriety of free-air reduction in Stokes's formula is as follows:

The integral equation between N and Δg when there are some masses external to the geoid is obtained by Lambert 5) in the following form:

$$g N - 2 U_e = \frac{1}{2\pi} \iint \frac{\Delta g_P \, d\omega}{r} + \frac{1}{2\pi} \iint \frac{3}{2a} g_N \frac{d\omega}{r} \tag{17}$$

$$= -\frac{6\varrho}{\varrho_m} G \left(h_0 + \frac{h_1}{3}\right) + \frac{1}{4\pi} \iint (a \, \Delta g_P + 3 U_e) F(\psi) \, d\omega$$

where U_e is the potential due to masses between the geoid and the earth. If the topography be condensed as a coating of surface density on σ the geoid, then

$$U_e = \iint \frac{\sigma \, d\omega}{r} + \Delta U_e,$$

where ΔU_e is the change in potential due to condensation reduction.

(17) may therefore be written as

$$g N - 2 \Delta U_e = \frac{1}{2\pi} \iint \left(\frac{\Delta g_P + 4\pi\sigma}{r}\right) d\omega + \iint \frac{3}{4\pi a} \frac{g_N}{r} d\omega \tag{18}$$

$$= \frac{1}{2\pi} \iint \frac{\Delta g_F}{r} d\omega + \frac{3}{4\pi a} \iint \frac{g_N}{r} d\omega.$$

If we neglect ΔU_e, we can easily get the usual quadrature formula from the above equation. Hence free-air anomalies can be used in equation (4).

The above is tantamount to saying that for determination of N for practical purposes, it is simplest to use condensation reduction. Instead of condensation reduction, we may use Hayford's Isostatic reduction, and deduce the rise of the compensated geoid from the Hayford anomalies Δg_H's. To get N of the natural geoid, one extra step is involved. The mass-transfer implied in Isostatic reduction causes a considerable deformation u of the natural geoid. This has to be computed and added to the rise deduced from formula (4). u can be obtained conveniently from the tables published in special Publication N° 199 of U.S.C. & G.S.

The question now arises "Which is a better method for determining N, the Condensation or Isostatic?" From the nature of formula (4) we see that Δg's over the whole globe are needed for computing N at each station. For practical computations, the earth is divided into a number of elementary areas and a mean value of Δg is estimated for each area by interpolation and extrapolation from Δg's at observed stations. Hence for practical purposes that reduction is the best, which enables an average value for each elementary area to be obtained more correctly. In other words, that reduction is preferable which gives anomalies which are tractable to interpolation.

Now $\Delta g_c = \Delta g_F$ in flat terrain, and $= \Delta g_F + \Delta g_R$ in mountainous country. Δg_F's and Δg_H's for some squares of $2^\circ \times 2^\circ$ extent in latitude and longitude in India are exhibited in the following table. The squares are taken at random amongst the plains.

Square	Station	Δg_F	Δg_H
(10° — 12°Φ)	297	—22	—43
(76 — 78 λ)	298	+43	—51
	299	+26	—72
	300	+14	—68
	303	—37	—67
	304	—34	—65
	305	—31	—67
	306	—14	—63
	307	—35	—67
	301	+ 1	—35
		Range 80	37
(12° — 14°Φ)	177	—22	—45
(78 — 80λ)	199	—57	—73
	178	—24	—42
	42	+64	+ 9
	198	+ 2	—28
		Range 121	82
(22° — 24°Φ)	61	+ 1	+ 7
(80 — 82 λ)	62	+27	+29
	63	+21	+ 8
	68	+28	+30
(26° — 28°Φ)	213	+19	+25
(74 — 76 λ)	214	—18	—16
	215	+71	+62
	224	+49	+44
(20° — 22°Φ)	65	— 2	— 3
(80 — 82 λ)	66	— 4	— 3
	238	— 1	— 8
(26° — 28°Φ)	103	—19	— 5
(76 — 78 λ)	104	— 1	+17
	105	— 4	+15
	106	— 9	+11
	107	—28	— 8
(26° — 28°Φ)	119	—35	—13
(78 — 80 λ)	120	—52	—29
	122	—73	—39
	263	—58	—26

Square	Station	Δg_F	Δg_H
(24° — 26°Φ) (72 — 74 λ)	114	+43	+47
	115	+114	+29
	117	+16	+23
	118	+10	+20
	364	+14	+24
	365	+49	+49
(24° — 26°Φ) (76 — 78 λ)	98	+26	+19
	99	+50	+39
	102	+38	+29
	223	—30	—21
(24° — 26°Φ) (78 — 80 λ)	95	— 5	— 2
	96	+26	+26
	100	+ 7	+14
(22° — 24°Φ) (72 — 74 λ)	111	—15	— 8
	112	+31	+36
	349	+ 9	+12
(22° — 24°Φ) (74 — 76 λ)	47	— 2	—11
	48	+ 9	—15
	49	—28	—19
	217	+27	+13
	348	+18	+11
	347	— 5	+ 5

Hirvonen (6) has estimated, that if Δg is known to ± 20 mgals for every 1° square, we should be able to get N accurately enough to about 8 feet. The above figures show, that while Δg_H's are on the whole more regular, Δg_F's in plain areas are not too bad, and may be interpolated to ± 20 mgals. The situation is rather different for mountainous areas. Table II shows the results for some Himālayan stations in India.

TABLE II.

Station	Height Feet	Δg_F cm./sec²	Δg_H cm./sec²
Murree	6885	+.032	—.025
Domel	2239	—.167	—.048
Shādipur	5193	—.116	—.030
Gandarbal	5200	—.094	+.010
Hayan	6084	—.105	+.017
Sonāmarg	9050	—.013	+.043
Churawan	8151	—.056	+.032
Minmarg	9351	—.033	+.035
Deosai I	13311	+.146	+.090
Deosai II	12805	+.094	+.062
Deosai III	12391	+.111	+.095
Lālpur	5633	—.045	+.017
Srīnagar	5198	—.070	+.021
Pingalan	5227	—.073	+.012
Yūs Maidan	7867	+.024	+.008
Korag	10952	+.149	+.034
Tosh Maidān	10315	+.135	+.050
		Range 316	143

The Δg_F's are ragged, and Δg_H's are much smaller and smoother. If there were no compensation then one can see in a common-sense way, that this would be the case, and that Δg_F would be very unrepresentative. Thus, at a point D on the top of a hill about 1 mile high, gravity would be about 0.17 gals too high on account of the extra mass under it. It would not be possible to obtain the value at C by interpolating it from A and B, if one only uses free-air reduction. If on the top of this, gelände-reduction Δg_R is added to the value at D, the error of interpolation from A and B will be still more enhanced. This argument is however too simple, and in actual practice, Δg_F's at mountainous stations are by no means positive, as Table II shows, indicating compensation of some sort.

Yet another instance about the interpolation of Δg's in mountainous areas is afforded by considering the 9 Indian gravity stations along the meridian of 88°30'.

The raggedness of Δg_F's is apparent.

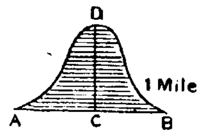

Fig. 2

To get mean free-air anomaly over a 1° square to an accuracy of

Station	Height Feet	Δg_F	Δg_H
Chātra	64	— 14	+ 5
Kisnapur	113	+ 12	+39
Rāmchāndpur	132	— 19	+21
Kesarbāri	204	— 60	+ 3
Silīguri	387	—149	—39
Kurseong	4913	0	+10
Darjeeling	6966	+ 55	+32
Sandakphu	11766	+189	+48
		Range 338	87

about ±20 mgals in mountainous areas, one requires a station every 10 miles or so, which amounts to a density of about 50 per degree square. This density would give the desired result, if one deliberately spaces these stations equally or else observes truly at random. It would not do to put the stations all in valleys or in accessible places. This in an impracticable programme in mountainous areas.

The conclusion therefore is, that for determining N of natural geoid, free-air anomalies should be used for plane areas. For mountainous areas Δg_H's should first be computed for the stations of observation. From these, mean Δg_F's for the middle points of the elementary areas should be obtained by interpolation and extrapolation. At such places, effect of topography and compensation should be computed and added on the mean Δg_H to get the mean Δg_F.

The above argument assumes a close mesh of gravity stations on the globe, which is a desideratum at the present moment. If one however wants to arrive at the order of magnitude of N with the meagre gravity data now available, one has to resort to highly precarious interpolations and extrapolations and Isostatic anomalies are much to be preferred to the free-air ones.

It might be remarked, that the condensation method is only an artifice, which enables us to get rid of the masses external to the geoid, thus enabling us to make use of formula (4). If we knew the actual law of variation of density in the earth's crust, that hypothesis would be the best one to use for getting rid of these masses. In suggesting the Hayford's Isostatic hypothesis as a via media for obtaining the rise of the natural geoid, it is by no means implied, that it is true to nature. It assumes point to point compensation, implying that the crust offers no resistance to deformation, which cannot be true. Seismological evidence provides a direct contradiction to this hypothesis. More rational hypotheses of regional compensation have been brought forward, but from a consideration of the gravity anomalies in the Himālayan stations of India, the writer has shown 7), that departures in nature from any form of isostasy are much greater than the differences between the various systems. In spite of what some people might claim for the Hayford hypothesis, one must get reconciled to the fact that in India, geodetic work has revealed mass anomalies from this hypothesis which are equivalent to a thickness of 2000 to 3000 feet of rock. Bullard's work in E. Africa shows also abnormalities of the same extent. The choice of Hayford reduction from amongst the many that are available is mainly governed by practical convenience.

At this stage, it is proper to indicate, which height is to be used for the ordinary height correction, the geoidal or the spheroidal. So far as the deduction of the undulations of the natural geoid from the free-air anomalies Δg_F's is concerned, it is obvious that observed values of gravity g on the earth need only be corrected for the geoidal height of the stations. When the earth is idealized, and the undulations of the corrected geoid are required, g is to be brought on to the corrected geoid. In particular, when Hayford reduction is used, and we want to find the rise of the compensated geoid, observed gravity on the earth has to be reduced from the earth to the natural geoid, and from the natural geoid to the compensated geoid. The latter reduction is termed Bowie reduction. Ordinarily the anomalies $\Delta g_H = g_H - \gamma_0$ are used also as a measure of the subterranean mass-anomalies, although g_H and γ_0 refer to two different surfaces. This is possible only, because we know that the deviations of compensated geoid from its reference spheroid are small. If however N between compensated geoid and its reference spheroid can amount to 1000 m. as some people still affirm, then $(g_H - \gamma_0)$ will be useless for the determination of the mass-anomalies.

In conclusion we will consider another point of view about the determination of N which has been put forward by Hopfner in various articles. He vigorously denounces all other reductions except Prey's, and asserts that this is the only reduction, that can be used for determining the geoidal rise. Prey's reduction at first sight has indeed much to commend it. It does not involve any displacement of masses, and gives true values of the gravity anomaly on the geoid.

The correct formula to be used for getting the undulations from Prey's anomalies is (14). We see that as in the case of no external masses, a knowledge of the distribution of density inside the geoid is not required. But to get U_e and Δg_P, it is essential to know the precise arrangements of masses external to the geoid. Hence if there are masses inside and outside a level surface, its from cannot be determined from a knowledge of the values of gravity on it alone. It is essential to know the external masses. The situation is therefore precisely the same as when there are no external masses.

The rigid computation of Prey's anomaly is by no means less troublesome or less inaccurate than Hayford's anomaly, and there is no particular advantage in using it for the computation of N. It might however be put to the following two uses :

If S is the natural geoid, and g_P the value of gravity on it (due to actual topography), then $\iint g_P \, dS = 4\pi M$, where M is the sum of masses inside the geoid.

Again if V_1 denotes the potential due to internal masses, then

$$V_1 = \frac{1}{4\pi}\iint g_P \frac{dS}{r} + \frac{\omega^2}{2\pi}\iiint \frac{d\tau}{r}.$$

Hence, if g_P is known, we can obtain the total masses inside the geoid, as well as their potential without knowing the internal law of density. From the point of view of the geoidal rise, this reduction has received exaggerated importance at Dr. Hopfner's hands.

Hopfner 8) also derives a very simple formula for the geoidal rise namely.

$$N = \frac{2a}{3G}\Delta g_P,$$

from which he claims one can get the geoidal rise without resorting to any *ad hoc* hypotheses like the Isostatic one. Jung 9) has shown that this formula is inadmissible, as it takes no count of masses surrounding the station.

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