CLASSIFICATION OF TIDES IN FOUR TYPES

by

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(Extract from « Conférences sur les Marées » issued by the Service Hydrographique de la Marine Française, Paris, 1938).

(Translated from the French).

1.— The tabulation of the tide harmonic constants for the various ports in the world shows a very large diversity in the values of amplitudes and phase lags of the different component tides. As a result, a classification of the tides presents only a conventional character.

Van der Stok (Tides and Tidal Streams in the Indian Archipelago, Batavia 1897) gives a classification of the tides in several types based on the value of the coefficient $\frac{K_1+O_1}{M_2+S_2}$ representing the ratio between the semi-amplitude of the diurnal tide at the time of the diurnal springs (tropics of the moon) and the semi-amplitude of the semi-diurnal tide at the time of the semi-diurnal springs (Syzygy).

The values as considered by Van der Stock are as follows : 0, 0,25, 1,5.∞ By adding the term 3, the following series is obtained :

$$\frac{K_1 + O_1}{M_2 + S_2} : \quad 0 \quad 0{,}25 \quad 1{,}5 \quad 3 \quad \infty$$

This series permits the definition of four types of tide possessing the following approximate characteristics, based on the number and the greater or less regularity of high waters and low waters during the same day.

1° type : $0 < \frac{K+ O_1}{M_2 + S_2} < 0{,}25$	Ports with regular *semi-diurnal tides.* 2 H W and 2 L W each day, the heights of which are very similar.
2° type : $0{,}25 < \frac{K_1 + O_1}{M_2 + S_2} < 1{,}5$	Ports with diurnal *inequalities.* 2 H W and 2 L W each day of unequal heights.
3° type : $1{,}5 < \frac{K_1 + O_1}{M_2 + S_2} < 3$	*Ports with mixed tides* showing sometimes 2 H W and 2 L W and sometimes a single H W and a single L W per day.
4° type : $3 < \frac{K_1 + O_1}{M_2 + S_2}$	*Ports showing diurnal tides.* 1 H W and 1 L W per day.

2. *Proof of the above classification.*

Let us prove the approximate characteristics as indicated above.

If we substitute in the ratio $\frac{K_1 + O_1}{M_2 + S_2}$ the most usual value $S_2 = \frac{M_2}{3}$ instead of the value S_2, the following is obtained ;

$$\frac{K_1 + O_1}{M_2 + S_2} = \frac{3}{4}\,\frac{K_1 + O_1}{M_2}$$

As a result, the assumed classification corresponds in its general lines to that obtained by the consideration of the coefficient $\frac{K_1 + O_1}{M_2}$ together with the following series :

$$\frac{K_1 + O_1}{M_2} : \quad 0 \quad \frac{1}{3} \quad 2 \quad 4 \quad \infty$$

Now, it is known that the tide resulting from the three components M_2 K_1 O_1 can be written in the following approximate form :

$$(175) \quad y = A_0 + M_2 \cos (2\,AH - K_{M_2}) + (K_1 + O_1) \sin S \cos \left(AH - \frac{K_{K_1} + K_{O_1}}{2}\right)$$

$$\text{with } S = s - \frac{K_{K_1} - K_{O_1}}{2}$$

This expression is obtained, assuming that the components K_1 and O_1 have nearly the same amplitude ($K_1 = O_1$) ; besides, it is strictly correct whatever may be the values of K_1 and O_1 at the time of the spring tides of the diurnal group (K_1 O_1).

In order to find the maxima and minima of these expressions in terms of the variable * AH, we shall make use of the "astroïd" or "hypocycloïd" with four points of inflection. (A. Courtier. Note sur la Prédiction des marées par le calcul à l'aide de la formule harmonique, Annales Hydrographiques 1908-1909), as generated by a straight line of length 4 based on the axes of the co-ordinates.

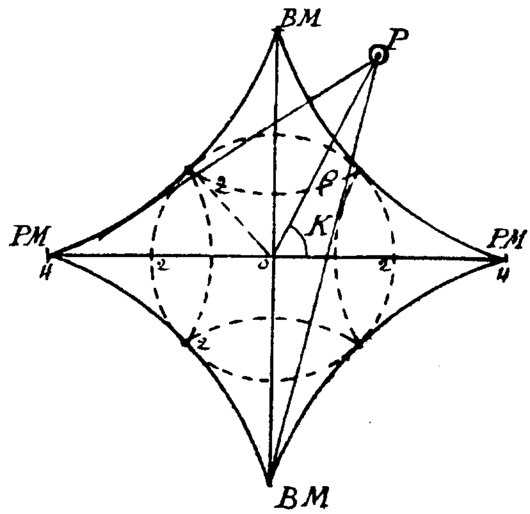

FIG. 77.

Let us make the point P of the polar co-ordinates ρ and K, such that :

(176) $$\begin{cases} \rho = \dfrac{K_1 + O_1}{M_2} \sin S \\ K = \dfrac{K_{M_2} - K_{K_1} - K_{O_1} + n.\ 720^\circ}{2} \end{cases}$$

and selecting $n = 0, 1, 2...$ so that K be a positive quantity comprised between 0° and 360°.

The maxima and minima of the tide formula (175) are obtained by drawing through point P the tangents to the astroïd ; those directed towards the lateral horns corresponding to the high waters and those directed towards the upper and lower horns corresponding to the low waters. The examination of the figure shows that :

when $\rho < 2$ — 2 H W and L W always occur.

$2 < \rho < 4$ — 1 H W and 1 L W or 2 H W and 2 L W obtain according to the different conditions.

$\rho < 4$ — only 1 H W and 1 L W occur.

The classification adopted is therefore justified at the time of the diurnal springs ($S = 90°$).

Let us now consider the case of tides occurring at any time.

We have just seen that with $\rho < 2$, that is to say when $\dfrac{K_1 + O_1}{M_2} < \dfrac{2}{\sin S}$, 2 H W and 2 L W always occur ; it is all the more so when $\dfrac{K_1 + O_1}{M_2} < 2$.

Consequently, the characteristics as indicated for the two first types of tides are justified at any period of the lunation.

We have yet to prove them with regard to the 3° and 4° types.

Let us suppose that $\dfrac{K_1 + O_1}{M_2} = 4$

* The symbol AH in the formula represents the hour angle HA.

a) When S equals 90° (diurnal springs) so that $\frac{K_1 + O_1}{M_2} \sin S = 4$ from which $\rho = 4$ is deduced, there is a single high water and a single low water in the port under consideration.

b) When S is comprised between 45° and 90°, that is during the half of the time of the revolution of the moon along its orbit, the following is valid : $\frac{K_1 + O_1}{M_2} \sin S > 4 \times 0{,}707$ that is $\rho > 2{,}83$.

It is easy to verify by means of the astroïd that under the present conditions, the likelihood of the occurrence of 2 high waters and 2 low waters is only 2,5 %.

c) When S is comprised between 30° and 45°, ρ is comprised between 2 and 2,83, and there is a probability of about 50 % to register 2 high waters and 2 low waters.

In consequence, the following result is obtained when $\frac{K_1 + O_1}{M_2}$ is equal to 4 ; diurnal tides are always predominant during the lunation and the semi-diurnal tides are only noticeable when the moon is in the vicinity of the Equator, that is when the tide is in general very small.

REMARK. — From the use of the astroïd it is seen that in the case when 2 high waters and 2 low waters occur, the heights of the two high waters are equal as long as point P is situated on the line o y, i.e. $2\ K = (2\ n + 1)\ 180°$; the heights of the two low waters are equal as long as point P is situated on o x, i.e. : $2K = 2\ n\ 180°$.

It is indeed easy to prove that the height of the tide (above the mean level) is proportionate to the distance separating point P from the inner astroïd forming the evolute of the first (fig. 77).

3. *Study of the four types of tides.*

Let us examine in succession the four types of tides as indicated above and describe, briefly, the properties of each one and its basic formulae, when considering only the principal components of the harmonic development. It is assumed that the results obtained have only an approximate character. They provide only average indications which, in general, are sufficient for practical purposes. When greater accuracy is required, a detailed study of each port must be made, based on the consideration of the numerical value of all the component tides.

1° TYPE : *Ports with regular semi-diurnal tides.*

$$\frac{K_1 + O_1}{M_2 + S_2} < 0.25$$

In these ports, two high waters and two low waters occur each day ; the heights of the high waters and low waters in the same day are nearly equal and the same thing applies to the heights of the low waters.

High water follows the meridian transit of the moon with an almost constant interval (establishment), the mean value of which is given by the following formula :

$$E_m = \frac{K_{m2}}{28°.98} \text{ hours}$$

The time of high water loses 50 minutes 47 on an average from one day to another ; this retardation decreases at springs and increases at neaps.

Spring tides occur after Syzygias (full and new moons) after an interval of time equal to $T_2 = \frac{K_{S2} - K_{M2}}{1{,}016}$ hours (age of the tide). Neap tides occur after the quadrature of the same amount.

The mean amplitude of the tide is $2\ (M_2 + S_2)$ at springs and $2\ (M_2 - S_2)$ at neaps.

In the preceding chapter (not reproduced here) relative to the 12th Lecture, are given formulae for the accurate numerical values of the usual characteristics of the tide, which are : mean amplitude, amplitude at springs, amplitude at neaps, etc. Establishment, taking into consideration all the constituents. (*)

(*) These formulae, not reproduced here, are taken from the Manual of Tides, by Rollin A. Harris, page 140.

2° TYPE : *Ports with diurnal inequalities.*

$$0,25 < \frac{K_1 + O_1}{M_2 + S_2} < 1,5$$

In these ports, 2 high waters and 2 low waters per day generally occur ; however, the heights show great inequalities due to the presence of the diurnal tide and the same thing occurs concerning the intervals of time between high waters and low waters.

The inequalities in the heights for each day are at maximum when the diurnal tide is at maximum, that is to say $T_1 = \frac{K_{K_1} - K_{O_1}}{1,098}$ hours after the transit of the Moon through its tropics (maxima and minima of the declination).

The minimum occurs after a certain interval of time T, after the transit of the Moon through the Equator.

The inequality in the heights between the high waters of a same day, is given approximatively by the following formulae :

$$(177) \quad \begin{cases} \text{High Waters} \quad (H\,W\,Q) = 2\,(K_1 + O_1)\ \sin\left(s - \frac{K_{K_1} - K_{O_1}}{2}\right) \cos K \\ \text{Low Waters} \quad (L\,W\,Q) = 2\,(K_1 + O_1)\ \sin\left(s - \frac{K_{K_1} - K_{O_1}}{2}\right) \sin K \end{cases}$$

Spring tides occur at Syzygy, when the tides M_2 and S_2 are in conjunction, and neap tides occur at the time of the quadratures. The mean heights of the tide at these times are then approximately

$$(178) \quad \text{Spr.} \begin{cases} \text{Higher H W.} = A_o + M_2 + S_2 + 0,707\,(K_1 + O_1)\ /\cos K/ \\ \text{Lower L. W.} = A_o - M_2 - S_2 - 0,707\,(K_1 + O_1)\ /\sin K/ \end{cases}$$

/cos K/ and /sin K/ representing the absolute values of cos K and sin K.

Sequence of the tide.

In a given port, the daily curve representing the tide has a form which remains almost always the same at the time of the Moon's tropics (maximum of the diurnal inequality), and even at any other time : this is called the sequence of the tide (*Séquence de la Marée*) ; four different forms of sequence occur, depending on the value taken by the quantity

$$(179) \qquad K = \frac{K_{M_2} - K_{K_1} - K_{O_1} + n\ 720°}{2} \quad n = 0,1,\ldots \quad K > 0$$

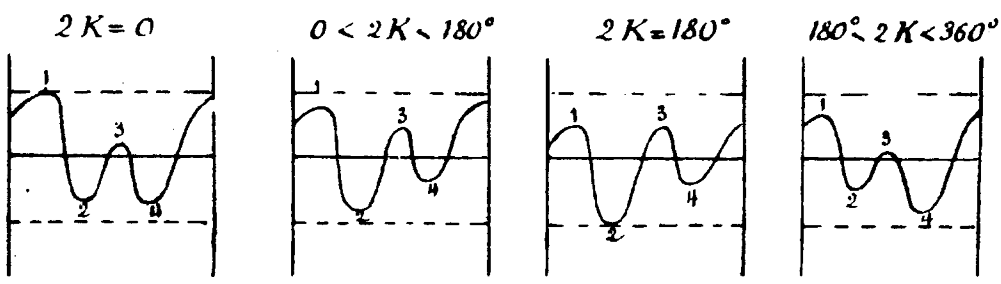

Fig. 78

Let us seek to prove the rule indicated above, assuming that the tide is limited to the components M_2 K_1 O_1 ..... we have seen that the height has very nearly the following expression at the time of lunar tropics (and at any other time in the case where the components K_1 and O_1, have approximately the same value) :

$$(179) \qquad y = A_o + M_2 \cos(2\,AH - K_{M_2}) + (K_1 + O_1)\ \sin\left(s - \frac{K_{K_1} - K_{O_1}}{2}\right) \cos\left(AH - \frac{K_{K_1} + K_{O_1}}{2}\right)$$

the latter can be written in the following way :

(180) $y = A_0 + M_2 \cos 2X + R_1 \cos (X + K)$

by putting $X = AH - \frac{K_{M_2}}{2}$

$$R_1 = (K_1 + O_1) \sin \left(s - \frac{K_{K_1} - K_{O_1}}{2} \right)$$

$$K = \frac{K_{M_2} - K_{K_1} - K_{O_1} + n.\,720}{2}$$

$n = 0,1.$

Let us select n in such a way that K is always positive and lies between 0° and 360°. A glance at the astroïd shows at once the result indicated above.

However, using a different method, and limiting ourselves to the consideration of the case when the diurnal tide is small (R_1 small), we will solve equation (180) by means of approximations. We thus obtain for the two high waters and two low waters of the same day, the following formulae :

(181)

1st H W	$X = -\frac{R_1}{4 R_2} \frac{1}{\sin 1^\circ} \sin K$	$Y = A_0 + R_2 + R_1 \cos K$
1st L W	$X = \frac{\pi}{2} + \frac{R_1}{4 R_2} \frac{1}{\sin 1^\circ} \cos K$	$Y = A_0 - R_2 - R_1 \sin K$
2nd H W	$X = \pi + \frac{R_1}{4 R_2} \frac{1}{\sin 1^\circ} \sin K$	$Y = A_0 + R_2 - R_1 \cos K$
2nd L W	$X = \frac{3\pi}{2} - \frac{R_1}{4 R_2} \frac{1}{\sin 1^\circ} \cos K.$	$Y = A_0 - R_2 + R_1 \sin K$

The above expressions enable us to draw the curve of the tides for the various values of K and to verify the results indicated concerning the *sequence*.

Let us note, in particular, that :
when Cos K = 0 i.e. 2 K = 180° the 2 high waters are equal ;
when sin K = 0 i.e. 2 K = 0 the 2 low waters are equal.

In the first case, the interval between the two low waters is 12h.25m ; in the second case, the two high waters occur at an interval of 12h.25m.

The presence in R_1 of the factor $\sin \left(s - \frac{K_{K_1} - K_{O_1}}{2} \right)$

gives as a result a variation of the inequalities in the heights and the times during the lunation, according to whether the declination of the Moon is North or South. This results from the fact that the term $\frac{K_{K_1} - K_{O_1}}{2}$ which represents the age of the lunar tide being in general very small, the following approximate formula is obtained :

$$\sin \left(s - \frac{K_{K_1} - K_{O_1}}{2} \right) = \frac{\sin D}{\sin I} \quad (1)$$

Formulae (181) relating to the heights Y permit the determination of the above expressions relating to the inequalities in the heights between high waters and low waters, and the heights of the mean spring tides.

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(1) Roughly :

$$\text{Sin } D = \sin I \,[s - \xi + 2 e \sin (s - p)]$$

i.e. roughly :

$$\text{Sin } s = \frac{\sin D}{\sin I}$$

in which formula I represents the maximum value of D during the month.

Establishment of high waters and low waters.

The times of the high waters are given by the following formula :

$$AH = \frac{K_{M2}}{2} + X$$

in which X equals either α or $\pi - \alpha$, α being the small quantity $-\frac{R_1}{4 R_2} \frac{1}{\text{Sin } 1^\circ} \sin K$

It follows, therefore, that the high waters follow the meridian transit of the Moon after an interval E_m which is practically constant:—

$$E_m = \frac{K_{M2}}{q_{M2}}$$

However, the interval differs according to whether one considers the higher high water or lower high water of the day, each one having its own establishment. In order to avoid ambiguities, it is advisable to refer all the establishments to the *upper transit* of the Moon and to reckon them from Oh. to 24h.50m. ; besides, it must be admitted that the establishment corresponds to the case where the Moon's declination is North, and this establishment should be increased by 12h.25m. when the declination is South.

(The U.S.A. tide-tables have adopted another convention. The establishments are reckoned from Oh. to 12h.25m., and they refer sometimes to the upper meridian and sometimes to the lower meridian).

The same remarks apply to the establishment of higher and lower low waters.

The approximate mean values of the establishments and heights of the higher high water and the lower low water are given in the following tabulation :

	Pleine mer supérieure		Basse mer inférieure	
	$E = \frac{K_{M_2}}{q_{M_2}}$	Hauteur $A_0 + M_2$	$E = \frac{K_{M_2}}{q_{M_2}} + 6^h 12^m 5$	Hauteur $A_0 - M_2$
$0 < K < 90$	$-1^h \frac{K_1+O_1}{M_2} /\sin S/ /\sin K/$	$+(K_1+O_1) /\sin S/ /\cos K/$	$+1^h \frac{K_1+O_1}{M_2} /\sin S/ /\cos K/$	$-(K_1+O_1) /\sin S/ /\sin K/$
$90 < K < 180$	$+12^h 25^m +$ — d° —	— — d° —	+ — d° —	— — d° —
$180 < K < 270$	$+12^h 25^m +$ — d° —	— — d° —	$-12^h 25^m$ — — d° —	+ — d° —
$270 < K < 360$	— — d° —	+ — d° —	$-12^h 25^m$ — — d° —	+ — d° —

In the above table, the terms /sin S/, /cos S/ represent the mean absolute values of the terms sin S and cos S, when S varies from O to π ; i.e. 0.707. The factor 1 h. appearing in the corrections of the establishments, represents the coefficient $\frac{1}{2\, q_{M2}} \quad \frac{1}{\sin 1^\circ}$ hours. The term 1 h. /sin S/ may be replaced by the value 42 minutes.

Tides at different times.

The mean tide at Syzygias, or more correctly when the constituents M_2 and S_2 are coincident (spring tides) is given by the approximate formula :

$$(M_2 + S_2)\cos(2t - K_{M2}) + 0{,}65\,(K_1 + O_1)\cos\left(t - \frac{K_{K1} - K_{O1}}{2}\right)$$

in which 0,65 (K_1+O_1) represents the mean semi-amplitude of the diurnal tide during the lunation. (1)

(1) We assume here 0,65 (K_1+O_1) in accordance with the formula given in the 12th Lecture. The expression (K_1+O_1) /sin S/ as indicated above had induced us to use 0,707 (K_1+O_1) in formula (178).

At springs, the higher high waters occur almost at the same time ; the same thing occurs with regard to lower low waters.

The *average tide during the moon's tropics,* and more exactly when the components K_1 and O_1 are coincident (maximum of diurnal inequalities) is given by the following formula :

$$(183) \qquad \left[M_2 + 0{,}27\,\frac{S_2^2}{M_2} - K_2\right] \cos\,(2\,AH - K_{M_2}) + 1{,}02\,(K_1 + O_1)\cos\left(AH - \frac{K_{K_1} - K_{O_1}}{2}\right)$$

By drawing a curve represented by the above expression, the so-called large mean tropical amplitude G_c, the small mean amplitude, S_c, the mean inequalities in heights at the monthly maxima of the diurnal inequalities (H W Q) (L W Q), are obtained.

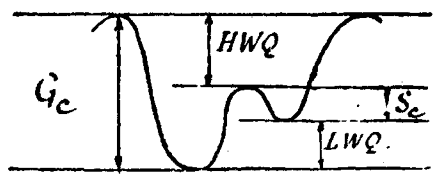

Fig. 79

The *mean tides for the year* correspond approximately to the following formula :

$$(184) \qquad M_2 \cos\,(2\,AH - K_{M_2}) + 0{,}65\,(K_1 + O_1)\cos\left(AH - \frac{K_{K_1} - K_{O_1}}{2}\right)$$

By drawing the curve represented by the above formula, the quantity Gt (great diurnal range) is obtained, the latter representing the mean difference between higher high water and lower low water for the various days of the year.

Lastly, let us note that in the ports of the 2° type, the higher high water occurs roughly in the morning during six months of the year, and in the evening during the other six months ; the lower low water shows the same peculiarity. This is due to the fact that the time of the higher high water lagging by 50 minutes on an average from one day to another, jumps suddenly 12h., as indicated above, at the time when the Moon transits through the Equator (diurnal neaps). Consequently, its repetition occurs nearly at the same hour of the 12-hour period. Taking into account, furthermore, the fact that the time of the higher high water at diurnal springs, primes by 2 hours each month and by 12 hours in six months, the result is that if the spring higher high water occurs in the morning during a certain period, it will occur in the evening six months later.

3° TYPE : *Ports with mixed tides.*

$$1{,}5 < \frac{K_1 + O_1}{M_2 + S_2} < 3$$

In these ports, sometimes a single high water and a single low water per day are observed during the month (at the Moon's tropics), and sometimes 2 high waters and 2 low waters per day (when the Moon is in the vicinity of the Equator).

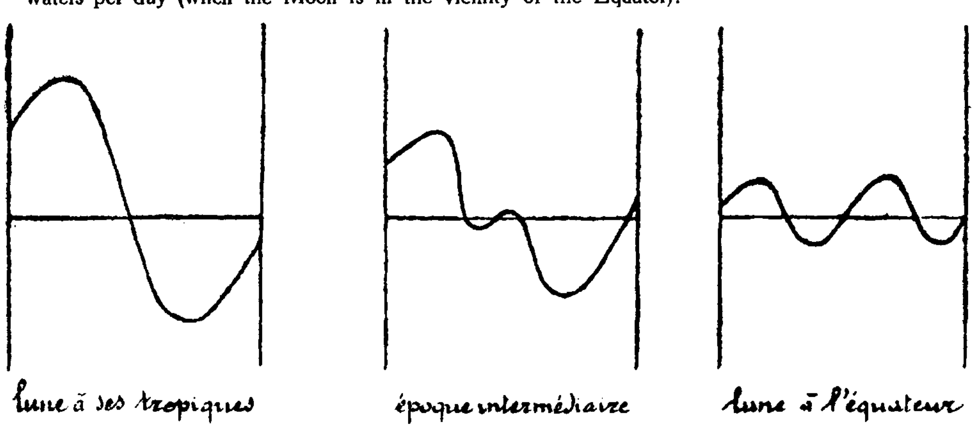

Fig. 80

In the ports of this type, the idea of *Establishment of the Port* has no definite meaning.

The large tides in the month occur at diurnal springs, that is at the time when the Moon has just crossed its tropics and when the constituents K_1 and O_1 coincide.

The mean heights of the high water and low water are then as follows :

(185) Springs
$$\begin{cases} H\ W = A_o + K_1 + O_1 + M_2 \cos \ (\kappa_{M_2} - \kappa_{K_1} - \kappa_{O_1}) \\ L\ W = A_o - K_1 - O_1 + M_2 \cos \ (\kappa_{M_2} - \kappa_{K_1} - \kappa_{O_1}) \end{cases}$$

At the time of these *tropical springs*, the time of the high water is given approximately by the formula :

$$t = \kappa_{K_1} - h + \frac{\pi}{2} + \frac{2}{\sin 1^\circ} \frac{M_2}{K_1 + O_1} \sin 2\,K \qquad (186)$$

in which t is reckoned from mean noon ; it is easy to see that at that time, the high water follows the upper meridian transit of the moon by a practically constant quantity equal to:

$$\frac{\kappa_{K_1} + \kappa_{O_1}}{q_{M_2}} + 475 \text{ minutes} \frac{M_2}{K_1 + O_1} \frac{\sin(\kappa_{M_2} - \kappa_{K_1} - \kappa_{O_1})}{/ \sin s /} + n.\ 12^h\ 25^m \qquad (187)$$

$$n = 0,1 \quad (1)$$

With regard to low water, the interval is obtained by adding 12h.25m. to the above expression and by changing the sign of the above 2nd term.

For the proof of the above formulae, which are only roughly approximate by reason of the existence of the neglected components, the tide at diurnal springs is written as follows :

$$y = A_o + (K_1 + O_1) \cos\left(AH - \frac{K_1 + O_1}{2} + n\,\pi\right) + M_2 \cos(2\,AH - \kappa_{M_2})$$

from which the following is deduced for the time of high water :

$$AH = \frac{\kappa_{K_1} + \kappa_{O_1}}{2} - \frac{2\,M_2}{K_1 + O_1} \frac{1^h}{\sin 1^\circ} \sin(\kappa_{K_1} + \kappa_{O_1} - \kappa_{M_2}) + n\,\pi$$

and also the following is deduced for the interval between the time of high water and that of the meridian transit of the Moon :

$$\frac{\kappa_{K_1} + \kappa_{O_1}}{q_{M_2}} + \frac{4\,M_2}{q_{M_2}(K_1 + O_1)} \times \frac{1^h}{\sin 1^\circ} \sin(\kappa_{M_2} - \kappa_{K_1} - \kappa_{O_1}) + n \times 12^h\ 25^m.$$

The term $\frac{4}{q_{M_2}} \times \frac{1^h}{\sin 1^\circ}$ expressed in minutes of time, equals $\frac{4}{29} \times 60 \times 57{,}3 = 475$ minutes.

The time of the high water springs primes by 2 hours every month ; consequently, if it occurs in the morning during a certain period of the year, it occurs during the evening six months later.

Generally speaking, in the case of ports showing mixed tides, the diurnal type prevails in summer and in winter (when P_1 is added to K_1 and O_1) and the semi-diurnal type becomes noticeable in the spring and the autumn (when K_2 is added to M_2 and S_2).

4° TYPE : *Ports having diurnal tides.*

$$3 < \frac{K_1 + O_1}{M_2 + S_2}$$

In these ports, a single high water and a single low water per day is usually observed, except at the times when the Moon having crossed the Equator, the diurnal tide has a tendency to disappear and the semi-diurnal tide becomes preponderant ; then two small high waters and two small low waters occur.

In ports where diurnal tides prevail, the high water follows the upper meridian transit of the following approximate quantity, reckoned from 0h. to 24h.50m.

$$\frac{\kappa_{K_1} + \kappa_{O_1}}{q\ M_2} \text{ hours} + n.\ 12^h\ 25^m. \qquad (188)$$

(*n* being constant)

when the declination of the Moon is plus, and of the same quantity increased by 12h.25m. when the declination is minus.

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(1) n being equal either to 0 or to 1, when the declination of the Moon is North, and to the other value when the declination is South.

Springs (K_1 and O_1 being coincident) occur twice monthly at the lunear tropics ; the time of the spring high water is given by the following formula :

$$(189) \qquad t = \kappa_{K_1} - h + \frac{\pi}{2}$$

t being reckoned from mean noon, and the tide primes by two hours per month and by 12 hours in six months. If it occurs in the morning during a certain month, it occurs in the evening six months later.

The height of the spring high water and low water is given by :

$$(190) \quad \text{Springs} \begin{cases} H\,W = A_0 + K_1 + O_1 \\ L\,W = A_0 - K_1 - O_1 \end{cases}$$

Neaps occur twice monthly when the Moon has crossed the Equator, and the height of the high water and the low water is given by the following formula :

$$\text{Neaps} \begin{cases} H\,W = A_0 + K_1 - O_1 \\ L\,W = A_0 - K_1 + O_1 \end{cases}$$

The action of the component P_1 results in the large spring tides of the year (at the time of the solstices).

The influence of tide M_2 is to modify the duration of the rising and falling tides and to create an inequality between the height of the high water and that of the low water, above the mean level.

Lastly, we must draw attention to the fact that the amplitude of the diurnal tide is noticeably reinforced every 18 & 2/3 years, when $N = O$, and is noticeably decreased when $N = 180°$; in the first case, the maximum declination of the Moon reaches the value of 28.6°, and in the second case, it reaches only 18.3°, so that the amplitudes of the diurnal tide at springs are respectively :

$$(191) \quad \begin{cases} 2 \ (1.1 \ K_1 + 1.18 \ O_1) & \text{for } N = 0 \\ 2 \ (0.88 \ K_1 + 0.80 \ O_1) & \text{for } N = 180 \end{cases}$$

the ratio of which is roughly equal to $\frac{2.29}{1.68} = 1.37$

In the preceding chapter (12th Lecture) which is not reproduced here, the formulae (*) show the numerical value of the usual characteristics of the tide in cases where the diurnal tide is preponderant.

(*) These formulae, not reproduced here, are taken from the Manual of Tides, by Rollin A. Harris.