VELOCITY OF SOUND IN SEA-WATER AND CALCULATION OF THE VELOCITY FOR USE IN SONIC SOUNDING

by

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INTRODUCTION.

In Japan, recently, the ships equipped with sonic sounding apparatus have considerably increased in number, and depths of the sea taken by the sonic method are accumulating year by year. Now it is necessary to determine the method for handling of these depths by investigating fundamentally the nature of the sonic sounding.

The general principle of the sonic sounding consists of evaluating the depth by means of the time taken by the sound signal in making a single journey from the surface to the bottom of the sea. Thus the depth is given by the product of the time required and the mean velocity of sound over the vertical depth (the mean vertical velocity). For the sounding by the sonic method, therefore, it is necessary to have the precise knowledge of the velocity of sound in sea-water.

The experiment to determine directly the mean vertical velocity was made by the U. S. Coast and Geodetic Survey steamer « Guide » (1). In 1923, the « Guide » engaged herself in the experiments under varying conditions encountered on an oceanographic cruise from New London, Conn., to San Diego, Calif., by way of Porto Rico and the Panama Canal. The mean vertical velocities of sound were obtained from the depths by the wire-sounding and the corresponding time intervals for the transmissions of sound-waves which were measured by a sonic sounding apparatus. By this method, however, it is difficult to obtain the accurate results on account of the fact that the drift of the observing ship introduces uncertainty into the depth.

The velocity of sound in sea-water is a function of the salinity, temperature and pressure, and the mean vertical velocity of sound for use in the sonic sounding may be calculated from the vertical distributions of these physical elements at the point of sounding which may be known from results of the oceanographic observations.

The velocity of sound in sea-water under definite physical conditions has been investigated by many persons from both experimental and theoretical grounds. In the experimental determinations, the experiments must be made on the velocity in the horizontal direction in a locality where the salinity and temperature are as nearly as may be uniform everywhere in such a deep water off the coast that the disturbances by echoes from the bottom and the coast have no confusing effects on the time measurements. In this case, the determination of the velocity relies upon the horizontal distance, and a large horizontal distance must be measured accurately. Such experiments are difficult to carry out in a great depth due to the technical difficulties, and the direct determination of the change in the velocity with pressure is almost impossible.

The horizontal experiment on the largest scale up to this time, was made by the U.S. Coast and Geodetic Survey for the purpose of hydrographic survey. Numerous attempts have been made to confirm the velocity in both shallow and deep waters. In November, 1933, it was verified for the first time that the sound-waves follow the well known physical laws of geometrical optics, by the Ship « Pioneer » under the command of Lieutenant-Commander C. C. Swainson, U.S.C. and G. S. assisted by the Ship « Guide » under the command of Lieutenant-Commander F. L. Peacock (2). The sound-waves are refracted and reflected according to the change in density of the medium, and in natural water, the density changes with depth at least on account of the increase in pressure. From these facts it may be seen that the experimental determination of the velocity is very difficult even in the case of determining the velocity only for a salinity or a temperature. The experiments in the laboratory would be disturbed by the effects of the walls of the vessel which holds the water.

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(1) U.S. Coast and Geodetic Survey, Special Publication, 108 (1924).

(2) "Recent Acoustic Work of the U.S. Coast and Geodetic Survey," by Paul A. Smith, Field Engineers Bulletin N° 8, pp. 60-75.

On the other hand, the relation between the velocity of sound and the physical constants of the medium is given from the theory of the thermodynamics. And the velocity under any définite conditions of water may be calculated accurately by the formula from certain properties of water which have been determined in the laboratory.

In the direction of the theoretical investigation, Dr. H. Maurer has calculated the sound-velocity for the water of salinity 32.3 0/00, taking the density and compressibility from Bjerknes' Hydrographic tables (1). The resulting velocity would not attain a high degree of accuracy from the fact that the significant figures for the compressibility which could be determined by using the specific volumes of water in the table were less. Moreover, it was a defect in his work that he assumed that the propagation of sound-waves in water takes place under isothermal conditions. On this account the velocities show large differences, especially at high temperatures and high pressures, from the results of observations.

Dr. A. Schumacher has prepared a velocity table by using Bjerknes' tables for the density as Dr. Maurer did, but he used the value of μ given by Ekman's formula for the compressibility (2). The value of μ, however, is not the actual compessibility of water under a pressure p, but it means the mean compressibility per decibar for the change in pressure from zero to p, In addition. μ is the mean compressibility for isothermal change, and the velocity of sound should be calculated from the adiabatic compressibility. But. Dr. A. Schumacher has neglected the error which would arise from the use of μ , as it was not important, Dr. Maurer has deduced the following empirical formula for the velocity at the surface of the sea, from Dr. Schumacher's results (3) :

$$V = 1445 + 4.46\ t - 0.0615\ t^2 + (1.2 - 0.015\ t)\ (S - 35).$$

After these investigations, N. H. Heck and J. H. Service, U.S. C. and G. S., have calculated the velocity first systematically (4). Through, as in the case of Dr. Maurer, they used Bjerknes' tables for density and compressibility, it was the first time that the application of the adiabatic correction to the velocity was investigated. By this investigation, the calculation of the velocity became much rigorous. The resulting velocity, however, would be of the same degree of accuracy as that of Maurer, since the compressibility has been calculated from Bjerknes' tables. In addition, they had some doubts about the use of the adiabatic correction.

The velocity table, which has been the most rigorously calculated up to the present, is that published by the British Admiralty (5). In the calculations of the tabular values, the formula for adiabatic propagation of sound-waves was adopted, and the actual compressibility deduced from Ekman's formula was used for the first time, and the changes in specific heat with the salinity, temperature and pressure were introduced into the calculation, the accuracy of the resulting velocity being much increased. However, in the table, no account has been taken of the fact that the pressure correction to the velocity is not the same for all salinities. The velocities have been calculated by determining the necessary physical constants at the intervals of 5° C. of temperature and 5 units of the value of σ_0 [$\sigma_0 = (\rho_0 - 1)10^3$, ρ_0 being the density of water at 0° C. under atmospheric pressure]. The velocities for other conditions were interpolated.

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(1) "Ueber Echolotungen der nordamerikanischen Marine," by H. Maurer, Ann. d. Hydro., April, 1924, pp. 75-87.

(2) "Hydrographische Bemerkungen und Hilfsmittel zur akustischen Tiefenmessung," by A. Schumacher, Deutsche Seewarte, Ann. d. Hydr., April, 1924, pp. 87-95.

(3) "Das englische Echolot," by H. Maurer, ditto, September, 1924, p. 221.

(4) "Velocity of Sound in Sea Water," by N. H. Neck and Jerry H. Service, U.S. Coast and Geodetic Servey, Special Publication N° 108, 1924.

(5) "Tables of the Velocity of Sound in Pure Water and Sea Water for Use in Echo-sounding and Sound-ranging," Hydrographic Department, Admiralty, H. D. 282, 1927.

The table has been used in most of the countries for sonic sounding and radio-acoustic ranging. Comparing with experiments at many times, it was proved that the velocities in the table are accurate within 0.2 % in error. Though not much value is to be ascribed to those check measurements based on the horizontal distance—in such tests the refraction, reflection and dispersion of the sound-waves will reduce the accuracy of these measurements—the accuracy of the theoretical velocity would be of this order. However, for our further investigations on the sounding, it was desirable for caution's sake to confirm the theoretical velocities in the British table.

The author has calculated the sound-velocity anew from the start in a somewhat different manner, and has made some discussions thereon in the present volume. His thanks are due to Mr. M. Seo, Assistant, who made chiefly the numerical calculations, and to Captain R. Sigemantu for much useful advice.

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METHOD OF PREPARING THE TABLES OF SOUND-VELOCITY IN SEA-WATER.

The velocity of sound in a perfect fluid is given from the theory of the hydrodynamics as follows :—

$$V=\sqrt{\frac{v\gamma}{k}}$$

where V = velocity of sound in the fluid,
v = specific volume of the fluid,
k = compressibility of the fluid,
γ = ratio of the specific heats of the fluid at constant pressure and constant volume.

The specific volume α_{stp} of sea-water is given by V. W. Ekman as follows (1) :—

$$\alpha_{stp}=\alpha_{sto}(1-\mu p)$$

and

$$10^9\mu=\frac{4886}{1+0\cdot0000183p}-[227+28\cdot33t-0\cdot551t^2+0\cdot004t^3]$$

$$+10^{-4}p[105\cdot5+9\cdot50t-0\cdot158t^2]-1\cdot5\times10^{-8}p^2t$$

$$-(\sigma_{s00}-28)[147\cdot3-2\cdot72t+0\cdot04t^2-10^{-4}p(32\cdot4-0\cdot87t+0\cdot02t^2)]10^{-1}$$

$$+(\sigma_{s00}-28)^2[4\cdot5-0\cdot1t-10^{-4}p(1\cdot8-0\cdot06t)]10^{-2},$$

where p is pressure in decibars ; μ, the mean compressibility per decibar for the pressures from zero to p ; α_{sto}, the specific volume of water of salinity S (0/00), temperature t (°C.) under the atmospheric pressure. α_{sto} has the following relation :—

$$\alpha_{sto}=\frac{1}{\sigma_{sto}10^{-3}+1}, \quad \text{and} \quad \sigma_{sto}=(\rho_{sto}-1)10^3,$$

where, ρ_{sto} is the specific weight of the sea-water.

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(1) Ekman. Publ. de Circon, N° 43, 1908

By Martin Knudsen,

$$\sigma_{st0}=\Sigma_t+(\sigma_{s00}+0\cdot1324)[1-A_t+B_t(\sigma_{s00}-0\cdot1324)],$$

where $$\Sigma_t=-\frac{(t-3\cdot98)^2}{503\cdot570}\,\frac{t+283}{t+67\cdot26},$$

$$A_t=t\,(4\cdot7867-0\cdot098185t+0\cdot0010843t^2)\,10^{-3};$$

$$B_t=t\,(18\cdot030-0\cdot8164t+0\cdot01667t^2)\,10^{-6},$$

$$\sigma_{s00}=-0\cdot069+1\cdot4708\,Cl-0\cdot001570\,Cl^2+0\cdot0000398\,Cl^3.$$

In the last formula, Cl is chlorinity. The constant relationship between the chlorinity Cl and salinity S is :

$$S = 0.030 + 1.8050\ Cl.$$

The actual compressibility k is given by

$$k=-\frac{1}{v}\,\frac{\partial v}{\partial P}=10^{-5}\frac{\mu+p\,\dfrac{\partial\mu}{\partial p}}{1-\mu p}$$

where P is pressure in C.G.S. units.

Though the value of γ has not been determined directly, it is given from the thermodynamical grounds as follows :—

$$\gamma=\frac{C_p}{C_v}=\frac{1}{1+\dfrac{T}{JC_p}\,\dfrac{\left(\dfrac{\partial v}{\partial t}\right)^2}{\dfrac{\partial v}{\partial P}}},$$

where T means the absolute temperature and J, the mechanical equivalent of heat, that is, $J = 4.186 \times 10^7$ erg $= 1$ cal_{15}.

Cp is the specific heat at constant pressure in calories per gram per degree C. and is a function of the salinity, temperature and pressure. A complete relationship between Cp and these physical quantities has not been determined. However, the specific heat of sea-water at atmospheric pressure and the temperature of 17°5 C. have been calculated by Krümmel (1) from the results of measurements made by Thoulet and Chevallier, where

Salinity	0	5	10	15	20	25	30	35	40
Specific Heat	1.000	0.982	0.968	0.958	0.951	0.945	0.939	0.932	0.926

The relation between Cp and temperature has been investigated only for distilled water, where (2)

$$C_p=1\cdot005-0\cdot0004226\,t+0\cdot000006321\,t^2.$$

Assuming that this relation is applicable to sea-water, the author deduced the following formula for Cp at 0° C. by the method of least squares with Krümmel's results :

$$Cp = 1.005 - 0.004136\ S + 0.0001098\ S^2 - 0.000001324\ S^3.$$

(1) Krümmel, "Ozeanographie," Bd. I, 1907.

(2) W. Jaeger u. H. v. Steinwehr, Sitzungsber. d. Berl. Akad. 1915, p. 424, or Hand. d. Phys., Bd. X, 1926, p. 323.

The value of Cp slightly decreases with increase in pressure. It is given from the thermodynamical grounds as follows :

$$\frac{\partial C_p}{\partial P} = -\frac{T}{J} \frac{\partial^2 v}{\partial t^2}$$

The velocities of sound in water under any definite physical conditions may be calculated by the relations described above.

For the practical computations, Taylor's expansion was applied. Using the formula written in the form convenient for the expansion,

$$V = v\left\{-\frac{\partial v}{\partial P} - \frac{T}{JC_p}\left(\frac{\partial v}{\partial t}\right)^2\right\}^{-\frac{1}{2}},$$

we calculated the necessary differential coefficients. For example, the partial differential coefficients of the velocity with respect to salinity may be calculated as follows :

$$\frac{\partial V}{\partial S} = \frac{1}{2} \frac{1}{H^{\frac{1}{2}}}\left(2\frac{\partial v}{\partial S} - \frac{v}{H}\frac{\partial H}{\partial S}\right),$$

where
$$H = -\frac{\partial v}{\partial P} - \frac{T}{JC_p}\left(\frac{\partial v}{\partial t}\right)^2,$$

and
$$\frac{\partial H}{\partial S} = -\frac{\partial^2 v}{\partial S \partial P} + \frac{T}{JC_p}\left\{\frac{1}{C_p}\frac{\partial C_p}{\partial S}\left(\frac{\partial v}{\partial t}\right)^2 - 2\frac{\partial v}{\partial t}\frac{\partial^2 v}{\partial S \partial t}\right\}$$

The differentials of the specific volume on the right-hand sides of these equations may be given by

$$\left(\frac{\partial v}{\partial S}\right)_{p=0} = \frac{\partial \alpha_{st0}}{\partial S} = -10^{-3}\alpha_{st0}^2 \frac{\partial \sigma_{st0}}{\partial S},$$

$$\left(\frac{\partial v}{\partial t}\right)_{p=0} = \frac{\partial \alpha_{st0}}{\partial t} = -10^{-3}\alpha_{st0}^2 \frac{\partial \sigma_{st0}}{\partial t},$$

$$\left(\frac{\partial^2 v}{\partial S \partial t}\right)_{p=0} = \frac{\partial^2 \alpha_{st0}}{\partial S \partial t} = -10^{-3}\left(2\alpha_{st0}\frac{\partial \sigma_{st0}}{\partial S}\frac{\partial \alpha_{st0}}{\partial t} + \alpha_{st0}^2 \frac{\partial^2 \sigma_{st0}}{\partial S \partial t}\right),$$

$$\frac{\partial v}{\partial P} = 10^{-5}\frac{\partial \alpha_{stp}}{\partial p} = -10^{-5}\alpha_{st0}\mu,$$

$$\frac{\partial^2 v}{\partial S \partial P} = 10^{-5}\frac{\partial^2 \alpha_{stp}}{\partial S \partial p} = -10^{-5}\left(\mu\frac{\partial \alpha_{st0}}{\partial S} + \alpha_{st0}\frac{\partial \mu}{\partial S}\right),$$

The differentials in the third members of these equations may be expressed as follows :

$$\frac{\partial \sigma_{stp}}{\partial S}=(1-A_t+2B_t\sigma_{s00})\frac{\partial \sigma_{s00}}{\partial S},$$

where
$$\frac{\partial \sigma_{s00}}{\partial S}=(1\cdot4708-0\cdot003140Cl+0\cdot0001194Cl^2)\frac{\partial Cl}{\partial S},$$

and
$$\frac{\partial Cl}{\partial S}=\frac{1}{1\cdot8060}.$$

$$\frac{\partial \sigma_{st0}}{\partial t}=\frac{\partial \Sigma_t}{\partial t}+(\sigma_{s00}+0\cdot1324)\left\{-\frac{\partial A_t}{\partial t}+(\sigma_{s00}-0\cdot1324)\frac{\partial B_t}{\partial t}\right\},$$

where
$$\frac{\partial \Sigma_t}{\partial t}=-\frac{1}{503\cdot570}\left\{215\cdot74-2(t-3\cdot98)-\frac{1094910}{(t+67\cdot26)^2}\right\},$$

$$10^3\frac{\partial A_t}{\partial t}=4\cdot7867-0\cdot196370t+0\cdot0032529t^2,$$

and
$$10^6\frac{\partial B_t}{\partial t}=18\cdot030-1\cdot6328t+0\cdot05001t^2.$$

$$10^5\frac{\partial \mu}{\partial S}=-\Big\{[147\cdot3-2\cdot72t+0\cdot04t^2-p\times10^{-4}(32\cdot4-0\cdot87t+0\cdot02t^2)]10^{-1}$$

$$+2\times10^{-2}(\sigma_{s00}-28)[4\cdot5-0\cdot1t-p\times10^{-4}(1\cdot8-0\cdot06t)]\Big\}\frac{\partial \sigma_{s00}}{\partial S}.$$

The change of the specific heat with respect to salinity is given by

$$\frac{\partial C_p}{\partial S}=-0\cdot004136+0\cdot00021965S-0\cdot000003972S^2.$$

Substituting the numerical values,

$$(v)_{35,0,0}=\alpha_{35,0,0}=0\cdot97264,\quad (\mu)_{35,0,0}=0\cdot4657\times10^{-5},\quad (C_p)_{35,0,0}=0\cdot938,$$

$$\left(\frac{\partial v}{\partial S}\right)_{\substack{s=35\\t=0\\p=0}}=-0\cdot76248\times10^{-3},\qquad \left(\frac{\partial v}{\partial t}\right)_{\substack{s=35\\t=0\\p=0}}=0\cdot50146\times10^{-3},$$

$$\left(\frac{\partial v}{\partial P}\right)_{\substack{s=35\\t=0\\p=0}}=-0\cdot45297\times10^{-10},\qquad \left(\frac{\partial^2 v}{\partial S\partial P}\right)_{\substack{s=35\\t=0\\p=0}}=0\cdot1509\times10^{-12},$$

$$\left(\frac{\partial C_p}{\partial S}\right)_{\substack{s=35\\t=0\\p=0}}=-0\cdot00131,\qquad \left(\frac{\partial^2 v}{\partial S\partial t}\right)_{\substack{s=35\\t=0\\p=0}}=0\cdot2798\times10^{-5},$$

and
$$(H)_{35,0,0}=0\cdot45279\times10^{-10},\qquad \left(\frac{\partial H}{\partial S}\right)_{\substack{s=35\\t=0\\p=0}}=-0\cdot15287\times10^{-12}.$$

Hence

$$\left(\frac{\partial V}{\partial S}\right)_{\substack{s=35\\ t=0\\ p=0}}=130 \cdot 7.$$

In the similar manner,

$$\left(\frac{\partial^2 V}{\partial S^2}\right)_{\substack{s=35\\ t=0\\ p=0}}=-0 \cdot 0306.$$

Therefore the expansion for velocity (in meters per second) with respect to salinity becomes :

$$V=1445 \cdot 5+1 \cdot 307(S-35)-0 \cdot 00015(S-35)^2+\cdots$$

In the case with the salinity, the terms of higher than the first degree may be neglected for sea-water.

In this manner, we have expanded the velocity in power series of the three variables—salinity, temperature and pressure—at the three of the standard states, (S = 35 0/00, t = 0° C., p = 0), (S = 35 0/00, t = 4°, C., p = 0) and (S = 35 0/00, t = 15° C., p = 0), in order that the terms of higher degree may be neglected. However due to the complicated forms of the changes in velocity with temperature and pressure, the terms up to the sixth and sometimes the seventh degree were necessary to calculate the velocities for all the possible conditions of sea-water. These expansions with the different standard states were all transformed into the equivalent ones referred to the common normal state (S = 35 0/00, t = 0° C., p = 0). Then we calculated the velocities for various conditions by the three transformed expansions, using each for a certain range of conditions near its initial standard state correspondingly.

The terms of the expansion were summed up into $V_{35,0,p}$, C_s, C_t and C_{stp}, and are shown in Tables from 1 to 4 respectively, where $V_{35,0,p}$ is the sound-velocity for a sea-water at 0° C. and of 35 salinity under any pressure p, « the normal velocity » ; Cs, the salinity correction ; Ct, the temperature correction ; and C_{stp}, the correction which consists of the product terms relating simultaneously to any two of, or to all of the elements salinity, temperature and pressure.

Table 1

$V_{35,0,p}$: Velocity of sound (in meters per second) in Sea-water.
S = 35 0/00. t = 0° C.

第 1 表

$V_{35,0,p}$：海水中ニ於ケル音波ノ速度（米/秒）鹽分 35 ‰. 溫度 0°C.

P (dbs.)	0	100	200	300	400	500	600	700	800	900
0	1445·5	1447·3	1449·1	1450·9	1452·7	1454·5	1456·3	1458·1	1459·9	1461·7
1000	63·4	65·2	67·0	68·8	70·6	72·4	74·2	76·0	77·8	79·5
2000	81·3	83·1	84·9	86·6	88·4	90·2	92·0	93·7	95·5	97·3
3000	99·0	1500·8	1502·6	1504·3	1506·1	1507·8	1509·6	1511·3	1513·1	1514·8
4000	1516·6	18·3	20·1	21·8	23·6	25·3	27·0	28·8	30·5	32·2
5000	33·9	35·7	37·4	39·1	40·8	42·5	44·2	46·0	47·7	49·4
6000	51·1	52·8	54·5	56·2	57·9	59·5	61·2	62·9	64·6	66·3
7000	68·0	69·6	71·3	73·0	74·6	76·3	78·0	79·6	81·3	82·9
8000	84·6	86·2	87·9	89·5	91·1	92·8	94·4	96·0	97·6	99·3
9000	1600·9	1602·5	1604·1	1605·7	1607·3	1608·9	1610·5	1612·1	1613·7	1615·3
10000	16·9	18·5	20·1	21·6	23·2	24·8	26·3	27·9	29·5	31·0

Proportional Table.

比 例 表

p(dbs.) 差	0	10	20	30	40	50	60	70	80	90
1·5	0·00	0·15	0·30	0·45	0·60	0·75	0·90	1·05	1·20	1·35
1·6	0·00	0·16	0·32	0·48	0·64	0·80	0·96	1·12	1·28	1·44
1·7	0·00	0·17	0·34	0·51	0·68	0·85	1·02	1·19	1·36	1·53
1·8	0·00	0·18	0·36	0·54	0·72	0·90	1·08	1·26	1·44	1·62

Table 2

C_t : Temperature Correction to the Velocity $V_{35,0,p}$ in Table 1.

第 2 表

C_t：溫度ニ對スル修正値

t°c.	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
-2	-9·3									
-1	-4·6	-5·1	-5·6	-6·0	-6·5	-7·0	-7·4	-7·9	-8·4	-8·9
-0	0·0	-0·5	-0·9	-1·4	-1·8	-2·3	-2·8	-3·2	-3·7	-4·2
0	0·0	0·5	0·9	1·4	1·8	2·3	2·7	3·2	3·6	4·1
1	4·5	5·0	5·4	5·9	6·3	6·8	7·2	7·7	8·1	8·6
2	9·0	9·4	9·9	10·3	10·8	11·2	11·6	12·1	12·5	12·9
3	13·4	13·8	14·2	14·6	15·1	15·5	15·9	16·4	16·8	17·2
4	17·6	18·0	18·5	18·9	19·3	19·7	20·1	20·6	21·0	21·4
5	21·8	22·2	22·6	23·0	23·4	23·9	24·3	24·7	25·1	25·5
6	25·9	26·3	26·7	27·1	27·5	27·9	28·3	28·7	29·1	29·5
7	29·9	30·3	30·7	31·1	31·4	31·8	32·2	32·6	33·0	33·4
8	33·8	34·1	34·5	34·9	35·3	35·7	36·1	36·4	36·8	37·2
9	37·6	37·9	38·3	38·7	39·1	39·4	39·8	40·2	40·5	40·9
10	41·3	41·6	42·0	42·4	42·7	43·1	43·4	43·8	44·2	44·5
11	44·9	45·2	45·6	45·9	46·3	46·6	47·0	47·3	47·7	48·0
12	48·4	48·7	49·1	49·4	49·8	50·1	50·4	50·8	51·1	51·5
13	51·8	52·1	52·5	52·8	53·1	53·5	53·8	54·1	54·5	54·8
14	55·1	55·5	55·8	56·1	56·4	56·8	57·1	57·4	57·7	58·0
15	58·4	58·7	59·0	59·3	59·6	59·9	60·2	60·6	60·9	61·2
16	61·5	61·8	62·1	62·4	62·7	63·0	63·3	63·6	63·9	64·3
17	64·6	64·9	65·2	65·5	65·8	66·1	66·4	66·6	66·9	67·2
18	67·5	67·8	68·1	68·4	68·7	69·0	69·3	69·6	69·9	70·1
19	70·4	70·7	71·0	71·3	71·6	71·8	72·1	72·4	72·7	73·0
20	73·2	73·5	73·8	74·1	74·3	74·6	74·9	75·2	75·4	75·7
21	76·0	76·2	76·5	76·8	77·0	77·3	77·6	77·8	78·1	78·4
22	78·6	78·9	79·1	79·4	79·7	79·9	80·2	80·4	80·7	81·0
23	81·2	81·5	81·7	82·0	82·2	82·5	82·7	83·0	83·2	83·5
24	83·7	84·0	84·2	84·5	84·7	85·0	85·2	85·5	85·7	86·0
25	86·2	86·4	86·7	86·9	87·2	87·4	87·6	87·9	88·1	88·4
26	88·6	88·8	89·1	89·3	89·5	89·8	90·0	90·2	90·5	90·7
27	90·9	91·2	91·4	91·6	91·9	92·1	92·3	92·5	92·8	93·0
28	93·2	93·5	93·7	93·9	94·1	94·4	94·6	94·8	95·0	95·3
29	95·5	95·7	95·9	96·1	96·4	96·6	96·8	97·0	97·2	97·5
30	97·7									

Table 3

C_s : Salinity Correction to the Velocity $V_{35,0,p}$ in Table 1.

第 3 表

C_s : 鹽分ニ對スル修正値

S (‰)	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
30	-6·5	-6·4	-6·3	-6·1	-6·0	-5·9	-5·8	-5·6	-5·5	-5·4
31	-5·2	-5·1	-5·0	-4·8	-4·7	-4·6	-4·4	-4·3	-4·2	-4·1
32	-3·9	-3·8	-3·7	-3·5	-3·4	-3·3	-3·1	-3·0	-2·9	-2·7
33	-2·6	-2·5	-2·4	-2·2	-2·1	-2·0	-1·8	-1·7	-1·6	-1·4
34	-1·3	-1·2	-1·0	-0·9	-0·8	-0·7	-0·5	-0·4	-0·3	-0·1
35	0·0	0·1	0·3	0·4	0·5	0·7	0·8	0·9	1·0	1·2
36	1·3	1·4	1·6	1·7	1·8	2·0	2·1	2·2	2·4	2·5
37	2·6	2·7	2·9	3·0	3·1	3·3	3·4	3·5	3·7	3·8
38	3·9	4·1	4·2	4·3	4·4	4·6	4·7	4·8	5·0	5·1
39	5·2	5·4	5·5	5·6	5·7	5·9	6·0	6·1	6·3	6·4
40	6·5	6·7	6·8	6·9	7·1	7·2	7·3	7·4	7·6	7·7

Table 4

C_{stp} : Correction to the Velocity $V_{35,0,p}$ in Table 1 for any Simultaneous Changes of Salinity, Temperature and Pressure.

第 4 表

C_{stp} : 鹽分、溫度並ニ壓力ノ中ノ二要素以上ノモノノ同時變化ニ對スル修正値

p (dbs.)	S (‰) \ t (°C.)	-2°	-1°	0°	1°	2°	3°	4°	5°	10°	15°	20°	25°	30°
0	30	-0·1	0·0	0·0	0·0	0·1	0·1	0·2	0·2	0·5	0·8	1·0	1·2	1·4
	35	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
	40	0·1	0·0	0·0	0·0	-0·1	-0·1	-0·2	-0·2	-0·5	-0·8	-1·0	-1·2	-1·3
1000	30	-0·3	-0·2	-0·1	-0·1	0·0	0·0	0·1	0·1	0·4	0·6	0·8	1·2	1·6
	35	-0·1	0·0	0·0	0·0	0·0	0·0	0·0	0·0	-0·1	-0·1	-0·2	-0·1	0·2
	40	0·2	0·2	0·1	0·1	0·1	0·0	0·0	-0·1	-0·5	-0·8	-1·1	-1·3	-1·1
2000	30	-0·4	-0·3	-0·2	-0·2	-0·1	-0·1	0·0	0·0	0·1	0·3	0·5	1·0	
	35	-0·1	0·0	0·0	0·0	0·0	0·0	0·0	0·0	-0·2	-0·4	-0·5	-0·2	
	40	0·3	0·3	0·2	0·2	0·2	0·1	0·0	-0·1	-0·6	-1·1	-1·4	-1·4	
3000	30	-0·5	-0·4	-0·3	-0·2	-0·2	-0·2	-0·2	-0·2	-0·2	-0·2	0·1		
	35	0·0	0·0	0·0	0·0	0·0	-0·1	-0·1	-0·2	-0·6	-0·8	-0·9		
	40	0·4	0·3	0·3	0·2	0·2	0·1	0·0	-0·2	-0·9	-1·5	-1·8		
4000	30	-0·4	-0·4	-0·3	-0·3	-0·3	-0·3	-0·3	-0·4	-0·6	-0·7			
	35	0·0	0·0	0·0	0·0	-0·1	-0·2	-0·2	-0·4	-1·0	-1·4			
	40	0·4	0·4	0·3	0·2	0·1	0·0	-0·2	-0·4	-1·3	-2·1			
5000	30	-0·3	-0·3	-0·3	-0·3	-0·4	-0·4	-0·5	-0·6					
	35	0·1	0·1	0·0	-0·1	-0·2	-0·3	-0·4	-0·6					
	40	0·5	0·4	0·3	0·1	0·0	-0·2	-0·4	-0·7					
6000	33	0·1	0·0	-0·1	-0·2	-0·4	-0·5	-0·7						
	35	0·2	0·1	0·0	-0·2	-0·3	-0·5	-0·7						
	37	0·3	0·2	0·1	-0·1	-0·3	-0·5	-0·7						
7000	33	0·3	0·1	0·0	-0·2	-0·5	-0·7	-0·9						
	35	0·4	0·2	0·0	-0·2	-0·5	-0·7	-0·9						
	37	0·4	0·3	0·0	-0·2	-0·5	-0·7	-1·0						
8000	33	0·5	0·3	0·0	-0·3	-0·5	-0·8	-1·1						
	35	0·5	0·3	0·0	-0·3	-0·6	-0·9	-1·2						
	37	0·5	0·3	0·0	-0·4	-0·7	-1·0	-1·4						
9000	33	0·8	0·5	0·1	-0·2	-0·6	-1·0	-1·3						
	35	0·7	0·4	0·0	-0·4	-0·8	-1·2	-1·6						
	37	0·6	0·3	-0·1	-0·5	-1·0	-1·4	-1·8						
10000	33	1·1	0·7	0·2	-0·2	-0·7	-1·2	-1·6						
	35	0·9	0·5	0·0	-0·5	-1·0	-1·5	-1·9						
	37	0·7	0·2	-0·2	-0·8	-1·3	-1·8	-2·2						

Table 5

Pressure (in decibars) at Each Depth in the Sea
Lat. = 35°. S = 35 0/00. t = 0° C.

第 5 表

各深度ニ對スル海水ノ壓力

緯度 35°. 鹽分 35 ‰. 溫度 0°C.

深度(米)	0	100	200	300	400	500	600	700	800	900
0	0	101	202	302	403	504	605	706	808	909
1000	1010	1111	1212	1314	1415	1517	1618	1720	1821	1922
2000	2023	2125	2228	2330	2432	2533	2635	2737	2839	2941
3000	3044	3146	3249	3350	3453	3555	3657	3761	3863	3966
4000	4068	4171	4273	4376	4479	4582	4684	4787	4891	4994
5000	5097	5200	5303	5406	5510	5613	5715	5819	5923	6026
6000	6129	6233	6336	6440	6545	6648	6752	6856	6959	7063
7000	7167	7272	7376	7480	7584	7688	7792	7897	8001	8105
8000	8210	8314	8418	8524	8628	8733	8837	8942	9047	9153
9000	9257	9361	9466	9571	9675	9781	9885	9990	10096	10201

(metres)

Table 6

C_G : Gravity Correction to the Sound-Velocity calculated for Depths by using Table 5.

第 6 表

C_G：第5表ノ壓力ヲ用ヒテ計算シタ音波速度ノ

重力ニ對スル修正値

緯度 深度(米)	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
1000	-0·1	0·0	0·0	0·0	0·0	0·0	0·0	0·1	0·1	0·1
2000	-0·1	-0·1	-0·1	0·0	0·0	0·1	0·1	0·1	0·1	0·1
3000	-0·1	-0·1	-0·1	0·0	0·0	0·1	0·1	0·2	0·2	0·2
4000	-0·1	-0·1	-0·1	0·0	0·0	0·1	0·2	0·2	0·2	0·3
5000	-0·2	-0·1	-0·1	0·0	0·0	0·1	0·2	0·2	0·3	0·3
6000	-0·2	-0·2	-0·1	0·0	0·1	0·1	0·2	0·3	0·4	0·4
7000	-0·2	-0·2	-0·1	-0·1	0·1	0·2	0·3	0·3	0·4	0·4
8000	-0·2	-0·2	-0·1	-0·1	0·1	0·2	0·3	0·4	0·4	0·5
9000	-0·3	-0·2	-0·2	-0·1	0·1	0·2	0·3	0·4	0·5	0·5
10000	-0·3	-0·3	-0·2	-0·1	0·1	0·2	0·4	0·5	0·6	0·6

(metres) (Latitude)

CALCULATION OF THE VELOCITY OF SOUND IN SEA-WATER.

In Table 1, the velocity is given referring to pressure (the atmospheric pressure is always considered to be zero). For practical purposes, however, it is convenient that the velocity be referred to depth. For this, Table 5 may be used for converting the arguments. In this case, the corrections in Table 4 may be used taking the pressure as directly equal to depth, without causing an appreciable error. Table 5 gives the water pressure at each depth, which is calculated on the assumptions that the latitude is 35° and that the water is at 0° C. and has a salinity of 35 0/00 throughout from the surface to the bottom. Though these assumptions are not quite correct for natural water, the resulting error in velocity is only about 0.2 m/sec. at the most, and it may be neglected for practical purposes. The correction on account of the change in gravity with latitude is given in Table 6. However, this correction must not be applied when the velocity is calculated with pressure independently of Table 5. The use of the tables is already evident from the above descriptions. A few examples are as follows :

Example 1. Required the velocity at the surface in water of 21°35 C. and 35 salinity—

From Table 1,	1445.5
» » 2,	76.9
	1522.4

Example 2. Required the velocity at 3°4 C. in water of 34.8 salinity under a pressure 7000 decibars—

From Table 1,	1568.0
» » 2,	15.1
» » 3,	—0.3
» » 4,	—0.8
	1582.0

Example 3. Required the velocity at 3°4 C. in water of 34.8 salinity at a depth 7000 m. in latitude 40°—

From Table 5 and 1,	1570.7
» » 2,	15.1
» » 3,	—0.3
» » 4,	—0.8
» » 6,	0.1
	1584.8

CALCULATION OF THE MEAN VERTICAL VELOCITY OF SOUND IN SEA-WEATER FOR USE IN SONIC SOUNDING.

The calculation of the velocity of sound in sea-water under any definite physical conditions is as shown in the former paragraphs. In the case of sonic sounding, however, the sound-waves run vertically between the surface and the bottom of the sea. Therefore the physical conditions of sea-water in the path of the sound-waves are not uniform and at least the pressure is always increasing with the depth even in the case of both the salinity and temperature having constant values, the velocity of sound being changed with depth accordingly. What is necessary for sonic sounding is the mean velocity over the vertical depth, and the depth is given by the product of the velocity and the time taken by the sound-signal in making a single journey from the surface to the bottom.

The mean vertical velocity V_S at a station may be given by

$$V_s = \frac{1}{D}\int_0^D V\, dD,$$

where V is the velocity of sound at any depth, D being the depth. The mathematical treatment of this integration is difficult for practical cases. But in practice, V_S may be given with sufficient accuracy by a mean value calculated by mechanical integral of the velocities over the depth whose distribution is determined from the oceanographical observations at intermediate depths at the station.

Before we begin the arithmetical work, now it is necessary to make some considerations about it.

V_S may be resolved into sum of the terms as follows :

$$V_s = \frac{1}{D}\int_0^D V_{35,0,p}\, dD + \frac{1}{D}\int_0^D (C_s + C_t + C_{stp})dD.$$

These integrals may be calculated mechanically by using the relation between **the depth D and the pressure p given in Table 5** (see above). In this case, however, we have to add the term of the gravity correction, and the expression for V_S becomes :

$$V_s = \frac{1}{D}\int_0^D V_{35,0,p}\, dD + \frac{1}{D}\int_0^D C_G\, dD + \frac{1}{D}\int_0^D (C_s + C_t + C_{stp})dD.$$

The first two terms of the second member of this equation are independent of the conditions of the sea-water in situ. Therefore, if we resolve the velocity into these terms the arithmetical work for computing the mean vertical velocity may be much simplified by omitting calculation of the means of $V_{35,0,p}$ and C_G. The mean of $V_{35,0,p}|$ over a depth is the mean vertical velocity of sound for the sonic sounding at any stations of the corresponding depth at latitude 35° and where the water is at 0° C. and has a salinity of 35 0/00 over the whole vertical depth, « the normal velocity of sound for sounding ». The normal velocities for various depths are given in Table 7. The second term gives the correction to the mean vertical velocity on account of the change in gravity with latitude which is shown in Table 8, being designated as C_G^s.

Accordingly, the mean vertical velocity over a depth at a station in question may be obtained by the sum of the normal mean vertical velocity for the depth taken from Table 7, corresponding gravity correction C_G^s in Table 8, and the mean of ($C_S + C_t + C_{stp}$) calculated over the depth according to the conditions of the waters in situ. An example of the calculation follows :

Specimen of the Calculation of the Mean Vertical Velocity of Sound for Each Depth at an Oceanographic Station

Lat. = 38° 18' N., Long. = 146° 38' E.

海洋觀測ノ地點ニ於ケル各深度ニ對スル

測深音速ノ計算例

Lat. = 38°18′ N. Long. = 146°38′ E.

深度 (米)	溫度 (°C.)	鹽分 (‰)	C_t	C_s	C_{stp}	$C = C_t + C_s + C_{stp}$	$\overline{C}$	$\Delta D \times \overline{C} \times 10^{-1}$	$\Sigma\Delta D \times \overline{C} \times 10^{-1}$	Corr.	C_G^s	V_s^n	V_s
0	20·77	34·02	75·3	-1·3	0·2	74·2							
							74·2	74·2					
10	20·77	·00	75·3	-1·3	0·2	74·2			74·2	74·2	0·0	1445·6	1519·8
							74·2	111·3					
25	20·77	·00	75·3	-1·3	0·2	74·2			185·5	74·2	0·0	1445·7	1519·9
							68·4	171·0					
50	16·64	·29	63·4	-0·9	0·1	62·6			356·5	71·3	0·0	1446·0	1517·3
							55·9	279·5					
100	12·39	·45	49·8	-0·7	0·1	49·2			636·0	63·6	0·0	1446·4	1510·0
							44·6	223·0					
150	9·86	·31	40·7	-0·9	0·1	39·9			859·0	57·3	0·0	1446·9	1504·2
							35·9	179·5					
200	7·82	33·95	33·1	-1·3	0·1	31·9			1038·5	51·9	0·0	1447·3	1499·2
							27·4	274					
300	5·61	·86	24·3	-1·5	0·0	22·8			1312·5	43·8	0·0	1448·2	1492·0
							18·9	189					
400	3·66	34·07	16·2	-1·2	0·0	15·0			1502	37·6	0·0	1449·1	1486·7
							15·0	150					
500	3·67	·04	16·3	-1·3	0·0	15·0			1652	33·0	0·0	1450·0	1483·0
							16·2	162					
600	4·26	·02	18·7	-1·3	0·0	17·4			1814	30·2	0·0	1450·9	1481·1
							16·1	322					
800	3·54	·33	15·7	-0·9	0·0	14·8			2136	26·7	0·0	1452·8	1479·5
							13·8	276					
1000	3·04	·40	13·6	-0·8	0·0	12·8			2412	24·1	0·0	1454·6	1478·7
							12·0	240					
1200	2·63	·45	11·8	-0·7	0·0	11·1			2652	22·1	0·0	1456·4	1478·5
							10·5	315					
1500	2·33	·54	10·5	-0·6	0·0	9·9			2967	19·8	0·0	1459·1	1478·9
							9·1	455					
2000	1·95	·60	8·8	-0·5	0·0	8·3			3422	17·1	0·0	1463·6	1480·7
							7·6	760					
3000	1·59	·72	7·2	-0·4	0·0	6·8			4182	13·9	0·0	1472·7	1486·6
							6·6	660					
4000	1·51	·70	6·8	-0·4	-0·1	6·3			4842	12·1	0·0	1481·7	1493·8
							6·4	640					
5000	1·54	·70	7·0	-0·4	-0·2	6·4			5482	11·0	0·0	1490·7	1501·7
							6·3	189					
5300	1·51	·70	6·8	-0·4	-0·2	6·2			5671	10·7	0·0	1493·4	1504·1

Table 7

V^n_s : Normal Velocity of Sound in Sea-Water for Sounding.
Lat. = 35°. S = 35 0/00. t. = 0° C.

第 7 表

V^n_s : 正規測深音速

緯度= 35°. 鹽分= 35 ‰. 溫度 =0°C.

深度 (米)	0	100	200	300	400	500	600	700	800	900
0	1445·5	1446·4	1447·3	1448·2	1449·1	1450·0	1450·9	1451·9	1452·8	1453·7
1000	1454·6	1455·5	1456·4	1457·3	1458·2	1459·1	1460·0	1460·9	1461·8	1462·7
2000	1463·6	1464·5	1465·4	1466·4	1467·3	1468·2	1469·1	1470·0	1470·9	1471·8
3000	1472·7	1473·6	1474·5	1475·4	1476·3	1477·2	1478·1	1479·0	1479·9	1480·8
4000	1481·7	1482·6	1483·5	1484·4	1485·3	1486·2	1487·1	1488·0	1488·9	1489·8
5000	1490·7	1491·6	1492·5	1493·4	1494·3	1495·2	1496·1	1497·0	1497·9	1498·8
6000	1499·7	1500·6	1501·5	1502·3	1503·2	1504·1	1505·0	1505·9	1506·8	1507·7
7000	1508·6	1509·5	1510·3	1511·2	1512·1	1512·0	1513·9	1514·8	1515·7	1516·5
8000	1517·4	1518·3	1519·2	1520·1	1520·9	1521·8	1522·7	1523·6	1524·5	1525·3
9000	1526·2	1527·1	1528·0	1528·8	1529·7	1530·6	1531·5	1532·3	1533·2	1534·1
10000	1534·9	1535·8	1536·7	1537·5	1538·4	1539·3	1540·1	1541·0	1541·8	1542·7

(metres)

Table 8

C^s_G : Gravity Correction to the Velocity V^n_s in Table 7

第 8 表

C^s_G : 測深音速ノ重力ニ對スル修正値

深度(米) \ 緯度	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
1000	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·0
2000	0·0	0·0	0·0	0·0	0·0	0·0	0·0	0·1	0·1	0·1
3000	-0·1	0·0	0·0	0·0	0·0	0·0	0·1	0·1	0·1	0·1
4000	-0·1	-0·1	0·0	0·0	0·0	0·0	0·1	0·1	0·1	0·1
5000	-0·1	-0·1	-0·1	0·0	0·0	0·1	0·1	0·1	0·1	0·2
6000	-0·1	-0·1	-0·1	0·0	0·0	0·1	0·1	0·2	0·2	0·2
7000	-0·1	-0·1	-0·1	0·0	0·0	0·1	0·1	0·2	0·2	0·2
8000	-0·1	-0·1	-0·1	0·0	0·0	0·1	0·2	0·2	0·2	0·3
9000	-0·1	-0·1	-0·1	0·0	0·0	0·1	0·2	0·2	0·3	0·3
10000	-0·2	-0·1	-0·1	0·0	0·0	0·1	0·2	0·2	0·3	0·3

(metres) (Latitude)

CONCLUSION.

The velocity of sound in sea-water does not appear, at present, to be calculated more accurately than by the tables in the present volume. In the calculation, it is only assumed that the change in specific heat of sea-water at constant pressure with temperature is independent of salinity. But the effect of the specific heat on the velocity is very small, and the change in its third decimal place affects the decimal place of velocity in extreme conditions. It is supposed, therefore, that the assumption would not cause an error amounting to one m/sec. in the velocity.

The accuracy of the velocity calculated by the above method depends chiefly upon the accuracy of Ekman's empirical formula for the compressibility of sea-water. According to his estimation, the error in μ, if calculated by his formula, does not amount to 0.3 %. However, this error gives an error of about 3 m/sec. to the sound-velocity, and it is known that the accuracy of the calculated velocity is nearly 0.2 %.

Therefore for a more accurate determination of the sound-velocity by the indirect method in future it is necessary to reinvestigate the compressibility of sea-water first. Next, the specific heat of sea-water at constant pressure must be determined for various temperatures.

The correction to the sound velocity on account of the simultaneous changes in pressure and salinity, which is neglected in the British table, is about 0.3 m/sec. at the most for natural sea-water. For example, if we calculate the velocity at 0° C. in water of 38 0/00 salinity under a pressure of 3000 decibars, the British tables give 1502-9 m/sec., while the present tables give 1503.1 m/sec. This correction is of the same order of magnitude with the gravity correction C_G, and the corrections for simultaneous changes in the salinity and temperature or in temperature and pressure are comparable with this. Considering the accuracy of the theoretical velocity, not only the correction in question but the gravity corrections C_G and C_G^2 and the corrections C_{stp} which consist of the product terms in the expansion are negligibly small, as is seen from Table 4.

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