TIDES IN SHALLOW WATER

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An explanation of the complex phenomena associated with the tidal movements in shallow channels necessarily involves many things of technical interest only and the object of this paper is simply to give a general account of the problem and of the methods which may be used to give satisfactory predictions of tides in shallow water. It has been shewn by mathematicians that if a wave travels along a channel then the velocity of travel of the wave depends upon the depth of the channel. In very deep water the distance from crest to crest of the wave will be very great, more than a thousand miles, but in coastal waters a tidal wave will be much shorter in length. Also it can be shewn that the higher parts of the wave travel more quickly than the lower parts because of the greater depth of water. The wave-shape from low water to high water changes steadily because the high waters run on more quickly than the low waters, and for such a simple type of wave the front becomes steeper than the rear. A graphical representation of this effect is given in fig. 1, in which it is supposed that the wave is travelling from left to right, and in its first oscillation it exhibits a pure sinewave shape. The crosses indicate the positions of high and low waters if the wave were supposed to travel without distortion, and it is evident that at any place the high waters are accelerated relatively to the mean tide level, and low waters are retarded. This is characteristic of tides in an estuary, and it is generally experienced that the tide rises more quickly than it falls. The same theory applies to a wind-wave as it rushes along a beach, for the front of the wave becomes steeper than the rear and ultimately the front becomes so steep that a breaker is formed.

While the above effect is true only for progressive waves, similar distortions of wave shapes occur with all kinds of oscillations; and any deductions we may make from the progressive wave apply in general to the more complex motions, so that in this paper we shall confine ourselves to the simplest case.

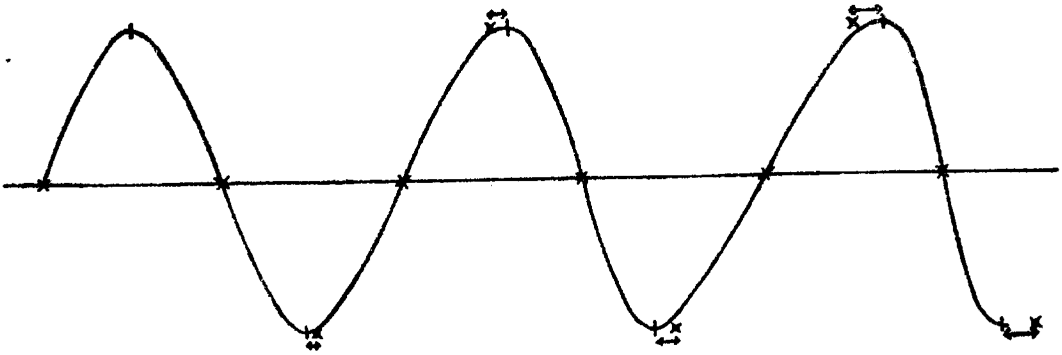


Fig. 1

Propagation of wave in shallow water

Speed $= \sqrt{g(h + 3\xi)}$ where h = mean sea level above the bed
ξ = elevation above h.

High water travels faster than low water, so that at any place high water is accelerated, low water is retarded, so that tide rises more quickly than it falls.

In fig. 2 we show in the curve (a) an oscillation with a shape like that of a simple sine-curve, such as would be found in very deep water, and in (b) we shew the oscillation derived from it as the wave travels in shallow water. The points M, for the mean level, are superposed so that the two curves can be compared. The original high-water point H has been accelerated to H' and the low-water point L has been retarded to L'. The difference between the two curves is shown by curve (c), and it is at once evident that this has two complete oscillations for every complete oscillation of (a). If therefore the original curve (a) gives a semi-diurnal oscillation as developed in deep water by astronomical forces then the curve (c) represents a quarter-diurnal oscillation derived from (a) by terrestrial distortions of the astronomic tide.

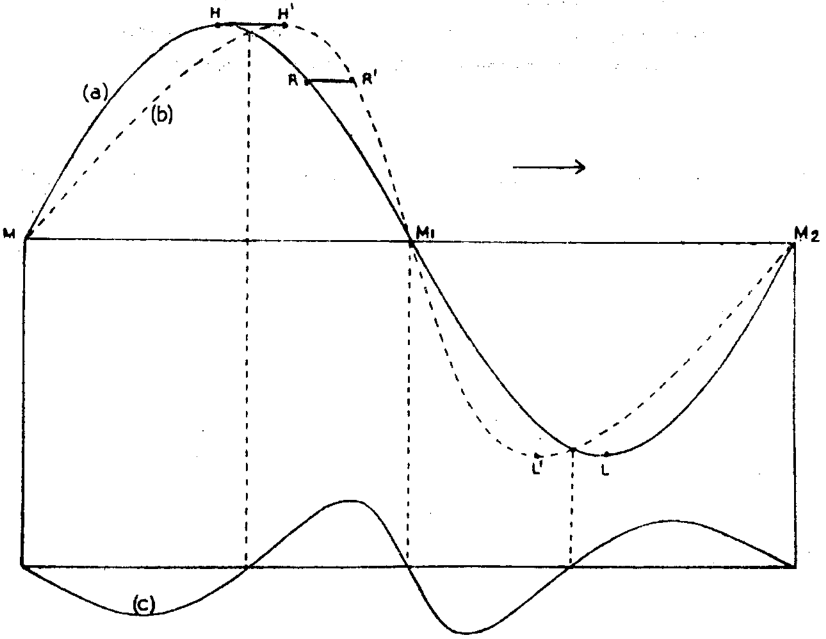


Fig. 2

Deduction of quarter diurnal tide from change of shape of progressive wave.

But as it is quite evident that the curve (a) does not represent a pure harmonic oscillation, in fig. 3 we redraw the curve and compare it with the more regular oscillation (d). The difference between the two curves (c) and (d) is shown by (e) and this curve has three times as many oscillations as the primary curve (a). If the primary oscillation (a) were a semi-diurnal tide then this effect (d) would represent a sixth-diurnal tide. If we repeated the above process of comparing the residual curve with a pure harmonic curve we should, in order, obtain eighth-diurnal, and tenth-diurnal, oscillations, and so on.

It is quite a simple exercise to shew that a curve which has only half the amplitude of that of fig. 2 will give a curve (c) with an amplitude only one quarter of that of curve (c) in fig. 2. That is, the amplitude of the quarter diurnal tide varies approximately as the square of the amplitude of the semi-diurnal tide. It can be shewn by the same method that the amplitude of the sixth-diurnal tide varies approximately as the cube of the amplitude of the semi-diurnal tide, and so on. This law is well exemplified by the results of tidal analyses all over the world, and it is fortunate that so simple a law can be utilised to estimate the relative sizes of constituents.

In Table 1 are shewn the amplitudes of the three quarter-diurnal constituents which can be derived from the principal semi-diurnal constituents M_2 and S_2. Taking relative values of S_2 to M_2 as 0.50 then the square of the amplitude of semi-diurnal tide shews that MS_4 will be equal in amplitude to M_4. When we come to the sixth-diurnal tide the constituent M_6 is not the greatest of the sixth-diurnal constituents, and the constituent M_8 is even less important compared with other constituents of the eighth-diurnal species. It is very necessary that this should be appreciated for too much emphasis is often laid upon the terms of the lunar series. When we include the constituent N_2 the complexity increases very greatly, and we see that though S_2 and N_2 may be very much less than M_2 (so that M_2 may be considered to be dominant) there are no really dominant terms in the higher species of constituents.

Table 2 shews the same thing in another way. Using the law of amplitudes described above we see that at spring tides the eighth-diurnal tides are five times as great as they are for average tides and that for neap tides this species of tides is of negligible amplitude.

The tables just discussed only give relative values of constituents within a species.

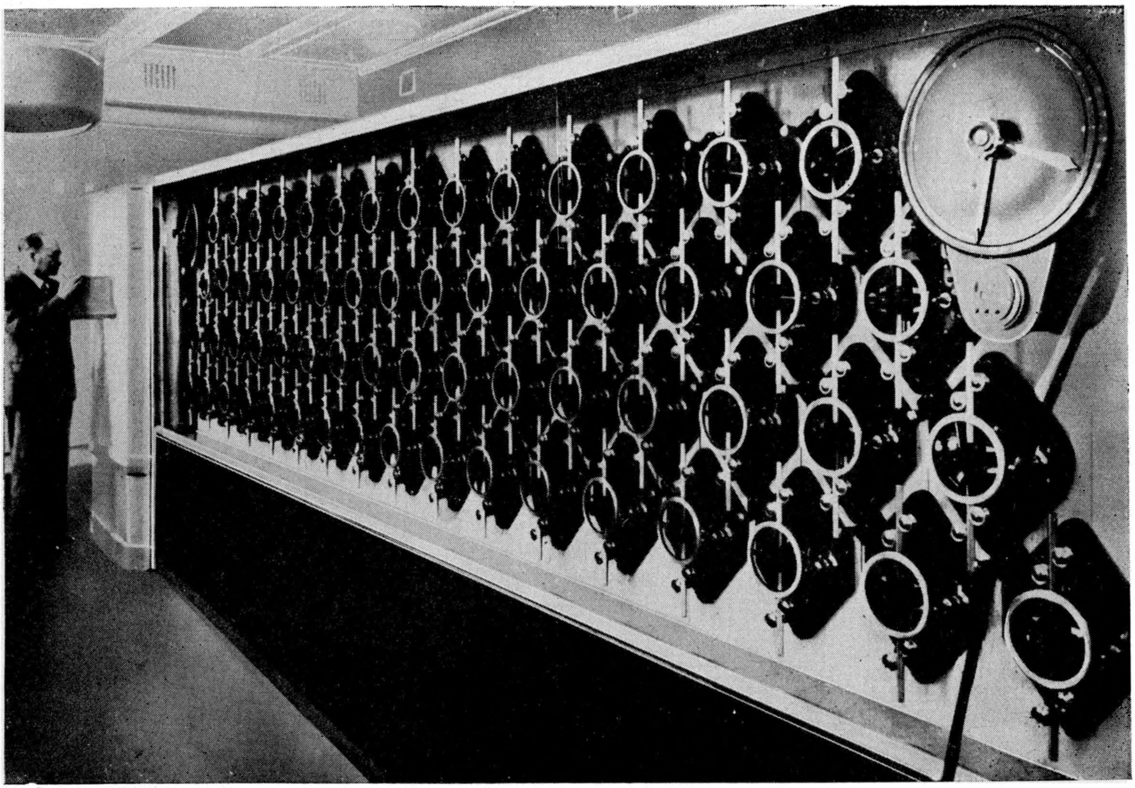


Fig. 4

The new German tide predicting machine.

For each species there is an over-all factor which depends upon observation. Table 3 gives an analysis of the mean spring tide for Avonmouth, Bristol.

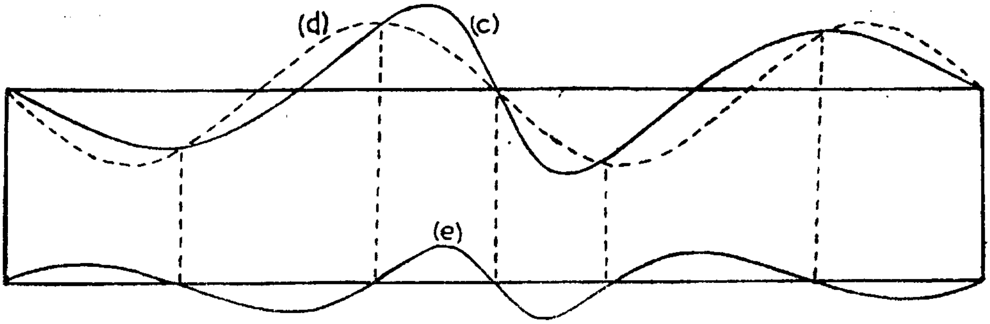


Fig. 3

Deduction of higher species of shallow water tides from change of shape of progressive wave.

An important indication from this table is that the amplitudes of species of tides diminish very slowly. We have what is called "slow convergence", and the consequence is that for such a place as Avonmouth it is necessary to consider shallow-water species of a much higher order than is usually imagined. The Tables shew the errors of predictions if the tide-predicting machine includes only the sixth-diurnal species of constituents. Just after low water the error would be more than one foot. If all the eighth-diurnal constituents were included the error would still be 0.7 ft. The error in the time of low water would be 13 minutes in the one case and eight minutes in the other case. This time error would affect the use of reduction tables, and cause serious errors. The errors noted assume that all the constituents in each species are on the machine, an assumption which is not justified. In fact, there is not a single machine in existence which adequately represents even the sixth-diurnal tides.

Table 4 gives a summary of the shallow-water constituents on the larger tide-predicting machines. The machines are arranged in order of date of manufacture and it is clearly shewn that there has been a definite trend of design to include more and yet more of these constituents. The latest machine built in Germany has made provision for far more of these constituents than any machine previously in existence. The sixth-diurnal and eighth-diurnal constituents on the U.S.A. machine are totally inadequate to represent these two species.

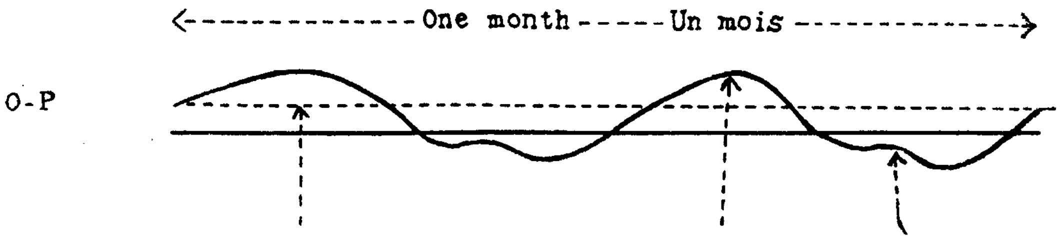
It will be noted that the two Tidal Institute machines do not have any eighth-diurnal constituents. When these machines were designed, in the case of the Kelvin machine, or reconstructed, in the case of the Légé machine, it was the considered judgment of the Institute that if the eighth-diurnal constituents were of any degree of importance then it was quite likely that the higher species of constituents were each of nearly the same degree of importance, so that as the convergence was slow it would be impracticable to use the machines at all so as to give in a direct way the final prediction. Other factors arise also. Any great increase in the number of constituents will swell the size of the machine out of all proportion to the advantage to be gained. Fig. 4 shews the immense size of the new German machine. The mechanical difficulties such as the cumulative effect of the component pulleys on the summation tape by stretching can be overcome by one means or another. It is a matter of capital cost relative to the advantage gained.

Undoubtedly there are occasions when a large machine will just cope with a problem which is beyond the capacity of smaller machines, but there will be problems which will be beyond the capacity of the large machines. If the demand for hourly heights of tide increases then larger machines will have to be regarded as a necessity, for there is no very satisfactory method of computing hourly heights by other means. The only practicable method is to use reduction tables which give the height of tide between low water and high water for given ranges of tide. As a rule these are not usable if there is much diurnal tide, and the computations are very tedious. It would be possible to develop methods which would give the tide in lunar time in a similar way to that in connection with the method of correction later explained for high and low waters. This again would be rather tedious. If it were possible to design shallow water components which would have variable amplitudes according to the amplitude of the semi-diurnal tide then the problem would be greatly simplified, but such a solution is not yet in sight. For the higher species of constituents, for special problems it might be worth while having a single constituent for each species whose amplitudes might be varied daily according to the amplitude of the semi-diurnal tide. While this would mean stopping the machines daily to make the adjustments, the loss of time, expressed in terms of wages, would be less costly than the costs of capital sunk in a very large machine.

Fig. 5

Explanation of shallow-water correction constituents.

P : Prediction by diurnal and semi-diurnal constituents.
O : Observed values.



Mean Difference (Constant)	Variation with period of about 14 lunar days	Variation with period of about 7 lunar days
M4	MS4	S4
M6	2MS6	2SM6
M8	3MS8	2(MS)8
M10	4MS10	(2S3M)10
etc. (1)	etc. (2)	etc. (3)

(1) Combined to make one constituent C (00).
(2) Combined to make one constituent C (25) with phase increment of 25°24 per lunar day.
(3) Combined to make one constituent C (50) with phase increment of 50°47 per lunar day.

The constituents combine in different ways for HWH, HWT, LWH, LWT, but the method of analysis deals separately with these and no attention needs to be paid to the component parts. Easch constitent C (n) expresses the resultant effect of a large number of small contributions.

Pending a mechanical solution to the problem, the Tidal Institute has developed a powerful method of dealing with it so far as the prediction of high and low waters is concerned. This method is called the Method of Harmonic Shallow-water Corrections. Referring to fig. 5 if the available machine is used to prepare a prediction using only the astronomic constituents for the diurnal and semi-diurnal tide, then the difference with observation will shew periodic changes of some complexity but of obvious regularity. The principal variation would have a period of a fortnight, recurring with some approximately constant relation to the time of spring tides. Other variations would have periods of a month or of a fraction of a month. We can readily shew that such constituents as M_4. M_6, ...would have an average or constant effect upon the high water height, say, while a group of constituents associated together with MS_4, $2MS_6$,... would have a period equal to the period of spring tides. For each variation a constituent could be found by analysis which would represent the effects of a large number of normal shallow-water constituents, and Table 5 gives a list of the more important constituents, with a brief indication of the normal constituents represented. It can be shewn that the observations can be compared with the primary prediction from the machine and that the difference can be analysed as though the high waters were equally spaced in lunar time. Consequently the corrections can be obtained by a special use of the tide-predicting machine by setting up the constituents on those machine components which have the same increment of angle in a lunar day, and thence by reading the machine at intervals of half a lunar day (as shewn by M_2) the corrections can be read off and allocated to the actual primary prediction.

By an expansion of the theory we can shew that it is not necessary to limit the primary prediction in the way indicated. In fact it is advisable to set up on the machine for the primary prediction all the components available on the machine as this will reduce the size of the correction. It is important to notice, however, that the primary prediction and the corrections go together, so that it is essential to know the primary constituents in order to use the corrections.

An important modification also arises, for as the theory is explored it becomes evident that if any suitable primary prediction has been made the basis the corrections give the right prediction. The primary prediction may be the deep water tide at the entrance to a long shallow estuary and river, and the corrections may be made on the one place so as to give predictions for places up the estuary. This method was singularly successful in the case of tidal predictions for Basra and Abadan. These are obtained by shallow-water corrections on the Shatt-al-arab Bar. The same principle is used to obtain London Bridge predictions from those for Southend, though it is necessary for Southend predictions themselves to use the primary prediction as for Southend and to correct it for Southend. There is thus no limit to the usefulness of this very flexible method.

The question may be raised as to whether non-harmonic methods will give results with the same accuracy as the shallow-water corrections. The answer is that a non-harmonic method is usually based upon the variations in the period of spring tides and that the corrections for parallax would need to be considerably amplified to deal with the shallow water effects. Probably the complications would become such as to render such methods intractable. The main difficulty, of course, arises when there are diurnal tides of some importance. Non-harmonic methods are of no use whatever for Avonmouth, where the diurnal tide is small, and they are useless for Basra, where the diurnal tide is large. In fact, at Basra we are faced with the existence of fifth and seventh diurnal tides for which no provision has been made on any machine.

Table 6 gives the amplitudes of the shallow water correction constituents for Antwerp. The constituents used for the primary prediction are also given (only the amplitudes are here referred to but of course each constituent has its proper phase-lag). The table shews that the sixth-diurnal tides are little less important than the quarter-diurnal tides and it may be taken that the higher species of tides will converge very slowly. The value of C (00) for the Low Water Height Correction is—0.093 ft. shewing that M_8, M_{10}, M_{12}, ... must all be comparable in size with M_6. The large corrections for the times of Low Water are also indicative of the same deduction. Table 7 gives details of the prediction of tides for Antwerp for March 1948. The primary prediction from the machine is given first and is followed by the correction. The final table of predictions gives the sums of these. The size of the corrections shews the absolute necessity of their use.

The Antwerp authorities have recently reported on the accuracy of the predictions, with which they are well satisfied. Indeed the same satisfaction has been expressed wherever the method has been applied, but while this is gratifying it is not held by me that the Method represents a final attainment in this difficult problem, and research is continually in progress towards simpler methods which will be equally accurate, and which will avoid the present necessity of computing separate corrections to be added to the primary predictions. The ideal would be for the machine used to produce the final result.

TABLE I.

Relative importance of shallow-water constituents.

Semidiurnal constituents		Quarterdiurnal constituents		Sixthdiurnal constituents		Eighthdiurnal constituents	
M2	1.00	M4	1.00	M6	1.00	M8	1.00
S2	0.50	MS4	1.00	2MS6	1.50	3MS8	2.00
		S4	0.25	2SM6	0.75	2(MS)8	1.50
				S6	0.12	3SM8	0.50
						S8	0.06

Even with only two main constituents the terms of comparable size within each species increase with the species number. The complications increase greatly when there are more semidiurnal constituents to be considered.

N2	0.20	MN4	0.40	2MN6	0.60	3MN8	0.80
		SN4	0.20	2SN6	0.38	2(MN)8	0.24
				2NM6	0.12	2MSN8	1.20
						2SMN8	0.60

Similar combinations arise with K2, L2, etc., and the diurnal and semidiurnal constituents combine to give third-and fifth-diurnal constituents.

TABLE II.

Relative amplitudes at springs and neaps.

Species-number	2	4	6	8
Springs	1.50	2.25	3.38	5.06
Average tides	1.00	2.00	1.00	1.00
Neaps	0.50	0.35	0.12	0.06

This shews the increase of importance of these constituents as the tide increases in range, and confirms the indications of the upper table that one single constituent cannot represent a species.

TABLE III.

Avonmouth mean spring tide.

Amplitudes of shallow tides.	
Semidiurnal	18.53 ft.
Quarterdiurnal	2.50 ft.
Sixthdiurnal	0.75 ft.
Eighthdiurnal	0.40 ft.
Tenthdiurnal	0.20 ft.
Twelfthdiurnal	0.15 ft.

Note the slow convergence. This shews that the higher species of tides are not negligible.

	Errors (in feet) after including all species up to the			
Time	6th	8th	10th	12th
0:00	0.3	0.0	0.1	0.0
0:30	0.1	0.2	0.1	0.1
1:00	—0.3	0.1	—0.1	0.0
1:30	—0.4	—0.1	—0.1	—0.1
2:00	0.1	0.0	0.2	0.0
2:30	0.3	0.0	0.0	0.0
3:00	0.3	0.0	—0.2	0.0
3:30	0.1	0.1	0.0	0.0
4:00	—0.3	0.1	0.2	0.0
4:30	—0.5	—0.2	0.0	0.0
5:00	—0.1	—0.2	—0.1	0.0
5:30	0.5	0.1	—0.1	—0.1
6:00	0.6	0.3	0.2	0.0
6:30	0.0	0.1	0.2	0.2
7:00	—1.1	—0.7	—0.5	—0.4
7:30	0.0	0.3	0.3	0.3
8:00	0.4	0.4	0.2	0.0
8:30	0.3	—0.1	—0.2	—0.2
9:00	0.1	—0.2	—0.1	0.1
9:30	—0.1	0.0	0.1	0.1
10:00	—0.3	0.0	0.0	—0.2
10:30	—0.1	0.2	0.0	0.0
11:30	0.1	—0.3	—0.1	—0.1
Error in low water time	13 min.	8 min.		
If a reduction table is used for heights at times between LW and HW the error at MTL will be about	1.3 ft.	0.7 ft.		

TABLE IV.

The trend of design of tide-predicting machines.

		USA	TI (K)	TI (L)	G
Diurnal	NO1	...	...	...	1
	SO1	...	...	...	1
	MP1	...	...	...	1
Semidiurnal	MNS2	...	...	1	1
	KJ2	...	...	...	1
	2SM2	1	1	1	1
	MSN2	...	...	...	1
	OP2	...	...	...	1
	MKS2	...	...	...	1
Thirddiurnal	M3	1	1	1	1
	MK3	1	1	1	1
	MO3	1	1	1	1
	SK3	...	...	1	1
	SO3	...	...	...	1

		USA	TI (K)	TI (L)	G
Quarterdiurnal	M4	1	1	1	1
	MS4	1	1	1	1
	MN4	1	1	1	1
	MK4	1	...	1	1
	S4	1	1	1	1
	SN4	...	...	1	1
	SK4	...	...	...	1
Sixthdiurnal	M6	1	1	1	1
	2MS6	...	1	1	1
	MSN6	...	...	1	1
	2MN6	...	1	1	1
	2SM6	...	...	1	1
	2MK6	...	...	1	1
	MSK6	...	...	1	1
	2SN6	...	...	1	...
	S6	1	...	...	...
Eighthdiurnal	M8	1	...	...	1
	3MS8	...	...	...	1
	2(MS)8	...	...	...	1
	2MSN8	...	...	...	1
	3MN8	...	...	...	1
	2MSK8	...	...	...	1

USA : United States Coast and Geodetic Survey, 37 components, 1894-1910.

TI (K) : Tidal Institute, Kelvin, 26 components, 1924.

TI (L) : Tidal Institute, Légé, 40 components, 1908-1936.

G : Hydrographic Institute, Hamburg, 61 components, 1938.

TABLE V.

Harmonic shallow-water constituents.

Symbol	Increment of phase in degrees per lunar day	Constituents represented	
C(00)	00.000	M4, M6, M8, M10, M12, ...	
C(01)	01.020	Sa, MST6, MTS6, ...	
C(02)	02.040	Ssa, MKS2, MSK2, 2MKS4, 2MSK4, OP2, ...	
C(11)	11.713	SN4, MSN6, 2MSN8, SNM2, ...	
C(13)	13.523	MN4, ML4, 2MN6, 3MN8, ...	
C(25)	25.236	MS4, 2MS6, 3MS8, 4MS10, ...	
C(27)	27.276	MK4, 2MK6, 3MK8, ...	
C(36)	36.949	2SN6, 2SMN8, ...	
C(38)	38.759	MSN2, MNS2, 2MSN4, 2MNS4, ...	
C(50)	50.472	S4, 2SM2, 2SM6, 2(MS)8, ...	
C(52)	52.513	SK4, MSK6, 2MSK8, ...	
C'(11)	11.598	MP1, SO3	also 5th diurnals and 7th diurnals.
C'(12)	12.618	MS1	
C'(13)	13.638	MK3, MO3	
C'(27)	27.161	MJ3, MQ3	
C'(36)	36.834	SP3,	
C'(38)	38.874	SO1, SK3	
C'(40)	40.915	KO3	

Each constituent can be set up on the tide-predicting machine by using one of the constituents represented or by any of the diurnal or semidiurnal constituents which contribute to the compound constituent. Thus C(27) can be set up on MK4 or on K2 and the readings of the machine are taken at intervals of 12 lunar hours. They are attributed to the corresponding high water or low water and the machine needs to be run four times to give HWH, HWT, LWH, LWT.

By an expansion of the theory the machine predictions for the primary prediction can include the use of all the constituents which are available on the machine, so reducing the size of the corrections.

TABLE VI.

Example of shallow-water corrections

Antwerp

Amplitudes of constituents for primary prediction (values of H)

Set on Kelvin 26 component machine.

Ssa	0.09	K1	0.20	M2	6.44	M3	0.02	M4	0.43	M6	0.28
Sa	0.20	O1	0.30	S2	1.56	MK3	0.14	MS4	0.28	2MS6	0.23
		P1	0.15	N2	0.98	MO3	0.14	MN4	0.16	2MN6	0.15
		Q1	0.12	M2	0.72			S4	0.01		
		J1	0.03	L2	0.62						
		S1	0.03	K2	0.51						
				V2	0.47						
				2N2	0.30						
				T2	0.13						
				2SM2	0.18						

Amplitudes of shallow-water correction constituents.

	Period	HWH	LWH	HWT	LWT
C(00)	(Constant)	0.29 feet	0.93 feet	— 6.3 min.	27.5 min.
C(01)	One year	0.05	0.15	2.6	1.1
C(02)	Half year	0.25	0.08	2.0	2.5
C(11)	Month	0.24	0.29	3.3	4.2
C(13)	Month	0.08	0.11	5.6	3.2
C(25)	Month/2	0.17	0.32	3.7	6.5
C(27)	Month/2	0.12	0.23	3.8	5.0
C(36)	Month/3	0.08	0.21	0.4	1.1
C(38)	Month/3	0.28	0.25	0.5	2.7
C(50)	Month/4	0.23	0.25	2.7	3.3
C(52)	Month/4	0.10	0.05	4.1	2.1

TABLE VII (TABLEAU VII)

Example of prediction for Antwerp, March, 1948
(Exemple de prédiction pour Anvers, mars 1948)

	High water (Pleine mer)								Low water (Basse mer)							
	Time (Heure)		Height (Hauteur)		Time (Heure)		Height (Hauteur)		Time (Heure)		Height (Hauteur)		Time (Heure)		Height (Hauteur)	
1	0706	— 5	17.0	0.1	1934	— 5	16.3	.0	0117	35	1.3	— .4	1346	31	0.5	— .5
2	0750	— 7	16.4	0.0	2019	— 9	15.5	— .2	0159	29	1.7	— .7	1429	25	1.4	— .9
3	0842	— 10	15.5	— .1	2114	— 10	14.4	— .3	0244	22	2.3	— 1.1	1516	20	2.4	— 1.4
4	0949	— 10	14.5	— .1	2230	— 7	13.7	— .3	0344	20	3.0	— 1.6	1620	20	3.2	— 1.8
5	1114	— 4	14.2	— .1	2354	0	13.8	— .1	0501	22	3.3	— 1.9	1740	26	3.6	— 1.9
6	...	...	...	...	1228	4	14.7	.0	0623	30	3.0	— 1.8	1854	34	3.3	— 1.6
7	0100	8	14.5	— .1	1327	10	15.5	.0	0730	39	2.2	— 1.4	1954	41	2.7	— 1.2
8	0155	11	15.3	.0	1415	10	16.2	.0	0825	44	1.3	— 1.0	2044	44	2.1	— .9
9	0240	7	15.9	.0	1459	4	16.5	.1	0912	44	0.8	— .8	2127	41	1.9	— .8
10	0319	— 1	16.0	.2	1536	— 6	16.5	.4	0955	39	0.8	— .8	2202	35	1.9	— .9
11	0353	— 11	15.9	.6	1608	— 14	16.4	1.0	1030	33	1.0	— 1.0	2235	30	2.0	— 1.2
12	0421	— 16	15.7	.8	1639	— 16	16.3	1.0	1100	30	1.3	— 1.3	2301	29	2.0	— 1.3
13	0450	— 16	15.9	1.1	1709	— 15	16.4	1.1	1126	29	1.4	— 1.3	2331	30	1.7	— 1.1
14	0519	— 15	16.3	1.0	1740	— 14	16.7	.8	1155	30	1.3	— 1.0	...	...	...	...
15	0551	— 14	16.9	.7	1815	— 14	16.9	.4	0001	30	1.4	— .9	1225	29	1.0	— .6
16	0629	— 16	17.0	.2	1852	— 19	16.8	.0	0036	26	1.2	— .5	1259	23	1.0	— .4
17	0709	— 20	17.0	.0	1934	— 21	16.2	— .1	0114	19	1.3	— .4	1339	14	1.3	— .5
18	0757	— 23	16.3	.0	2024	— 22	15.1	.1	0159	11	1.9	— .6	1426	7	2.1	— .9
19	0857	— 20	15.2	.4	2130	— 17	13.8	.5	0255	6	2.7	— 1.1	1529	5	3.1	— 1.4
20	1017	— 14	14.3	.7	2303	— 10	13.2	.8	0411	8	3.5	— 1.5	1656	10	3.8	— 1.6
21	1151	— 6	14.4	.9	...	...	...	...	0549	15	3.5	— 1.6	1829	19	3.5	— 1.5
22	0033	— 4	13.8	.9	1304	— 2	15.5	.9	0710	25	2.5	— 1.4	1941	29	2.4	— 1.1
23	0137	— 3	15.0	.7	1402	— 4	16.8	.5	0815	32	1.2	— .9	2040	35	1.6	— .6
24	0229	— 6	16.3	.6	1454	— 10	17.6	.4	0909	36	0.0	— .4	2130	37	1.0	— .2
25	0315	— 14	17.0	.4	1539	— 15	17.9	.2	0957	37	- 0.5	— .1	2214	36	0.7	.0
26	0359	— 16	17.5	.3	1621	— 16	18.0	.3	1040	36	- 0.7	.0	2257	36	0.6	.0
27	0440	— 16	17.7	.4	1703	— 14	17.7	.4	1121	35	- 0.5	.0	2336	35	0.7	— .1
28	0520	— 12	17.7	.5	1744	— 9	17.4	.5	...	...	...	...	1201	34	- 0.2	— .1
29	0600	— 8	17.5	.5	1824	— 6	16.9	.3	0014	33	0.8	— .1	1239	31	0.3	— .2
30	0641	— 6	17.3	.2	1901	— 7	16.3	.0	0050	29	1.0	— .3	1314	26	1.0	— .4
31	0724	— 9	16.6	— .1	1944	— 10	15.5	— .3	0129	24	1.3	— .6	1351	20	1.7	— .7

The first column gives the primary machine prediction, and the second column gives the correction.
(La première colonne donne la prédiction primaire à la machine et la deuxième colonne donne la correction).

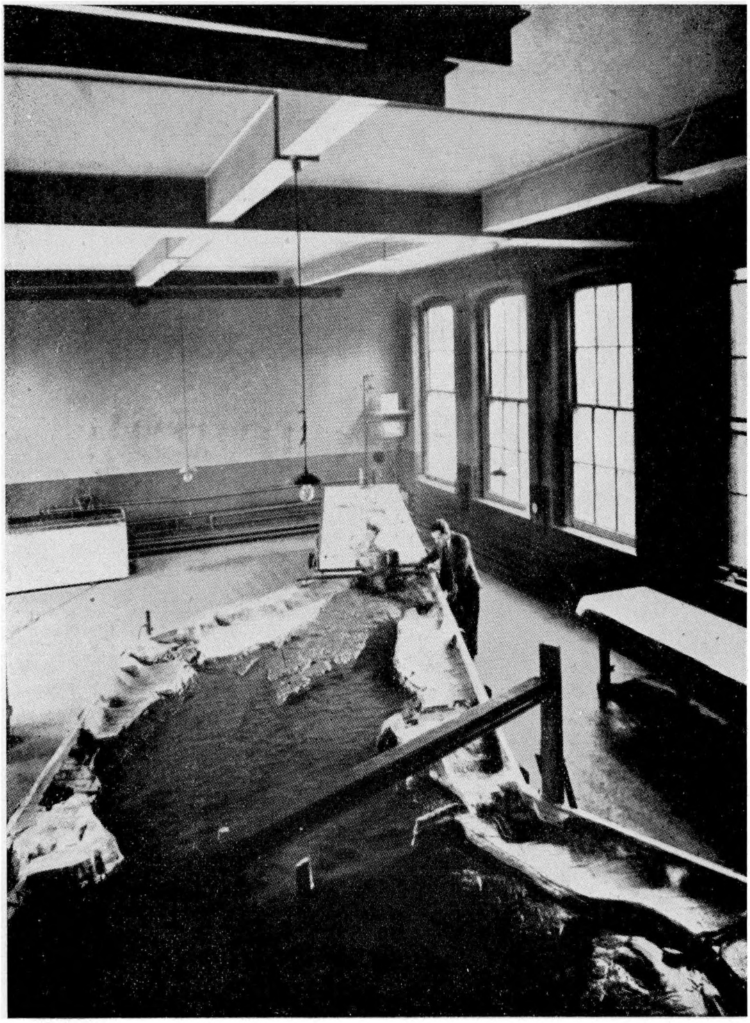


Fig. 1
General view of Severn Model,

Fig. 2
Severn Model showing plunger.