DEFLECTORS AND THEIR USE IN RELATION TO MAGNETIC COMPASSES.

A critical survey of the generally accepted method of using these instruments.

By

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SLOUGH, November 1947.

(The mathematical theory of the directive force at the compass is concisely stated. At this stage it is assumed that the deflector gives an accurate measure of the directive force, and upon this assumption the method of employment of the deflector is developed.

Resulting from various unavoidable and accidental inaccuracies in either reading or setting the instrument, certain fractional errors occur in the assessment of the directional force. The relation between these inaccuracies and the resulting error is examined mathematically for two well-known types of deflector.

As a result of these investigations the following conclusions are reached :—

(a) How to use the instruments so that the unavoidable errors of operation upset the accuracy of the measurement by the least amount.

(b) How to avoid conditions which cause these unavoidable errors of operation to upset the accuracy of the measurement to a serious degree.

(c) How to assess the result of correcting a compass by means of a deflector when no subsequent swing has taken place for the purpose of obtaining an accurate deviation card.)

SECTION I.

The theoretical basis of the use of the deflector.

There are various forms of deflector of which two well-known examples are the Thompson deflector and the de Colongue deflector.

In each of these instruments the measurement read off from the scale of the adjusting device is roughly proportional to the directive force H' on the heading on which the measurement is made.

When using either of these instruments or the vibrating needle, if (for the evaluation of λ) the actual magnitude of the force H' is required as compared with H the earth's horizontal field, an observation must be made at a suitable shore station remote from magnetic material. In the case of the two deflectors this entails landing the compass bowl as well as the instrument.

Three examples are given of the use of these observations of the magnitude of the horizontal component of the total force.

1. **Determination of approximate coefficients B°, C°, D° and E°.**—To a first-order accuracy, we have from deviation theory :—

$$\frac{H'}{\lambda H} \cos \delta = 1 + \sin B^\circ \cos \zeta' - \sin C^\circ \sin \zeta' + \sin D^\circ \cos 2\zeta' - \sin E^\circ \sin 2\zeta'$$

where B°, C°, D° and E° are approximate coefficients measured in degrees and ζ' is the compass course.

Taking observations on compass N., S., E., and W. gives *

$$kR_n = \frac{H'_n}{\lambda H} = 1 + \sin B° + \sin D° \qquad (1)$$

$$kR_s = \frac{H'_s}{\lambda H} = 1 - \sin B° + \sin D° \qquad (2)$$

$$kR_e = \frac{H'_e}{\lambda H} = 1 - \sin C° - \sin D° \qquad (3)$$

$$kR_w = \frac{H'_w}{\lambda H} = 1 + \sin C° - \sin D° \qquad (4)$$

When R_n, R_s, R_e and R_w are the readings of the deflector (or the reciprocals of the squares of the periods of the vibrating needle) and k is the constant of the instrument for the particular values of λ and H.

Adding the four equations gives :—

$$k = \frac{4}{R_n + R_s + R_e + R_w} \qquad (5)$$

from which k is found. Making use of this value of k : B°, C° and D° may be evaluated from

$$\text{Sin } B° = \frac{k}{2}(R_n - R_s) \qquad (6)$$

$$\text{Sin } C° = \frac{k}{2}(R_w - R_e) \qquad (7)$$

$$\text{Sin } D° = \frac{k}{4}[(R_n + R_s) - (R_e + R_w)] \qquad (8),$$

To evaluate E° four more observations are required on headings NE., SE., SW., NW. These give :—

$$k\,R_{ne} = \frac{H'_{ne}}{\lambda H} = 1 + \frac{\sin B°}{\sqrt{2}} - \frac{\sin C°}{\sqrt{2}} - \sin E° \qquad 9)$$

$$k\,R_{se} = \frac{H'_{se}}{\lambda H} = 1 + \frac{\sin B°}{\sqrt{2}} + \frac{\sin C°}{\sqrt{2}} + \sin E° \qquad (10)$$

$$k\,R_{sw} = \frac{H'_{sw}}{\lambda H} = 1 - \frac{\sin B°}{\sqrt{2}} + \frac{\sin C°}{\sqrt{2}} - \sin E° \qquad (11)$$

$$k\,R_{nw} = \frac{H'_{nw}}{\lambda} = 1 - \frac{\sin B°}{\sqrt{2}} - \frac{\sin C°}{\sqrt{2}} + \sin E° \qquad (12)$$

Hence : $\sin E° = \frac{k}{4}[(R_{se} + R_{nw}) - (R_{ne} + R_{sw})].$ $\qquad$ (13)

2. **Correcting the compass by deflector.**—For this purpose only the deflectors are used, since the vibration needle is not suitable. To correct B° it is seen from equation (6) that for zero value of B° we require $R_n = R_s$. Since inserting permanent magnets in the fore-and-aft direction increases R_n by exactly the amount it decreases R_s or vice versa, the value of R_n and R_s are each made equal to $(R_n + R_s)/2$ by inserting fore-and-aft magnets until this result is achieved.

To correct C° we see from equation (7) that for zero value of C° we require $R_e = R_w$, and after correction these each become $(R_e + R_w)/2$ the correction being made by placing permanent magnets athwartships so that they add to one the amount they subtract from the other.

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* Taking $\cos\delta = 1$ in each case. Written in full, equation (1) should read :—

$$kR_n \cos\delta_n = \frac{H'_n \cos\delta_n}{\lambda H} = 1 + \sin B° + \sin D°.$$

For correction of D°, equation (8) shows that we require $\frac{(R_n + R_s)}{2} - \frac{(R_e + R_w)}{2}$ equal to zero. Equation (8) shows also that when D° is positive $\frac{(R_n + R_s)}{2} - \frac{(R_e + R_w)}{2}$ is positive whereas it is negative when D° is negative. Hence when correcting D°, if $\frac{(R_n + R_s)}{2} - \frac{(R_e + R_w)}{2}$ is positive the spheres should be moved in and when it is negative the spheres should be moved out.

It is not usual to use the deflector for correcting coefficient E°, but equation (13) shows how this may be done : If $(R_{se} + R_{nw})$ exceeds $(R_{ne} + R_{sw})$ then the E° to be corrected is positive and for the correction of this the spheres must be slewed clockwise until $(R_{se} + R_{nw}) - (R_{ne} + R_{sw})$ is zero.

To estimate the D° which has to be corrected we have from (5) and (8) :—

$$\text{Sin D}^\circ = \frac{(R_n + R_s) - (R_e + R_w)}{R_n + R_s + R_e + R_w}$$

Having noted the size and distance of the spheres fitted, we find from the tables, the D° already being corrected. Adding the D° now present to the D° already corrected gives the "Total D°".

From equation (13) and the sum of the four equations (9), (10), (11) and (12) we have :—

$$\text{Sin E}^\circ = \frac{(R_{se} + R_{nw}) - (R_{ne} + R_{sw})}{R_{se} + R_{nw} + R_{ne} + R_{sw}}$$

From which E° may be calculated.

Then if M° be the angle of slew

$$\text{Tan } 2\text{M}^\circ = \frac{\text{E}^\circ}{\text{Total D}^\circ}$$

or

$$\text{M}^\circ = \frac{1}{2} \tan^{-1}\left(\frac{\text{E}^\circ}{\text{Total D}^\circ}\right)$$

Coefficient A° cannot be corrected by means of a deflector since λ HA is a force *always at right angles* to magnetic North.

3. **To evaluate λ.** From equation (1) if the observation reading RA, be taken ashore, we have :—

$$k\,\text{R}_\text{A} = \frac{H}{\lambda H} = \frac{1}{\lambda} \quad \text{where } \lambda = \frac{1}{k\,\text{R}_\text{A}}$$

$$\text{and } \lambda = \frac{1}{4} \frac{(R_n + R_s + R_e + R_w)}{\text{R}_\text{A}}$$

i. e. the mean reading of the directive force taken in the ship divided by the reading taken ashore.

If the deviations are large this becomes :—

$$\lambda = \frac{\frac{1}{4}(R_n \cos \delta_n + R_s \cos \delta_s + R_e \cos \delta_e + R_w \cos \delta_w)}{\text{R}_\text{A}}$$

SECTION II.

The Thompson Deflector.

(a) *General description.*—The Thompson deflector consists of a hinged magnet system the moment of which may be varied, the relative values of the moment being indicated on a scale of arbitrary divisions.

The magnet system is mounted on a framework which fits on the top of the compass bowl. The framework, which is carefully centred, may be turned in azimuth. A pointer shows the direction of the magnetic force at the compass needles due to the hinged magnet system of the device.

(b) *The instructions for use.*—With the ship maintained steady on a given course, by means of another compass or otherwise, the deflector is turned in azimuth, and the strength of its field is varied until the card has been deflected through 90°; the pointer of the deflector being kept throughout over some fixed reading of the deflected card, say 80°.

NOTE. — In a widely distributed instruction book for the use of the deflector, the fact that the instrument should deflect the card through 90° was expressed in the words "obtain **normal** deflection". It cannot be too strongly emphasised that the word **normal** in this context was and is used in the strict mathematical sense where its meaning is "at right angles". This somewhat unfortunate use of a common word in a strict technical sense must be appreciated therefore, as the effective operation of the instrument depends entirely upon the fact that the deflection produced is 90°.

(c) *Theory of the instrument.*—Figure 1 shows the triangle of forces in which the vector H' represents the magnetic force directed towards compass North. The vector D represents the magnetic force due to the deflector and the vector F represents the resultant of H' and D along which the magnetic needles align themselves. The instrument having been used correctly the angle between the vectors H' and F is 90°.

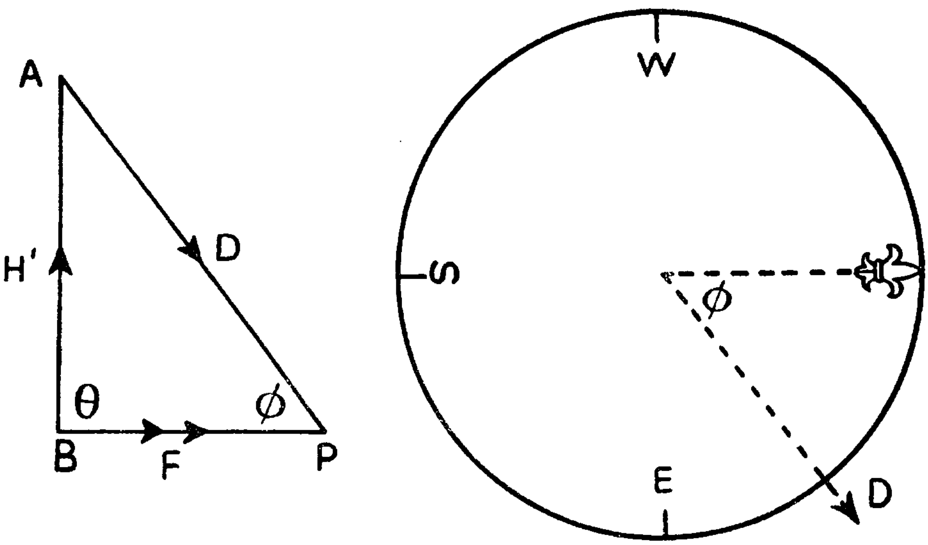

FIG. 1

It will be noted that $\emptyset$, the angle between the vectors D and F, is the particular angle on the card over which the pointer of the deflector is kept throughout the operation of setting. The angle $\emptyset$ is therefore constant on each occasion and

$$D = \frac{H'}{\sin \emptyset}$$

In the construction of the instrument if we assume that the reading of the scale is proportional to D, then we may write : D = kS where S is the scale reading associated with D.

So that $k\,S = \dfrac{H'}{\sin \emptyset}$ and $S = k_1 H'$.

Thus $k_1 = \dfrac{1}{(k \sin \emptyset)}$ = constant and S is proportional to H' as required.

(d) *Errors of the instrument.*—Consider the general case in which the deflector produces a magnetic force D, which causes a clockwise deflection θ, when directed at an angle $\emptyset$ to clockwise of the North point of the deflected compass card. Figure 2 illustrates this.

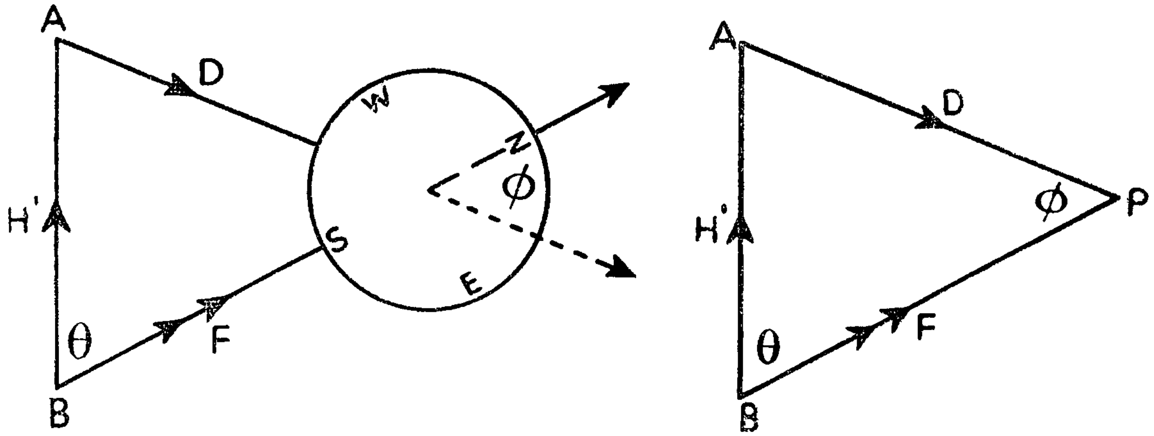

FIG. 2

If no consideration were given to errors it might appear that any configuration of the force triangle would be satisfactory provided that it was repeated on each heading.

Thus by the sine formula :—

$$\frac{D}{\sin\theta} = \frac{H'}{\sin\emptyset} \quad \text{so that } D = \frac{H'\sin\theta}{\sin\emptyset}$$

The angles θ and $\emptyset$ being constants, D is directly proportional to H' and the variations of H' may be assessed by the variations in D.

It is desirable, however, to investigate the relative merits of the various forms which the force triangle may take. In the following pages the various forms of error to which the deflector is subject are discussed together with their probable sizes and the best methods of minimising them.

(e) *ERROR I : The fractional error in D due to the limit of accuracy of the observation of the angle $\emptyset$.*—The angle $\emptyset$ is assessed by deciding when a pointer above the glass of the compass bowl is in line with a reading on the card say half an inch below the glass. In order to modify the angle $\emptyset$, without altering θ, it is necessary :

(1) To alter the setting of the hinged magnet system which causes $\emptyset$ to alter.

(2) To alter the attitude of the deflector with respect to the lubber's point which again causes $\emptyset$ to alter.

Unfortunately then the operation of the instrument does not consist merely of reading the angle $\emptyset$, but of making it have a specified reading without upsetting θ. Fortunately, however, the action of altering the attitude of the deflector with respect to the lubber's point can be arranged to have practically no effect upon the angle θ, provided θ is as near 90° as possible. This is explained in error II.

Practical experience shows that it is difficult to be sure that $\emptyset$ is less than 2° in error so we will assume expert operation of the instrument and calculate the effects of a 1° uncertainty of the angle $\emptyset$.

In figure 3 the angle $\emptyset$ has erroneously been adjusted to $\emptyset + \varepsilon$ thus causing the vector D to change to D_I since the direction of F has not altered.

The fractional error in D will be : $\dfrac{D_I - D}{D}$

By the sine formula :—

$$\frac{D}{\sin\theta} = \frac{H'}{\sin\emptyset} \quad \text{and} \quad \frac{D_1}{\sin\theta} = \frac{H'}{\sin(\emptyset + \varepsilon)}$$

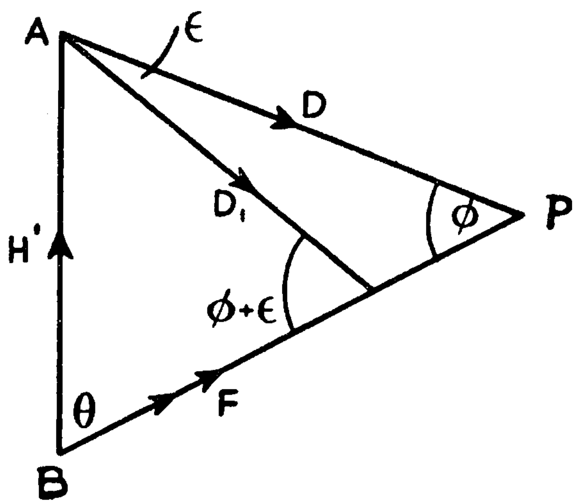

FIG. 3

Hence :—

$$\frac{D_1 - D}{D} = \left(\frac{H' \sin \theta}{\sin (\emptyset + \varepsilon)} - \frac{H' \sin \theta}{\sin \emptyset}\right) \Big/ \left(\frac{H' \sin \theta}{\sin \emptyset}\right)$$

Multiplying numerator and denominator by $\frac{\sin \emptyset \sin (\emptyset + \varepsilon)}{H' \sin \theta}$

$$\frac{D_1 - D}{D} = \frac{\sin \emptyset - \sin (\emptyset + \varepsilon)}{\sin (\emptyset + \varepsilon)} = \frac{\sin \emptyset - \sin \emptyset \cos \varepsilon - \cos \emptyset \sin \varepsilon}{\sin \emptyset \cos \varepsilon - \cos \emptyset \sin \varepsilon}$$

Taking $\cos \varepsilon = 1$, and dividing through by $\cos \emptyset$:—

$$\frac{D_1 - D}{D} = \frac{- \sin \varepsilon}{\tan \emptyset + \sin \varepsilon} \text{ to second order accuracy.}$$

Taking $\varepsilon = 1°$ and $\emptyset = 60°$ the fractional error due to the readings of $\emptyset$ will be :—

$$\frac{0{,}0175}{1.7321 - 0.0175} = 0.01 \text{ or } 1\ \%.$$

Table I shows the percentage error produced in the value of D per degree error in $\emptyset$ for various values of $\emptyset$. Since D is assumed proportional to H' this may be considered a percentage error in the estimation of H'.

TABLE I.

$\emptyset$	*Percentage error in the estimation of H' per degree error in $\emptyset$.*
80°	0,3 %
70°	0,6 %
60°	1,0 %
50°	1,5 %
40°	2,0 %

Thus if this were the only error, $\emptyset$ should be kept as large as possible.

(f) *ERROR II : The fractional error in H' due to the limits of accuracy of the estimation of the angle* θ.—The angle θ is observed by noting the difference between the reading of the card which lies opposite the lubber's point before and after the compass has been deflected. These two readings are each subject to observational error and their difference is subject to the addition of any constant or varying changes which have occurred in the direction of the ship's head during the operation of setting the deflector.

The error in this angle has then a constant part made up of the two errors of observation and any discrepancy between the course at the time of the first observation and the actual course upon which the ship is steadied. Here under sea-going conditions it would seem reasonable to assess the errors of observation at say ¼° each and the error of the course at say ½° thus allowing the constant error in the estimation of θ to be 1°. This error has also a variable part which is continually altering due to the unavoidable yaw and the operation of steering the ship.

Provided this yaw is sufficiently slow for the compass to respond to its effects, its instantaneous value may be added to the constant part of the error of θ. Under these conditions the error may be termed slow yaw.

Let $\theta + \alpha$ be the actual value of the angle which is measured to be θ. Let α_1 be the angle of slow yaw and α_2 be the angle of error of the actual steady course with relation to the ideal steady course, both α_1 and α_2 being measured positive to clockwise. Let α_3 be the amount by which θ has been under-estimated due to errors of reading the card. Then :—

$$\alpha = \alpha_1 + \alpha_2 + \alpha_3.$$

In setting the instrument the angle $\emptyset$ will be adjusted in the ordinary way since there is nothing to indicate that the angle θ has not been correctly set. In figure 4 the correct form of the force triangle A B P becomes changed to A B Q.

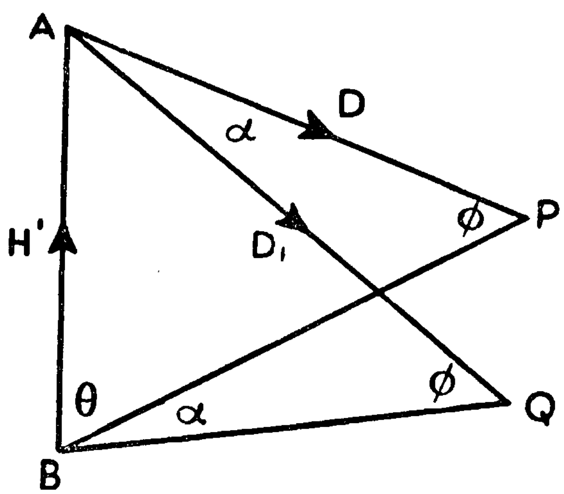

FIG. 4

In the triangle A B P, by the sine formula :—

$$\frac{D}{\sin\theta} = \frac{H'}{\sin\emptyset}$$

Similarly in triangle A B Q :—

$$\frac{D_1}{\sin(\theta + \alpha)} = \frac{H'}{\sin\emptyset}$$

so that

$$\frac{D_1}{\sin(\theta + \alpha)} = \frac{D}{\sin\theta}$$

and

$$D_I = \frac{D \sin(\theta + \alpha)}{\sin \theta}$$

The fractional error in D is :—

$$\frac{D_I - D}{D} = \left(\frac{D \sin(\theta + \alpha)}{\sin \theta} - D \right) / D$$

Multiplying the numerator and denominator by $\frac{\sin \theta}{D}$, gives

$$\frac{D_I - D}{D} = \frac{\sin(\theta + \alpha) - \sin \theta}{\sin \theta} = \frac{\sin \theta \cos \alpha + \cos \theta \sin \alpha - \sin \theta}{\sin \theta}$$

$$= \cot \theta \sin \alpha + \cos \alpha - 1$$

Hence the percentage error in the estimation of D (and therefore of H') is :—

$$(\cot \theta \sin \alpha + \cos \alpha - 1) \times 100\ \%.$$

Table II gives the value of this percentage error for various negative values of α, when θ is + 90°, + 80°, + 70°, + 60°, + 50° and + 40°. The negative values of α are chosen because when θ and α have opposite signs $\cot \theta \sin \alpha$ is negative so that, since $(\cos \alpha - 1)$ must always be negative, the error is slightly greater under these conditions than when θ and α have the same signs.

TABLE II.

Table of percentage error in the estimation of H' due to errors in θ for various positive values of θ and negative values of α.

θ	= 90°	= 80°	= 70°	= 60°	= 50°	= 40°
α						
— 1°	— 0,02	— 0,33	— 0,66	— 1,0	— 1,5	— 2,1
— 2°	— 0,06	— 0,67	— 1,4	— 2,0	— 3,0	— 4,2
— 3°	— 0,14	— 1,1	— 2,0	— 3,0	— 4,5	— 6,3
— 4°	— 0,24	— 1,5	— 2,9	— 4,3	— 6,1	— 8,6
— 5°	— 0,38	— 1,9	— 3,6	— 5,4	— 7,7	— 10,8
— 6°	— 0,55	— 2,4	— 4,3	— 6,6	— 9,3	— 13,0
— 7°	— 0,75	— 2,9	— 5,2	— 7,7	— 11,0	— 15,2
— 8°	— 0,97	— 3,4	— 6,0	— 9,0	— 12,6	— 17,6
— 9°	— 1,23	— 4,0	— 6,9	— 10,2	— 14,3	— 18,8
— 10°	— 1,52	— 4,6	— 7,8	— 11,2	— 16,1	— 22,2

Since errors in θ are by far the largest and may be quite undetectable, it is vital that the effect such errors have upon the setting of D shall be as small as possible. From the table it appears that the condition required is that $\theta = 90°$. That this is the value of θ for which the errors are least is confirmed when we consider the formula for the error :

$$\cot \theta \sin \alpha + \cos \alpha - 1$$

$$\text{Here if } \theta = 90° \quad \cot \theta = 0$$

and the error is reduced to the second-order value $(\cos \alpha - 1) \times 100\ \%$.

The table illustrates the astonishing rate at which the error grows as θ departs from its optimum value, 90°.

(g) *ERROR III : The uncertainty of setting of both θ and Ø due to the effects of a normal yaw.*—Before dealing with the consideration of the movements of the compass card it is of primary importance to study the effect of a yaw upon the resultant field when the deflector is set to some particular value of D and is left to lie on the glass of the compass bowl so following the yaw of the ship.

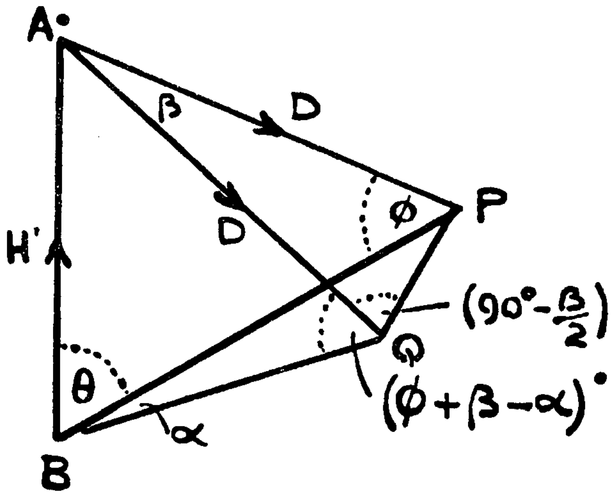

FIG. 5

Figure 5 illustrates the case. Here as a result of the yaw, the vector D changes position from AP to AQ because the deflector moves with the ship. As a result of this the angle of deflection θ is increased by the angle α. However, the lubber's point has moved round through the angle β so that the change in reading opposite the lubber's line due to the yaw is $(\beta - \alpha)$. In other words the apparent value of θ changes from θ to $(\theta + \beta - \alpha)$ when the yaw is β.

In the figure the angle AQB is $(\phi + \beta - \alpha)$ and the angle AQP is $(90° - \frac{\beta}{2})$. Hence the angle PQB is $(90° + \phi + \frac{\beta}{2} - \alpha)$.

Now PQ is $D \times 2 \sin 1/2\beta$ which if β is a small angle may be written $D\beta$ to second-order accuracy. Here β is in radians.

In the triangle ABP we have by the sine formula :—

$$\frac{\overline{BP}}{\sin(180 - \phi - \theta)} = \frac{D}{\sin\theta} \text{ whence } \overline{BP} = \frac{D \sin(\theta + \phi)}{\sin\theta}$$

In triangle BPQ by the sine formula :—

$$\frac{\overline{BP}}{\sin B\hat{Q}P} = \frac{\overline{PQ}}{\sin P\hat{B}Q}$$

So to second-order accuracy :—

$$\frac{D \sin(\theta + \phi)}{\sin\theta \sin(90 + \phi + \frac{\beta}{2} - \alpha)} = \frac{D\beta}{\alpha}$$

Hence :—

$$\frac{\beta}{\alpha} = \frac{\sin\theta \cos\phi + \cos\theta \sin\phi}{\sin\theta \cos(\phi + \frac{\beta}{2} - \alpha)} = \frac{\cos\phi + \cot\theta \sin\phi}{\cos\phi \cos(\frac{\beta}{2} - \alpha) - \sin\phi \sin(\frac{\beta}{2} - \alpha)}.$$

so that

$$\frac{\beta}{\alpha} = \frac{\cot\phi + \cot\theta}{\cot\phi - \frac{\beta}{2} + \alpha}.$$

and $$\beta\left(\cot \emptyset - \frac{\beta}{2}\right) + \alpha\,\beta = \alpha\,(\cot \emptyset + \cot \theta),$$

$$\alpha = \frac{\beta\left(\cot \emptyset - \frac{\beta}{2}\right)}{\cot \emptyset + \cot \theta - \beta},$$

$$\beta - \alpha = \beta\left[\frac{\cot \emptyset + \cot \theta - \beta - \cot \emptyset + \frac{\beta}{2}}{\cot \emptyset + \cot \theta - \beta}\right];$$

$$\beta - \alpha = \frac{\beta}{2}\left[\frac{2\cot \theta - \beta}{\cot \emptyset + \cot \theta - \beta}\right] \text{ radians.}$$

If θ be 90° :—

$$\beta - \alpha = \frac{\beta}{2}\left[\frac{-\beta}{\cot \emptyset - \beta}\right]$$

In the table below $(\beta - \alpha)$ the apparent change in θ is shown for various values of $\emptyset$ and β.

TABLE III.

Table of the apparent change of θ when the ship yaws slowly through the angle β The results being given for various values of the angle $\emptyset$; θ is 90°.

β \ $\emptyset$	= 80°	= 70°	= 60°	= 50°	= 40°
1°	— 0.1	— 0	— 0	— 0	— 0
2°	— 0.2	— 0.1	— 0.1	— 0	— 0
3°	— 0.7	— 0.2	— 0.2	— 0.1	— 0
4°	— 1.3	— 0.4	— 0.3	— 0.2	— 0.1
5°	— 2.5	— 0.7	— 0.4	— 0.3	— 0.2
6°	— 4.4	— 1.2	— 0.7	— 0.4	— 0.3
7°	— 7.9	— 2.5	— 1.0	— 0.6	— 0.4
8°	— 15	— 4.6	— 1.3	— 0.8	— 0.5

Consideration of the figures in the table show that for small angles of yaw the direction of the deflected resultant F will move round so that its attitude with respect to the lubber's point is effectively constant and independent of the yaw. This stability of the resultant F with respect to the lubber's point enables the deflector to be accurately set even under conditions of small but continuous slow yaw.

There is however a limit beyond which a slow yaw upsets the setting of the instrument. To find the conditions which must be satisfied if the reading of θ is not to be disturbed by more than say n° it is necessary to equate $(\beta - \alpha)$ to n.

Expressing both sides of the equation in radians we have, for θ = 90°

$$\frac{\beta}{2}\left[\frac{-\beta}{\cot \emptyset - \beta}\right] = \frac{n\,\pi}{180},$$

$$\beta^2 = \frac{2\,n\,\pi}{180}\cot \emptyset - \frac{2\,n\,\pi\,\beta}{180}$$

$$\cot \emptyset = \beta\left[1 - \frac{\beta}{2\,n} \times \frac{180}{\pi}\right]$$

expressing β in degrees : $\text{Cot } \emptyset = \sin \beta^{o}\left[1 - \frac{\beta^{o}}{2\,n}\right]$. Since β is a small angle.

For example if it be decided that $n°$ shall be — ¼° = and β is 3° :—

$$\text{Cot } \emptyset = 3 \times \frac{\pi}{180} (1 + 6) = 0.366 \text{ and } \emptyset = 70°$$

Table IV shows the greatest permissible values of $\emptyset$ which reduce the effect of a slow yaw (β) to less than one quarter of a degree when θ is set at 90°.

TABLE IV.

β Angle of yaw.	Maximum value of $\emptyset$ for ($\beta-\alpha$) less than ¼ degree.	Maximum value of $\emptyset$ for ($\beta-\alpha$) less than ½ degree.
1°	86°	
2°	80°	
3°	70°	
4°	58°	
5°	49°	62°
6°		54°
7°		46°

For yaws greater than 5° the reduction of ($\beta-\alpha$) to less than 1/4° is impractical and the figures in the righthand column are applicable for a reduction to less than half a degree.

Now although this would appear as an argument for decreasing the angle $\emptyset$ this would only be true if the yaw were very slow. In fact if there be any yaw at all the effect of the rate of yaw quite overshadows any gain produced by decreasing $\emptyset$ below 80°.

Inertia effects.

Although in what has gone before it has been shown that if $\emptyset$ has certain values and θ is 90° the card should almost stand still under the lubber's point as the ship moves, nevertheless in fact all that has been shown is that the resultant F of the vectors D and H' moves as has been calculated.

It is part of the object of the design of compasses to ensure that they shall not respond to any periodic force which has the periodicity of the yaw roll or pitch of the ship. This lack of response to these particular frequencies is achieved by having a weak magnetic moment and a large moment of inertia. The condition therefore when the deflector has turned the card through 90° is even less likely to cause the card to respond to the normal rate of yaw. This is because the value of the magnetic force F is considerably less than H', thus causing the natural frequency of the card to be even lower than under normal conditions and therefore even further from a condition of resonance with the yaw.

The result is that whereas a slow yaw interferes with the true value of θ and has negligible effect upon $\emptyset$, the moment the yaw ceases to be an almost imperceptible creep and becomes a characteristic movement of the ship, the desirable features of the card's behaviour break down and the uncertainty in the values of both θ and $\emptyset$ are effectively equal to the full amount of the yaw.

Thus the effect of errors I and II are aggravated approximately in the manner shown below for conditions when using $\emptyset$, 80° and $\emptyset$, 60°.

ERROR I.
The effect of the error in $\emptyset$. (Unavoidable error 1°.)
(Figures taken from Table I).

Angle of Yaw	+ 1°	Percentage Error in the estimation of H'.	
β		$\emptyset$ = 80°	$\emptyset$ = 60°
0°	1°	0,3 %	1 %
1°	2°	0,6 %	2 %
2°	3°	0,9 %	3 %
3°	4°	1,2 %	4 %
4°	5°	1,5 %	5 %
5°	6°	1.8 %	6 %

ERROR II.

The effect of the error in θ. (Unavoidable error 1°.)
(Figures taken from Table II).

Angle of Yaw	+ 1°	Percentage Error in the estimation of H'.	
β		$\emptyset = 80°$	$\emptyset = 60°$
0°	1°	0,02 %	0,02 %
1°	2°	0,06 %	0,06 %
2°	3°	0,14 %	0,14 %
3°	4°	0,24 %	0,24 %
4°	5°	0,38 %	0,38 %
5°	6°	0,55 %	0,55 %

Thus if we allow say 2° of yaw as "good conditions" the combined effect of the errors of setting introduced in θ and $\emptyset$ would produce errors in the estimation of H', of the order of (0.9 + 0.14) = 1 % when using $\emptyset$ = 80° or 3 % when using $\emptyset$ = 60°.

If the conditions be bad it will still be worth while to use $\emptyset$ = 80° even though by so doing the values of θ and $\emptyset$ cannot be adjusted to nearer than 6° with certainty; for in spite of this, the total error in the estimation of H' will still hardly exceed 3 %.

This should be compared with the result of a reduction of the value of $\emptyset$ from 80° to say 60° under these same bad conditions. This gives a percentage error = 7.6 % (7 % + 0.6 %) in the estimation of H'.

We now consider the deviating effect of a force of strength 1 % of H'. This will be

$$\frac{1}{100} \times \frac{180}{\pi} = 0{,}6° \text{ deviation.}$$

Thus, under good conditions, using the instrument with θ = 90°, $\emptyset$ = 80°, the error of each reading will probably be of the order of 1 %, i.e. 0.6° deviation.

Under bad conditions still using θ = 90°, $\emptyset$ = 80°, the error of each reading will probably be 3 % or 2° of deviation.

If for any reason either of the angles, θ and $\emptyset$, departs from its optimum 90° and 80° respectively, then the errors introduced must be considerably greater. Thus in the example here given, when $\emptyset$ becomes 60° the error in the good case is about 2 1/2 % while in the bad case it is about 8 %, which correspond to 1.5° and 4,2° of deviation respectively.

Attention has already been called to the unnecessary increase in error resulting from using any angle for θ other than 90°.

Comments on the accuracy of correction of compasses by means of the deflector.

Consideration of the figures here given, shows that, under good conditions it is reasonable to assess the probable error of a single reading as a force which when turned at right angles to the magnetic meridian will cause a deviation of 0.6°.

Consider now the method of correcting coefficient B°. Here readings are taken on the North and South each subject to a probable error of 0,6°.

Thus the probable error of their mean will be $\frac{1}{2}\sqrt{0.6^2 + 0.6^2}$ say E_I. In assessing the effect of the correctors it is fair to assume a probable error of 0.6° = E_2. Hence the actual setting of the magnets is subject to a probable error

$$\sqrt{E_I^2 + E_2^2} = \sqrt{\frac{0.6^2 + 0.6^2}{4} + 0.6^2} = 0.73°$$

The probable error in the correction of coefficient C° will also be 0.73.

To assess the effect of coefficient D° requires four readings from which the mean is taken and a fifth to adjust the setting of the spheres.

The probable error of the mean of the four readings is :—

$$\frac{1}{4}\sqrt{0.6^2 + 0.6^2 + 0.6^2 + 0.6^2} = 0.3°.$$

Combining this with the error of the fifth reading gives :—

$$\sqrt{0.3^2 + 0.6^2} = 0.67°.$$

On the four intercardinal points the values of the deviation are made up numerically from 0.707 × B°, 0.707 × C° and D° with the signs as set out in the table below :—

	B°	C°	D°
On N.-E. we have :	+	+	+
S.-E. — :	+	—	—
S.-W. — :	—	—	+
N.-W. — :	—	+	—

It will therefore be clear that whatever the actual signs of the actual errors in B°, C° and D° there is always one intercardinal point on which the errors all have the same sign. Hence to assess the probable value of the maximum error we must add

$$(0.707 \times 0.73°) + (0.707 \times 0.73°) + 0.67°.$$

The sum of these is 1.7°.

We thus see that it is reasonable to state that under good conditions when using the deflector for adjusting the correctors the probable value of the maximum deviation caused by errors introduced by the device is about 2° and that under bad conditions this figure is about 4° or 5°.

Moreover it is clear that these figures assume that the instrument has been in the hands of an expert who has not departed from the values $\theta = 90°$ and $\varnothing = 80°$. As these errors are unpredictable in sign there is no means of telling on which of the intercardinal headings the maximum deviation will occur, nor can it be accurately forecast how these deviations will add with the other deviations which may exist and which are discussed in the next section.

SECTION III.

Total errors of a compass corrected by means of the deflector.

When by means of the deflector the directive force has been adjusted to a nearly equal value on heading North, South, East and West the rotating vectors which have been nullified are :—

$\lambda\ H\bar{B}$ rotating with the ship.

$\lambda\ H\bar{C}$ rotating 90° ahead of the ship.

$\lambda\ H\bar{D}$ rotating ζ ahead of the ship.

On these four headings the vector $\lambda\ H\bar{E}$ which rotates $90° + \zeta$ ahead of the ship has always been directed to East or West but has never contributed to the directive force at the time it was being measured. Furthermore the vector $\lambda\ H\bar{A}$ never does contribute to the directive force whatever the heading.

Although with a well-placed compass these two vectors are so small compared to λ H that their effect is seldom *corrected*, this does not prevent their combined effect being of the order of say 2° at maximum. It is possible to estimate the value of the deviation due to the vector $\lambda\ H\bar{E}$ by means of the deflector and even to correct it by slewing the spheres but this is very seldom done. Thus over and above the unavoidable error of 2° or 3° which is due to the use of the instrument there is also the uncorrected deviation due to the forces λ HA and λ HE.

Now the danger to ships does not lie in the fact that the compass has or does not have deviations, but in whether the amount of these deviations are known or unknown on a particular heading. Thus although the deflector may be used to reduce the deviations to reasonable values, perhaps in some instances better than could be achieved by a less expert operator using say reciprocal bearings; nevertheless, the latter produces a deviation card which enables the navigator to allow for the deviations, whereas the operator who uses the deflector for adjustment even under good conditions, without afterwards swinging to obtain a deviation card by bearings, can only say : " Your compass has been adjusted so that the deviation is unlikely to exceed 5° on any heading, but I expect it to have deviations of about 3° ".

Notes on the adjustment of the instrument.

It may be worth noting that when using the instrument the following statement is always true :

" Provided the ship does not yaw, and the deflector is not turned, the adjustment of the moment of the magnet system of the deflector causes θ and $\emptyset$ to change by *exactly* equal and opposite amounts."

This is true at all times and is merely a restatement of the fact that the sum of the three angles of a triangle is constant (180°). It is sometimes useful to make final adjustments by this means, for if say $\emptyset$ is 88° and θ is 70° then by turning the deflector it is possible to make the amount by which θ is short of 90° equal to the amount by which $\emptyset$ is greater than 80°.

In this example it is clear that if we increase $\emptyset$ to 92°, θ will still be 70°, so that the required condition will probably be reached when say $\emptyset = 102°$ and $\theta = 68°$, i. e. $\emptyset$ exceeds 80° by 22°, and θ is 22° short of 90°. When this state of affairs has been reached it is known that the *direction* of the deflector is right and only its strength requires increasing until θ becomes 90° and $\emptyset$ becomes 80°. Another way of saying it is : " The *direction* of the deflector is right when the sum of θ and $\emptyset$ is 170°. "

SECTION IV.

The de Colongue deflector.

This instrument consists of two main parts. The first is a horizontal permanent magnet system mounted so that its centre point can be slid up and down a vertical rod which is erected above the compass bowl so that its axis is in line with the pivot.

This main magnet system cannot rotate about the rod but its direction is defined by an indicating pointer on the base of the instrument. This base fits the compass bowl and may be locked at any required azimuth.

The second main part consists of an auxiliary magnet which is mounted horizontally in the base at right angles to the vertical plane defined by the magnetic axis of the main magnets and the axis of the vertical rod. The direction of the magnetic moment of the auxiliary magnet is 90° to clockwise of the magnetic moment of the main magnet.

Arrangements are provided for separating the base plate with its indicating pointer, from the vertical rod with its main and auxiliary magnetic system. A suitable locking device is used which unites the two parts so that the magnetic axis of the main magnet is directed in exactly the opposite direction to that of the pointer.

In use, the ship's head is maintained on the required heading by means of a steering compass or other means. The non-magnetic base plate is then carefully fixed to the rim of the compass bowl so that the pointer is accurately situated over the South point of the compass card. A prism is fitted which presents the image of the pointer at the level of the card and the line of sight is so arranged that parallax is avoided. During this part of the operation the magnetic parts of the deflector are kept sufficiently remote to avoid deflecting the compass, i. e. some eight feet or more away.

When the base plate is accurately fixed it is firmly clamped to the bowl and the magnetic parts of the deflector are mounted upon it with the rod end of the main magnet system pointing in the magnetic North direction. The main magnet system of the deflector is then slid up or down until the compass card aligns itself with the field of the auxiliary magnet which is at right angles to that of the main magnet.

If the compass card settles down in line with the auxiliary magnet then the directive force of the earth must have been accurately cancelled by the effect of the main magnet system. Clearly the weaker the auxiliary magnet the more sensitive the correction becomes.

It may be observed that, once set, the field of the device becomes effectively a single deflecting force inclined at some angle Ø to the North point of the card such that

$$\tan \text{Ø} = \frac{\text{strength of main magnetic field.}}{\text{strength of auxiliary magnetic field.}}$$

Hence as regards the effect of quick yaw the instrument is subject to uncertainty as to when it is accurately set, in the same way as the Thompson deflector.

The position of the main magnet system on the vertical rod may be read off from a scale on the rod; a vernier is provided.

Advantages of this type of deflector.

(1) The angle of deflection can only be 90° and is not variable, hence, the mistake of using other than 90° cannot be made.

(2) The setting of the instrument is achieved by one simple action only, i. e. the sliding of the main magnet system to its best position on the vertical rod.

(3) The scale on the rod is marked so that its reading gives the horizontal field at the compass position due to the main magnet system in arbitrary units. Thus it may be scaled to suit, at the compass, the "sideways on" field of a magnet system of the actual finite length of the main magnet system.

(4) The measurement made is independent of the auxiliary magnet field and thus is subject only to errors introduced by variations in the value of the angle of deflection, due to change of course since the base plate was clamped, plus the uncertainty of reading of the angle of deflection due to yaw.

These two sources of error may be dealt with under one heading. Thus if α represent the error in the angle of deflection at the time the reading is made, the error in the reading is : $(\cos \alpha - 1) \times 100\,\%$.

Thus for angles of yaw 2° the error is of the order of 0,06 % while for angles of yaw of 6° the error is still only 0,55 %.

This deflector gives readings which may be used either for correcting the compass or estimating the value of the approximate coefficients. It has slightly less errors than the Thompson deflector but still suffers from the criticism that when used for correcting a compass the residual errors are unknown.

CONCLUSIONS

The conclusions to be drawn from this mathematical investigation of the methods of using deflectors are self-evident.

They are :—

(1) It should be realised that even in the hands of an expert a deflector cannot give readings of forces which are not present when the measurement is made.

For example no appreciation of coefficient A can ever be obtained by using a deflector, nor can any measure of coefficient E be made without steadying the ship on at least one and preferably four intercardinal points.

(2) We also conclude that even if coefficients A and E are very small, there is still an unavoidable error in the results due to the limitations of the instrument and the conditions under which it is used.

(3) We further conclude that even if the instrument be in the hands of an expert these remaining unavoidable errors are always additive on one particular heading. On this heading the unknown deviation will be about 2° or 3° with a possibility of being as much as 5°.

(4) Another important conclusion is that the effect of deflecting the compass card through any angle other than 90° may easily double the unperceived residual errors without this being detected. The effect of using a deflecting force which is inclined to the meridian by more than about 10° is almost as bad and should be avoided.

NOTE. — The construction and method of use of the de Colongue deflector makes it impossible to deflect the card through any angle other than 90°. This is a definite advantage since it prevents the operator making use of unreliable readings.

(5) The final and most important conclusion of all is that ships should always be swung to obtain a deviation card after their compasses have been corrected. This applies with equal force whether the adjustment was actually made by means of a deflector or by the more orthodox methods such as reciprocal bearings, etc.