LARGE SCALE PLOTTING

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A position defined by horizontal sextant angles, and plotted by station pointer, in the manner usual in hydrographic survey, may be defined also as lying at the intersection of two circular arcs which are, respectively the locii of points representing the observed angles. Similar circular arcs may be plotted in the form of a graticule covering any area which it is proposed to survey. This technique for eliminating the station pointer was in use more than twenty years ago.

The method was included in the instructions to the United States Coast and Geodetic Survey[(1)], and has been described in its application to harbour work by Bostwick and Rosenzweig[(2)], and MacMillan[(3)], [(4)].

The positions of circle centres, and their radii, are found by plane trigonometry. In figure 1, an example of such work is shown. A, B and C, are objects

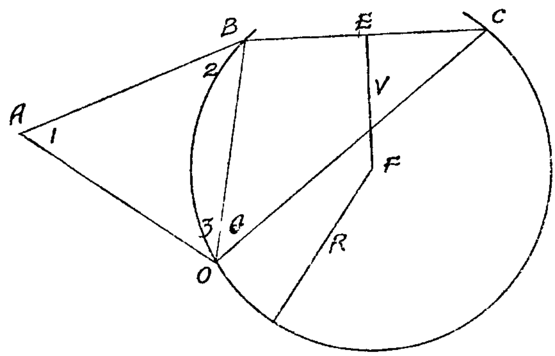

FIG. 1

observed. O, is the observer's position. The observed angles are denoted by 3 and 4. The centre of the circle corresponding to angle 4, lies at F along the line EF, which bisects BC at right angles.

In figure 1 : V = 1/2 BC. Cot. 4,
and R = 1/2 BC. Cosec. 4.

where V is the distance EF and R is the radius of the circum-circle to the triangle BCO, corresponding to the angle 4.

It is useful to tabulate these expressions for all required values of the observed angle.

An early American method of working, by small scale plot and photographic projection, is very probably still as convenient as any where photographic facilities are readily available. The use of two independent pairs of objects for subtending the observed angles gives the advantage that sounding marks may be so placed on the ground as to give the strongest graticule while reducing computation.

These applications have proved valuable alternatives to station pointer work. There remain however cases in which station pointer methods are practically impossible, and it is, unfortunately, in these cases that the old graphical technique for plotting circum-circles, outlined above, is at its weakest. Where, for instance, areas lying at considerable distances offshore are to be surveyed at large scales. and in some cases of large scale inshore work, the graphical plotting of the circular arcs requires long compass beams with resulting inconvenience and possibility of error.

The proper alternative would be Pothenot or Collins point solutions for each graticule intersection. This would entail tedious computation, though it would be no hardship in many of the deliberate, routine surveys for which it might be required. A further alternative, where acceptable, would be the computation of a limited number of accurate positions and the joining of these by arcs drawn on railway curves of approximately correct radius. In cases of this kind it would be more satisfactory to compute back to the observed angle from a pre-selected radius for which an accurately cut curve exists.

Thus, in figure 1: $4 = \mathrm{Sin}^{-1} \frac{BC}{2R}$ where R is the pre-selected radius.

For other angles the finite variations in sines, may be computed from the radius variations.

The following technique is, however, suggested as a suitable means of accurate working while avoiding the somewhat complex double angle solutions.

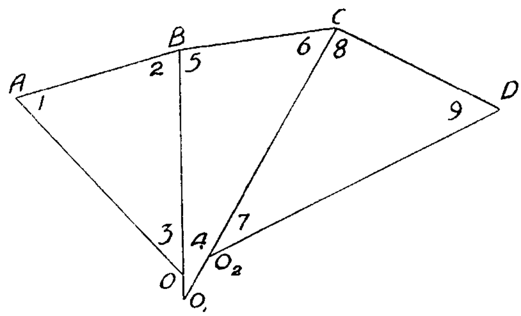

FIG. 2

In figure 2, A, B, C and D, are objects observed. The distances between them and the angles at B and C are known from the controlling survey. O is the position of the observer. From a small scale plot measure values for the angles 1, 2 3 and 4. The angles 3 and 4 may be assumed to the nearest convenient integral value, or they may be governed by some pre-selected radii. The value of 5 is now fixed by 2. Conclude a value for 6.

$$\text{Then, in figure 2: } BO = AB \frac{\mathrm{Sin}\ 1}{\mathrm{Sin}\ 3}$$

$$\text{and } BO_1 = BC \frac{\mathrm{Sin}\ 6}{\mathrm{Sin}\ 4}$$

$$\text{then } O \sim O_1 = BO \sim BO_1 .$$

Similarly an intercept such as $O_1 \sim O_2$ may be found if necessary. These intercepts may now be plotted, at any required scale, independent of the objects A, B, C or D. It now remains necessary to plot any required group of curves corresponding to angles observed.

In figure 3, A and B are objects observed, O is the observer's position. OG is a line parallel with the locus (EF of fig. 1) of circle centres. HJ is a line tangent to the curve locus of the angle at O (observed between A and B). The radius to the curve lies perpendicular to HJ.

The direction OG is defined by the angle 4, and angle 4 = 90° — angle 1.

The direction HJ is defined by the angle 5, and angle 5 = angle 2.

The parallel tangents and radii for any other required curves based on AB (fig. 3) may now be plotted, the intervals between circle centres being laid in the line OG (or OG produced), and the differences of radius being laid off towards or away from circle centres in a direction parallel with the radius at O.

In a similar way the tangents and radii for angles observed on another pair of objects may be deduced, the systems corresponding to each pair of objects being separated by the intercepts $O \sim O_1$, etc., of figure 2.

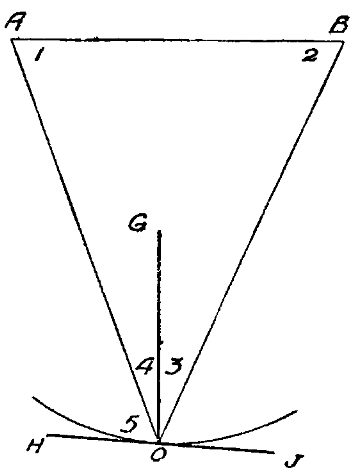

FIG. 3

The required curves may be plotted in relation to tangent and radius in any preferred way. If railway curves of specified radius are being used the variations will be as follows :—

$$\Delta \sin O = \frac{AB}{2 \Delta R}$$

and $\Delta V = 1/2 \ AB \ \Delta \cot. O.$

where O is the observed angle between objects A and B :—

Δ R is any finite variation of radius,

Δ V is a corresponding distance between circle centres.

and Δ sin O is the difference between the sines of successive angles.

Where integral variations of observed angle are being used :—

$$\Delta R = 1/2 \ AB \ \Delta \operatorname{cosec.} O,$$

where Δ cosec. O, is the difference between the cosecants of successive angles.

Each curve may, if preferred, be laid down by short chords, the direction of each chord being changed according to the angle subtended at circle centre. In large scale work such a figure may approximate very closely to the actual curve.

Where coordinates are required, the coordinates of each curve to axes tangent and radius may be computed, using sines and versines, and any required mathematical or graphical transformations can be performed. It will, of course, be appreciated that these operations, like most of those mentioned above, are of a quite minor nature, involving comparatively short distances.

REFERENCES :

(1) United States Coast and Geodetic Survey, *Special Publication* No. 143.

(2) BOSTWICK & ROSENZWEIG, *Dock and Harbour Authority*, London, 1939, April.

(3) MACMILLAN, *Dock and Harbour Authority*. London, 1939, June.

(4) MACMILLAN, *Hydrographic Review*, Vol. XVI, No. 2, Monaco, 1939 November.