COMPUTATION OF THE FIX AT SEA BY MEANS OF COMMANDANT GUYOU'S "POCKET TABLES"

By *Ingénieur Hydrographe en Chef* of the French Navy

André GOUGENHEIM.

1. Determination of the fix at sea comprises a succession of operations among which the solution by computation of a spherical triangle is perhaps neither the longest nor the most delicate. Yet it is almost wholly to this phase of the procedure that those seeking to facilitate the work of the navigator have applied themselves ; attracted by the admirable suppleness of the formulae of spherical trigonometry, many hoped, and still hope to see a miracle spring from it ; let it be added also that a pencil and a sheet of paper are sufficient for examination of the problem laid down while the improvement or the realisation of an observation instrument requires infinitely greater materiel means.

It is quite understandable why numerous solutions have been proposed for the computation of the fix ; some of them are very ingenious but the very fact that none of them has proved of such stable value as to be universally adopted, proves that they are not exempt from criticism.

The purpose of the present article is not to present any new solution but to draw the attention of users to a remarkable method worked out in France at the end of the last century and which, in spite of all the interest which it offers, has not enjoyed the favour which it would seem to deserve. The evolution of the question since that epoch, comparison with modern methods and the concessions these involve, will perhaps allow the advantages of the method under consideration to be more highly appreciated and tardy justice to be rendered to its author.

2. - THE MERIDIONAL PARTS.

All users of charts know the meridional part, that term of geographical latitude which serves to define the spacing of the parallels on a Mercator net and the values of which may be found in Navigational Tables.

In reality, the Mercator chart is only one particular example of the application of this term which is the principal element in the theory of the conformal representation of the revolution surfaces ; for spherical surfaces, the term takes a simple form encountered, moreover, under different names, in other mathematical or physics theories.

For the sphere, the meridional part is merely the logarithm of a trigonometrical tangent, so that tables giving its values may equally be used to apply the logarithmic formulae in which angles enter by their tangent. However, the more current tables express the meridional part in minutes from the equator ; the basis of these logarithms is distinct therefore from that of the vulgar logarithms and, in practice, the tables of meridional parts are not fitted to be employed concurrently with the other logarithmic tables.

3. - GUYOU'S METHOD.

As early as 1874, Hilleret had to some degree foreseen this use of the meridional parts, but it is to Guyou that is due the merit of having in 1884 indicated a method of computation of the fix at sea based upon the exclusive use of this term and of having published tables corresponding to its application [1]. His idea, however, does not seem to have been very favourably met and the complementary paper that he published in 1888 [2] did not contribute to any higher appreciation of the procedure recommended by him. It must be admitted that his explanations lacked clearness ; they did not tend to illuminate the broad outlines of the method nor its advantages ; even the practice of the procedure as shown was not without obscurity ; already the attention of the author had turned to extension of the application of the tables to accessory computations, especially to the study of curved altitude position lines to which he attached, like many of his contemporaries, an importance perhaps necessary.

A few years later, in 1895, the Englishman Goodwin, attracted by the Guyou method proposed to generalise it and to treat all problems of nautical astronomy by means of the Table of Meridional Parts only [3]. However, when the angles do not appear in the formulae by their tangent it is necessary to have recourse to auxiliary variables the use of which makes the calculation singularly more involved, besides which the advantage inherent in the use of one single Table is lost. It may well be thought then that, by this excess of zeal, Goodwin's explanation ran counter to the end sought for and did not contribute to make evident the simplicity characterising Guyou's solution.

The latter, moreover, examined at this same period a similar generalisation [4] and introduced into his method geometrical considerations which unnecessarily complicated it and led to his *Navigation Tables* (1911) in two bulky volumes far removed from the *Pocket Tables*.

4. - USE OF BORDA'S FORMULAE.

Having, on our side, rediscovered Guyou's method during some research work which followed quite a different path, its simplicity and elegance struck us forcibly and the wish arose to set it forth in a way that would bear relation to its essential qualities.

The quicker way is to take Borda's formulae as point of departure : it is known that these are relations calculable by logarithms and which allow the angles of a spherical triangle to be expressed in terms of its sides.

Applied to the computation of the fix at sea, Borda's formulae may therefore serve to compute the polar angle and the azimuth of a celestial body the altitude of which above the horizon at a given point of latitude has been measured; from the polar angle it is easy to deduce the longitude of the station point. If the latitude is only dead-reckoned, the calculation then gives the direction of the altitude position line and the position of one of its points. Subsequently it is possible to plot on a chart or on a diagram the geometrical locus of the vessel at the time of observation. As a result the corresponding methods of computation are placed among those known as the *estimated parallel* methods.

---

(1) Tables de poche donnant le point Observé et les Droites de Hauteur, Berger-Levrault, 1884.

(2) Calcul du point Observé à l'aide de la Table des Latitudes croissantes : *Annales Hydrographiques* 1888.

(3) A Nautical Astronomy of a New Type : *Nautical Magazine*, Nov.-Dec. 1895.

(4) Les problèmes de Navigation et la Carte Marine, *Annales Hydrographiques*, 1895.

5. - THE LAMBDA and COLAMBDA TERMS.

The meridional part α of a geographical latitude a is defined by the relation :

$$\alpha = \frac{10.800}{\pi} \operatorname{Log} \operatorname{tg} \left(45^\circ + \frac{a}{2} \right)$$

in which the abbreviation *Log.* designates the Neperian logarithms and the factor $\dfrac{10.800}{\pi}$ serves to express α in minutes of the equator.

To simplify the entry let us write :

$$\alpha = \lambda (a)$$

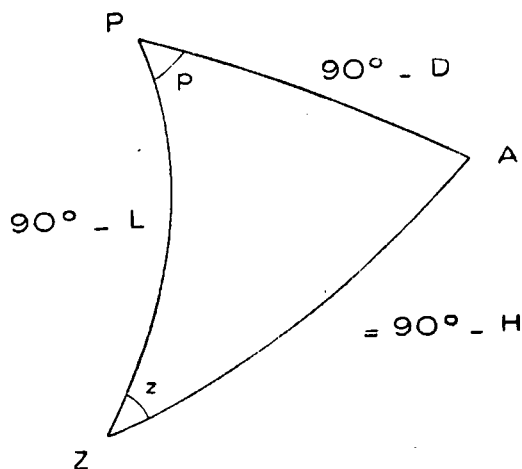and let us introduce like Guyou the notation co λ defined by :

$$\operatorname{co} \lambda (a) = \lambda (90^\circ \cdot a) = \frac{10.800}{\pi} \operatorname{Log} \operatorname{cot} \frac{a}{2}$$

Farther on we shall have to use the following property of the term λ, which it is very easy to establish : the relation :

$$\lambda (c) = \lambda (a) - \lambda (b)$$

is equivalent to :

$$\operatorname{cot} \frac{c}{2} = \frac{1 - \operatorname{tg} \dfrac{a}{2} \operatorname{tg} \dfrac{b}{2}}{\operatorname{tg} \dfrac{a}{2} - \operatorname{tg} \dfrac{b}{2}} = \frac{\cos \dfrac{a + b}{2}}{\sin \dfrac{a - b}{2}}$$

6. - BORDA'S FORMULAE and THEIR TRANSFORMATION.

If the given Latitude is L, D the declination of the celestial body and ς ($= 90^\circ - H$) its true zenith distance deduced from observation, the unknown quantities being the polar angle P and the azimuth Z, the Borda formulae applied to the spherical triangle PZA (pole, zenith of the station, celestial body) are written :

$$\cot^2 \frac{P}{2} = \frac{\cos\left(\frac{L}{2}+\frac{D-\varsigma}{2}\right)}{\sin\left(\frac{L}{2}-\frac{D-\varsigma}{2}\right)} \quad \bullet \quad \frac{\cos\left(\frac{D+\varsigma}{2}+\frac{L}{2}\right)}{\sin\left(\frac{D+\varsigma}{2}-\frac{L}{2}\right)}$$

$$\cot^2 \frac{Z}{2} = \frac{\cos\left(\frac{L}{2}+\frac{D-\varsigma}{2}\right)}{\sin\left(\frac{L}{2}-\frac{D-\varsigma}{2}\right)} \quad \bullet \quad \frac{\sin\left(\frac{D+\varsigma}{2}-\frac{L}{2}\right)}{\cos\left(\frac{D+\varsigma}{2}+\frac{L}{2}\right)}$$

If :

$$\cot \frac{c_1}{2} = \frac{\cos\left(\frac{L}{2}+\frac{D-\varsigma}{2}\right)}{\sin\left(\frac{L}{2}-\frac{D-\varsigma}{2}\right)} \qquad \cot \frac{c_2}{2} = \frac{\cos\left(\frac{D+\varsigma}{2}+\frac{L}{2}\right)}{\sin\left(\frac{D+\varsigma}{2}-\frac{L}{2}\right)}$$

relations which, according to the property of the term λ above-indicated, are equivalent to :

$$\lambda\,(c_1) = \lambda\,(L) - \lambda\,(D-\delta) \qquad\qquad \lambda\,(c_2) = \lambda\,(D+\delta) - \lambda\,(L)$$

the Borda formulae appear in the form :

$$\cot^2 \frac{P}{2} = \cot \frac{c_1}{2} \cot \frac{c_2}{2} \qquad\qquad \cot^2 \frac{Z}{2} = \frac{\cot \frac{c_1}{2}}{\cot \frac{c_2}{2}}$$

Or again, by taking the colambdas (Neperian logarithms multiplied by $\frac{10.800}{\pi}$)

$$2\,co\,\lambda\,(P) = co\,\lambda\,(c_1) + co\,\lambda\,(c_2) \qquad 2\,co\,\lambda\,(Z) = co\,\lambda\,(c_1) - co\,\lambda\,(c_2)$$

Finally if the zenith distance ς is replaced by the true altitude H, which is traditionally used by navigators, the computation of the fix at sea by the Borda formulae is realised in the four following relations ([1])

$$\lambda\,(c_1) = co\,\lambda\,(H+D) + \lambda\,(L) \qquad 2\,co\,\lambda\,(P) = co\,\lambda\,(c_1) + co\,\lambda\,(c_1)$$

$$\lambda\,(c_2) = co\,\lambda\,(H-D) - \lambda\,(L) \qquad 2\,co\,\lambda\,(Z) = co\,\lambda\,(c_2) - co\,\lambda\,(c_2)$$

which can be solved by means of the Table of Meridional Parts solely. The computation is further facilitated when the Table, like that shown in Friocourt's Navigational Tables, gives side by side for each value of latitude a, the values of $\lambda\,(a)$ and of $co\,\lambda\,(a)$; this result can be obtained by a special typographical arrangement without adding to the bulk of the Table.

In summing up it should be noted that to the advantage of simplicity derived from the use of *one only single-entry* Table is added that of the accuracy which accompanies the *determination of angles by their tangent.*

7. - HOW TO PROCEED IN THE COMPUTATION : EXAMPLES.

The relations that we have established are general if the signs corresponding to those of L, D, D — ς and D + ς are given to the terms λ, but in practice it is preferable to avoid use of the signs. *D* will therefore always be considered as an absolute value, in the same way as ς, and two cases

will be distinguished according to whether the latitude and the declination have or have not the same names ; furthermore in the final result it will be necessary to count the azimuths starting from the pole of the same name as the declination.

Finally, practical application must be proceeded with in the way indicated in Annex 1 which takes into account all the triangles that may be met with and in which the arrows indicate the use of the Table of Meridional Parts.

Although occupying only a few lines each, the examples given in Annex 3 do not sufficiently convey the brevity of the calculation, for the latter is greatly facilitated by the use of *one only single-entry* Table, in this case Friocourt's Table of Meridional Parts (See Annex 2).

Friocourt's Table would moreover be improved by the addition of time figures permitting the polar angles to be obtained directly in hours, minutes and seconds.

The fact that an accuracy of the order of one minute of arc could be maintained by neglecting the decimal of the meridional parts, may be verified.

February, 1949.

---

(1) It should be pointed out that these formulae are also arrived at starting from the NEPER analogies.

ANNEX 1.

Performance of Computation.

Formation de $\lambda(c_1)$ et $\lambda(c_2)$

	$\lambda(c_1)$		$\lambda(c_2)$
	$D+H<90°$	$D+H>90°$	
Latitude et Déclinaison — de mêmes noms	$co\lambda(D+H)+\lambda(L)$	$\lambda(L)-\lambda(D+H-90°)$	$co\lambda|D-H|-\lambda(L)$
de noms contraires	$co\lambda(D+H)-\lambda(L)$	n'a pas lieu	$co\lambda|D-H|+\lambda(L)$

$$L \rightarrow \lambda(L)$$

$$D+H \begin{cases} \text{Si } D+H<90° \rightarrow co\lambda(D+H) \\ \text{Si } D+H>90° \rightarrow \lambda(D+H-90°) \end{cases}$$

$$|D-H| \rightarrow co\lambda|D-H|$$

$$\underline{\quad \lambda(c_1) \quad \lambda(c_2) \quad}$$

$$\lambda(c_1) \rightarrow co\lambda(c_1)$$
$$\lambda(c_2) \rightarrow co\lambda(c_2)$$
$$\underline{\text{Somme}}$$

$$\tfrac{1}{2}\text{ Somme } = co\lambda(u) \rightarrow u$$

$$\text{Différence} \begin{cases} \text{si positive : } \tfrac{1}{2}\text{ Différence} = co\lambda(v) \rightarrow v \\ \text{si négative : } \tfrac{1}{2}|\text{Différence}| = \lambda(v-90°) \rightarrow v-90° \end{cases}$$

$$\begin{cases} \text{Si } D<H \quad P=u \quad Z=v \\ \text{Si } D>H \quad P=v \quad Z=u \end{cases}$$

Nota : Les Azimuts Z obtenus sont comptés à partir du pôle de même nom que la déclinaison.

Reproduction of a page of FRIOCOURT's Table of Meridional Parts.
The complete Table covers 9 pages.

VI — Latitudes Croissantes.

'	15° λ	15° coλ	16° λ	16° coλ	17° λ	17° coλ	18° λ	18° coλ	19° λ	19° coλ	'
0	910,5	6 970,3	972,7	6 745,7	1 035,3	6 534,4	1 098,2	6 334,8	1 161,5	6 145,7	60
1	911,5	6 966,5	973,8	6 742,1	1 036,3	6 531,0	1 099,3	6 331,6	1 162,5	6 142,6	59
2	912,5	6 962,6	974,8	6 738,5	1 037,4	6 527,6	1 100,3	6 328,4	1 163,6	6 139,6	58
3	913,6	6 958,8	975,8	6 734,9	1 038,4	6 524,2	1 101,4	6 325,1	1 164,7	6 136,5	57
4	914,6	6 954,9	976,9	6 731,3	1 039,5	6 520,8	1 102,4	6 321,9	1 165,7	6 133,4	56
5	915,6	6 951,1	977,9	6 727,7	1 040,5	6 517,4	1 103,5	6 318,7	1 166,8	6 130,4	55
6	916,7	6 947,2	979,0	6 724,0	1 041,6	6 514,0	1 104,5	6 315,5	1 167,8	6 127,3	54
7	917,7	6 943,4	980,0	6 720,4	1 042,6	6 510,6	1 105,6	6 312,3	1 168,9	6 124,3	53
8	918,7	6 939,6	981,0	6 716,8	1 043,7	6 507,2	1 106,6	6 309,0	1 169,9	6 121,2	52
9	919,8	6 935,7	982,1	6 713,2	1 044,7	6 503,8	1 107,7	6 305,8	1 171,0	6 118,2	51
10	920,8	6 931,9	983,1	6 709,6	1 045,8	6 500,4	1 108,7	6 302,6	1 172,1	6 115,1	50
11	921,9	6 928,1	984,2	6 706,1	1 046,8	6 497,0	1 109,8	6 299,4	1 173,1	6 112,1	49
12	922,9	6 924,3	985,2	6 702,5	1 047,9	6 493,6	1 110,8	6 296,2	1 174,2	6 109,0	48
13	923,9	6 920,5	986,3	6 698,9	1 048,9	6 490,2	1 111,9	6 293,0	1 175,2	6 106,0	47
14	925,0	6 916,7	987,3	6 695,3	1 049,9	6 486,9	1 112,9	6 289,8	1 176,3	6 103,0	46
15	926,0	6 912,9	988,3	6 691,7	1 051,0	6 483,5	1 114,0	6 286,6	1 177,4	6 099,9	45
16	927,0	6 909,1	989,4	6 688,2	1 052,0	6 480,1	1 115,0	6 283,4	1 178,4	6 096,9	44
17	928,1	6 905,2	990,4	6 684,6	1 053,1	6 476,7	1 116,1	6 280,2	1 179,5	6 093,9	43
18	929,1	6 901,5	991,5	6 681,0	1 054,1	6 473,4	1 117,2	6 277,0	1 180,5	6 090,8	42
19	930,1	6 897,7	992,5	6 677,5	1 055,2	6 470,0	1 118,2	6 273,9	1 181,6	6 087,8	41
20	931,2	6 893,9	993,6	6 673,9	1 056,2	6 466,7	1 119,3	6 270,7	1 182,7	6 084,8	40
21	932,2	6 890,1	994,6	6 670,4	1 057,3	6 463,3	1 120,3	6 267,5	1 183,7	6 081,8	39
22	933,3	6 886,3	995,6	6 666,8	1 058,3	6 460,0	1 121,4	6 264,3	1 184,8	6 078,8	38
23	934,3	6 882,6	996,7	6 663,3	1 059,4	6 456,6	1 122,4	6 261,2	1 185,8	6 075,7	37
24	935,3	6 878,8	997,7	6 659,7	1 060,4	6 453,3	1 123,5	6 258,0	1 186,9	6 072,7	36
25	936,4	6 875,0	998,8	6 656,2	1 061,5	6 449,9	1 124,5	6 254,8	1 188,0	6 069,7	35
26	937,4	6 871,3	999,8	6 652,6	1 062,5	6 446,6	1 125,6	6 251,7	1 189,0	6 066,7	34
27	938,4	6 867,5	1 000,8	6 649,1	1 063,6	6 443,2	1 126,6	6 248,5	1 190,1	6 063,7	33
28	939,5	6 863,8	1 001,9	6 645,6	1 064,6	6 439,9	1 127,7	6 245,3	1 191,1	6 060,7	32
29	940,5	6 860,0	1 002,9	6 642,1	1 065,7	6 436,6	1 128,7	6 242,2	1 192,2	6 057,7	31
30	941,6	6 856,3	1 004,0	6 638,5	1 066,7	6 433,3	1 129,8	6 239,0	1 193,3	6 054,7	30
31	942,6	6 852,5	1 005,0	6 635,0	1 067,8	6 429,9	1 130,9	6 235,9	1 194,3	6 051,7	29
32	943,6	6 848,8	1 006,1	6 631,5	1 068,8	6 426,6	1 131,9	6 232,7	1 195,4	6 048,7	28
33	944,7	6 845,1	1 007,1	6 628,0	1 069,9	6 423,3	1 133,0	6 229,6	1 196,4	6 045,7	27
34	945,7	6 841,3	1 008,1	6 624,5	1 070,9	6 420,0	1 134,0	6 226,5	1 197,5	6 042,7	26
35	946,7	6 837,6	1 009,2	6 621,0	1 072,0	6 416,7	1 135,1	6 223,3	1 198,6	6 039,8	25
36	947,8	6 833,9	1 010,2	6 617,5	1 073,0	6 413,4	1 136,1	6 220,2	1 199,6	6 036,8	24
37	948,8	6 830,2	1 011,3	6 614,0	1 074,1	6 410,1	1 137,2	6 217,0	1 200,7	6 033,8	23
38	949,9	6 826,5	1 012,3	6 610,5	1 075,1	6 406,7	1 138,2	6 213,9	1 201,7	6 030,8	22
39	950,9	6 822,8	1 013,4	6 607,0	1 076,2	6 403,4	1 139,3	6 210,8	1 202,8	6 027,8	21
40	951,9	6 819,1	1 014,4	6 603,5	1 077,2	6 400,1	1 140,4	6 207,7	1 203,9	6 024,9	20
41	953,0	6 815,4	1 015,4	6 600,0	1 078,3	6 396,9	1 141,4	6 204,5	1 204,9	6 021,9	19
42	954,0	6 811,7	1 016,5	6 596,5	1 079,3	6 393,6	1 142,5	6 201,4	1 206,0	6 018,9	18
43	955,0	6 808,0	1 017,5	6 593,0	1 080,4	6 390,3	1 143,5	6 198,3	1 207,1	6 016,0	17
44	956,1	6 804,3	1 018,6	6 589,6	1 081,4	6 387,0	1 144,6	6 195,2	1 208,1	6 013,0	16
45	957,1	6 800,6	1 019,6	6 586,1	1 082,5	6 383,7	1 145,6	6 192,1	1 209,2	6 010,0	15
46	958,2	6 796,9	1 020,7	6 582,6	1 083,5	6 380,4	1 146,7	6 189,0	1 210,2	6 007,1	14
47	959,2	6 793,2	1 021,7	6 579,2	1 084,6	6 377,2	1 147,7	6 185,9	1 211,3	6 004,1	13
48	960,2	6 789,6	1 022,8	6 575,7	1 085,6	6 373,9	1 148,8	6 182,8	1 212,4	6 001,2	12
49	961,3	6 785,9	1 023,8	6 572,2	1 086,7	6 370,6	1 149,9	6 179,7	1 213,4	5 998,2	11
50	962,3	6 782,2	1 024,9	6 568,8	1 087,7	6 367,3	1 150,9	6 176,6	1 214,5	5 995,3	10
51	963,4	6 778,5	1 025,9	6 565,3	1 088,8	6 364,1	1 152,0	6 173,5	1 215,6	5 992,3	9
52	964,4	6 774,9	1 026,9	6 561,9	1 089,8	6 360,8	1 153,0	6 170,4	1 216,6	5 989,4	8
53	965,4	6 771,2	1 028,0	6 558,4	1 090,9	6 357,6	1 154,1	6 167,3	1 217,7	5 986,4	7
54	966,5	6 767,6	1 029,0	6 555,0	1 091,9	6 354,3	1 155,1	6 164,2	1 218,8	5 983,5	6
55	967,5	6 763,9	1 030,1	6 551,6	1 093,0	6 351,1	1 156,2	6 161,1	1 219,8	5 980,6	5
56	968,6	6 760,3	1 031,1	6 548,1	1 094,0	6 347,8	1 157,3	6 158,0	1 220,9	5 977,6	4
57	969,6	6 756,6	1 032,2	6 544,7	1 095,1	6 344,6	1 158,3	6 154,9	1 221,9	5 974,7	3
58	970,6	6 753,0	1 033,2	6 541,3	1 096,1	6 341,3	1 159,4	6 151,8	1 223,0	5 971,8	2
59	971,7	6 749,4	1 034,3	6 537,9	1 097,2	6 338,1	1 160,4	6 148,8	1 224,1	5 968,8	1
60	972,7	6 745,7	1 035,3	6 534,4	1 098,2	6 334,8	1 161,5	6 145,7	1 225,1	5 965,9	0
	coλ	λ	coλ	λ	coλ	λ	coλ	λ	coλ	λ	'
	74°		73°		72°		71°		70°		

Annexe 3

Exemples

1er Exemple

$L : 17~19'N$ $\lambda(L) = 1.055,2$ $\omega\lambda(c_1) = 1529,0$
$D : 22~52~N$ $\omega\lambda(c_2) = 1.040,4$
$H : 10~15,4$ Somme : $2~569,4$ $\omega\lambda(u) = 1.284,7$ $u = Z : 69°04',2$
$D+H = 33~07,4$ $\omega\lambda(D+H) = 4.169,1$ Difference : $488,6$ $\omega\lambda(v) = 244,3$ $v = P : 85°55',9 = 5^h43^m44^s$
$D-H = 12~36,6$ $\omega\lambda(D-H) = 7.572,8$
           $\lambda(c_1) = 5.224,3$
           $\lambda(c_2) = 6.517,6$

2ème Exemple

$L : 44°20',5~N$ $\lambda(L) = 2.974,4$ $\omega\lambda(c_1) = 1.647,9$
$D : 43°18,5~N$ $\omega\lambda(c_2) = 4.738,0$
$H : 15~04,5$ Somme : $6.385,9$ $\omega\lambda(u) = 3.192,9$ $u = Z : 43°06',7$
$D+H : 58~23,0$ $\omega\lambda(D+H) : 2~001,3$ Difference : $-3.090,1$ $\lambda(v-90°) = 1.545,0$ $v = P : 114°55'4 = 7^h39^m42^s$
$D-H : 28~14,0$ $\omega\lambda(D-H) = 4.745,3$
           $\lambda(c_1) = 4~975,7$
           $\lambda(c_2) = 1.770,9$

3ème Exemple

$L : 44°20',5~N$ $\lambda(L) = 2974,4$ $\omega\lambda(c_1) = 2~793,6$
$D : 24~50~N$ $\omega\lambda(c_2) = 6~773,0$
$H : 60~06,5$ Somme : $9.566,6$ $\omega\lambda(u) = 4.783,3$ $u = P : 27°56',1 = 1^h51^m44^s$
$D+H : 84~56,5$ $\omega\lambda(D+H) = ° 303,9$ Difference : $-3.979,4$ $\lambda(v-90°) = 1.989,7$ $v = Z : 121°27',1$
$H-D : 35~16,5$ $\omega\lambda(H-D) = 3~939,3$
           $\lambda(c_1) = 3.278,3$
           $\lambda(c_2) = 964,9$

4ème Exemple

$L : 55°15'~N$ $\lambda(L) = 3994,2$ $\omega\lambda(c_1) = 2370,3$
$D : 62~06,5~N$ $\omega\lambda(c_2) = 9.536,7$
$H : 31~14$ Somme : $11.907,0$ $\omega\lambda(u) = 5953,5$ $u : Z : 20°04,2$
$D+H : 93~20,5$ $\lambda(D+H-90°) : 200,6$ Difference : $-7.166,4$ $\lambda(v-90°) = 3.583,2$ $v : P : 141°09,1 = 9^h24^m36^s$
$D-H : 30~52,5$ $\omega\lambda(D-H) = 4.423,8$
           $\lambda(c_1) = 3.793,6$
           $\lambda(c_2) = 429,6$

5eme Exemple

$L : 48°02'$ N $\lambda(L) : 3294,5$ $co\lambda(c_1) = 3189,7$

$D : 32 00,9 N$ $co\lambda(c_2) = 7030,0$

$H : \underline{64 57,4}$ Somme $= 10 219,7$ $co\lambda(u) : 5109,8$ $u = P : 25°29',4 = 1^h 41^m 57^s$

$D + H : \underline{96 58,3}$ $\lambda(D+H-90°) : 419,3$ Différence $= -3840,3$ $\lambda(v-90) : 1920,1$ $v = Z : 120°27',4$

$H - D : 32 56,5$ $co\lambda(H-D) : \underline{4189,1}$

$\lambda(c_1) : 2875,2$

$\lambda(c_2) : 894,6$

6eme Exemple

$L : 48°19'$ N $\lambda(L) : 3320,0$ $co\lambda(c_1) : 6941,1$

$D : 6 53,5 S$ $co\lambda(c_2) = \underline{431,9}$

$H : 25 36,4$ Somme : $7373,0$ $co\lambda(u) : 3686,5$ $u = P : 37°46',9 = 2^h 31^m 08^s$

$D + H : 32 29,9$ $co\lambda(D+H) : 4238,3$ Différence $= 6509,2$ $co\lambda(v) : 3254,6$ $v = Z : 42°24',8$ (à partir du Sud)

$H - D : 18 42,9$ $co\lambda(H-D) : \underline{6198,6}$ $Z : 137°35',2$ (à partir du Nord)

$\lambda(c_1) : \underline{918,3}$

$\lambda(c_2) : 9518,6$

$7^{ème}$ Exemple

$L : 5°18'$ S $\lambda(L) : 318,5$ $co\lambda(c_1) : 10.621,9$

$D : 50 44,2 N$ $co\lambda(c_2) : 1.227,8$

$H : \underline{28 47,7}$ Somme : $11.849,7$ $co\lambda(u) : 5924,8$ $u = Z : 20°14',4$

$D + H : 79 31,9$ $co\lambda(D+H) : 631,6$ Différence : $9.394,1$ $co\lambda(v) : 4697,0$ $v = P : 28°37',0 = 1^h 54^m 28^s$

$D - H : 21 56,5$ $co\lambda(D-H) : \underline{5640,2}$

$\lambda(c_1) : \underline{313,1}$

$\lambda(c_2) : 5958,7$

8eme Exemple

$L : 15°31'$ S $\lambda(L) : 942,6$ $co\lambda(c_1) : 5843,8$

$D : 23 11,7 N$ $co\lambda(c_2) : \underline{413,6}$

$H : \underline{32 14}$ Somme $= 6257,4$ $co\lambda(u) : 3128,7$ $u = P : 43°50',9 = 2^h 55^m 23^s$

$D + H : 55 25,7$ $co\lambda(D+H) : 2213,0$ Différence $= 5430,2$ $co\lambda(v) : 2715,1$ $v = Z : 48°49',8$

$H - D : 9 02,3$ $co\lambda(H-D) : \underline{8724,4}$

$\lambda(c_1) : 1270,4$

$\lambda(c_2) : 9667,0$