CONSOL NAVIGATION SYSTEM

A paper by Caradoc Williams, describing the Consol Navigation System in general terms, appeared in Volume XXIII, pages 66 to 69, of the "International Hydrographic Review".

One is aware that Consol is an improved British version of a wartime navigation aid developed by Germany and applied to submarine and aircraft navigation up to ranges of 1,500 miles. Consol provides a simple means of determining the great circle bearing of a transmitting station by counting a series of audible dots and dashes heard on an ordinary medium frequency receiver, and referring to a special table computed for each Consol transmitting station.

Before consulting the table or using the special Consol charts, it is advisable to average several bearings.

The great circle bearing from the Consol station is obtained by entering the appropriate table with the count of dots and dashes. The sector is called dot sector or dash sector according to the kind of signal received first during the cycle.

Each Consol station is identified by a call-sign transmitted on a continuous signal. Then the number of dots and dashes which follow are counted. A total of 60 dots and dashes should be heard in each keying cycle but in practice this is rarely the case, since the exact change from dots to dashes (or from dashes to dots) is masked by the width of the equisignal, also known under the English designation of "twilight zone".

This is due to the fact that the radiation pattern consists of alternate sectors of dot and dash signals, each sector being approximately 15° to 20° wide (see fig. 1). This pattern rotates so that the equisignal between the dot and dash sectors moves exactly through one sector's width during the keying cycle. The rotation ceases at the end of the cycle and the pattern returns to its original position for the next cycle. Consequently an observer will hear the equisignal once during each keying cycle, and his angular position in the pattern sector is determined by the number of dot or dash characters heard before the equisignal. If, following upon this, the hearer loses some of the characters at the time of the equisignal, the total number of counted characters should be subtracted from 60 and half the difference added to each of the dot and dash counts.

The Mercatorial bearing from the station is obtained by means of a conversion table in order to account for the Givry correction for meridian convergence, by entering it with the difference of longitude between the observer and the station and the observer's latitude ; any necessary interpolation is made and the resulting correction applied to the great circle bearing in accordance with the rules given in the conversion table. (See : *Radiosignaux à l'usage des navigateurs,* Vol. 1, Publication No. 2 of the *Service Hydrographique de la Marine,* Paris, pages 10 to 20, for the computation and practical application of the Givry correction).

No further attempt will be made here to describe *Consol* equipment, which has already been done in the previously mentioned paper appearing in the *International Hydrographic Review* ; or details for the obtaining of bearings from a Consol station, described in the second volume of the report of the International Meeting on *Radio Aids to Marine Navigation,* held at London in May 1946 under the auspices of the *Ministry of Transport* (Radio Navigation, Radar and Position Fixing Systems for use in Marine Navigation, published by His Majesty's Stationery Office, London, 1946).

Detailed information concerning Consol transmitting stations at present in operation at sea and in European territory will be found, on the other hand, in the usual publications on navigation radiosignals such as *The Admiralty List of Radio Signals,* published by the Hydrographic Department, London, 1949, Volume II, or *Radio Navigational Aids,* Publication No. 205 of the United States Navy Department Hydrographic Office, Washington, 1949.

The Consol medium frequency transmitter has a directional aerial system ordinarily produced by three aerials in line evenly spaced at a distance of the order of three times the wave-length of the transmitter. The accuracy obtained is maximum on the perpendicular bisector of the base line of the three aerials. It falls off as the base line extension is approached, so that useful coverage is restricted to two sectors of about 120° in extent (see (fig. 1) Moreover, the system is not usable within 25 miles from the station. Experience has shown that when interference need not be feared, ranges of at least 1,000 miles over the sea and 700 miles over flat land can be expected on about 90 per cent of occasions under daylight conditions. At night, ranges of at least 1,200 miles are obtainable. Consol is therefore essentially a long range navigational aid, for aircraft as well as for ocean navigation.

During landfall and coastal navigation, mariners are warned to check their Consol bearings, especially when closing danger. As much as 12 miles in error at night have been noted for some bearings. By day the probable error will be smaller than by night, but the possible extent of error should be borne in mind.

No. 22 of 28th May 1949 of the *Weekly Admiralty Notices to Mariners* gives useful information on the subject in the form of a table which is reproduced below. Probable position line errors are given in nautical miles at various ranges and bearings from the Consol station. The figures given are "95 % Position Line Errors", that is, the errors which, for a given range and bearings, are not exceeded in 95 % of a large number of bearings. Thus they are errors which it is prudent to assume present, although individual bearings may be more accurate than the figures quoted.

TABLE

Bearing from normal to aerial array	Day range in nautical miles					Night range in nautical miles		
	Over sea			Over flat land				
	250	500	1,000	250	500	100	300-1,000	1,500
	1 1/2	3	6	3	6	1/2	approx. 10	18
60°	3	6	12	6	12	1	— 20	36
75°	6	12	24	12	24	2	— 40	72

Consol stations are extremely powerful special radio beacons with transmitters having wave-lengths varying between 1500 and 600 metres corresponding to 200 and 500 kilocycle frequencies respectively. The power of the coastal transmitter reaches 2 kilowatts on a carrier wave. The receiver used on board ship for the signals must be extremely selective and tuned in on the wave-length of one of the stations in order that three separate transmissions repeating at intervals may be picked up. These three transmissions make up the entire transmission cycle, and enable the observer to determine his position line in relation to the radio beacon with great accuracy. The first transmission consists of the station's *call-sign* for indentification purposes in case there is a station in the vicinity having a nearly similar wave-length. The second transmission consists of a continuous signal ; it is *omnidirectional* and makes it possible for ships equipped with a radiogoniometer to avail themselves of the station by operating the instrument in the usual manner. The third transmission, called directional transmission, consists for all practical purposes of three separate groups that can be listened to as follows : (1) as a group of dots ; (2) continuous signal ; (3) group of dashes ; or, (1) group of dashes ; (2) continuous signal ; (3) group of dots.

The new Consol radio beacons at present in operation have the three types of transmission mentioned ; but the older ones are limited to the last two types.

The part of the signal beginning with dots gives rise to a noise in the background that gradually increases in intensity and ends by changing into dash signals ; the reverse occurs, dashes changing into dots. The transition from dots to dashes or vice versa cannot be picked up very clearly; some of the dots or dashes are masked by the background noise.

In either case, a continuous signal, which shall hereafter be called *equisignal* (signal of equality) separates the two dot and dash signal groups. Its existence is due to the fact that the change from dots to dashes or the reverse does not take place suddenly but during the zone of uncertainty described above ; during this period dots and dashes are not clearly distinguishable ; rather a combination of sounds corresponding to the two signals is heard which finally resolves into a continuous signal.

The directional transmission of the radio beacon invariably consists of 60 signals, so that if the zone of uncertainty did not exist the sum of the dots and dashed heard by the observer would always be 60. In practice the sum of the signals heard in each group is always under 60 and the difference between this number and 60 represents the width of the equisignal. If, for example, a signal consisting of 16 dots and 42 dashes has been observed, two of the characters have been lost, and the Consol signal will have to be brought up to 17 dots. The value of E, the error of the width of the equisignal, is given by the formula :

$$E = 60 - (X + Y)$$

and the numeral value S of the "Consol signal" is given by the formula :

$$S = X + \frac{60 - (X + Y)}{2}$$

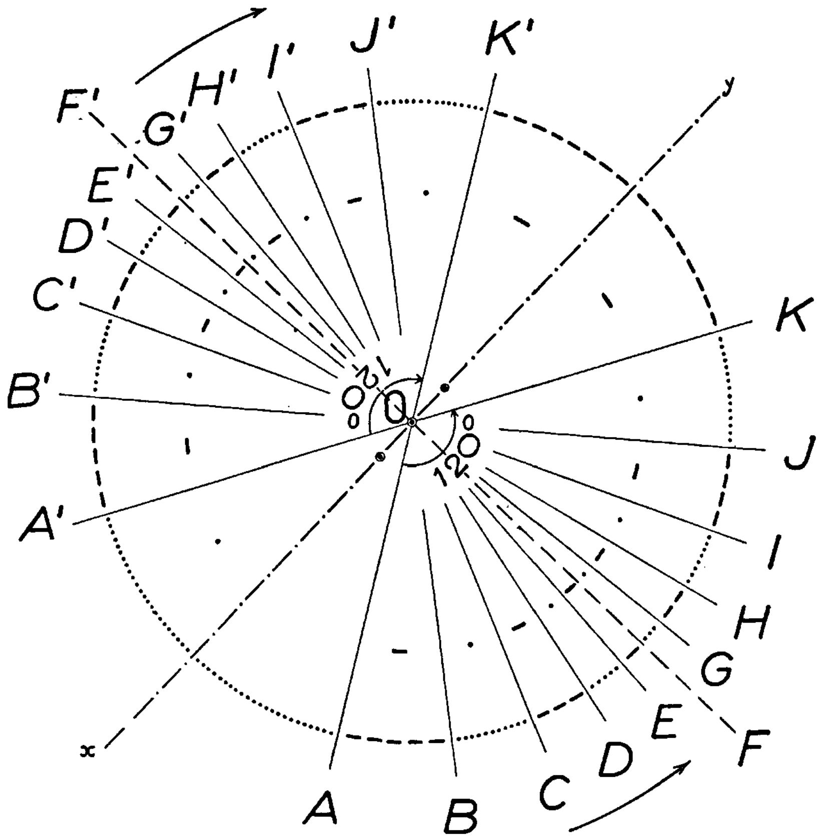

FIG. 1

in which X is the number of signals in the group beginning the directional transmission and Y the number of signals in the group ending the cycle ; it must be observed that during directional transmission it is absolutely necessary to count the signals heard in each group in order to determine the value of the width of the equisignal without which it would be impossible to define the Consol signal.

The Consol radio signal is the same along the great circle passing through the radio beacon and the observer. Two observers on this great circle will hear the same Consol signal whatever their distance from the station. A graphic representation of Consol signals can therefore be plotted on a chart by means of straight lines in gnomic projection and special Mercator projection curves.

Let x O y (Fig. 1) be the directional array of the Consol antenna, and OA, OC, OE, OI and OK the great circle bearings, computed from the radio beacon, of positions where a 60-dot Consol signal may be heard ; and Ox, OB, OD, OF, OH, OJ and Oy be the great circle bearings, computed from the radio beacon, of positions where a 60-dash Consol signal may be heard. At any point within the sectors limited by the bearings x and A, B and C, D and E, etc., dot signals are heard whose numerical value increases as the observer moves in the direction of the arrow ; that is, in the xOA sector, near Ox, only a few dots are heard, compared to many dashes, while in the vicinity of OA in the same sector, a large number of dots and few dashes are heard. Conversely, at all positions inside the sectors bounded by bearings A and B, C and D, etc., dash signals in increasing numbers are heard as the observer moves in the direction of the arrow, that is, near OA in the OAB sector, a few dashes and many dots are heard, and near OB, without however quite reaching it, many dashes and few dots.

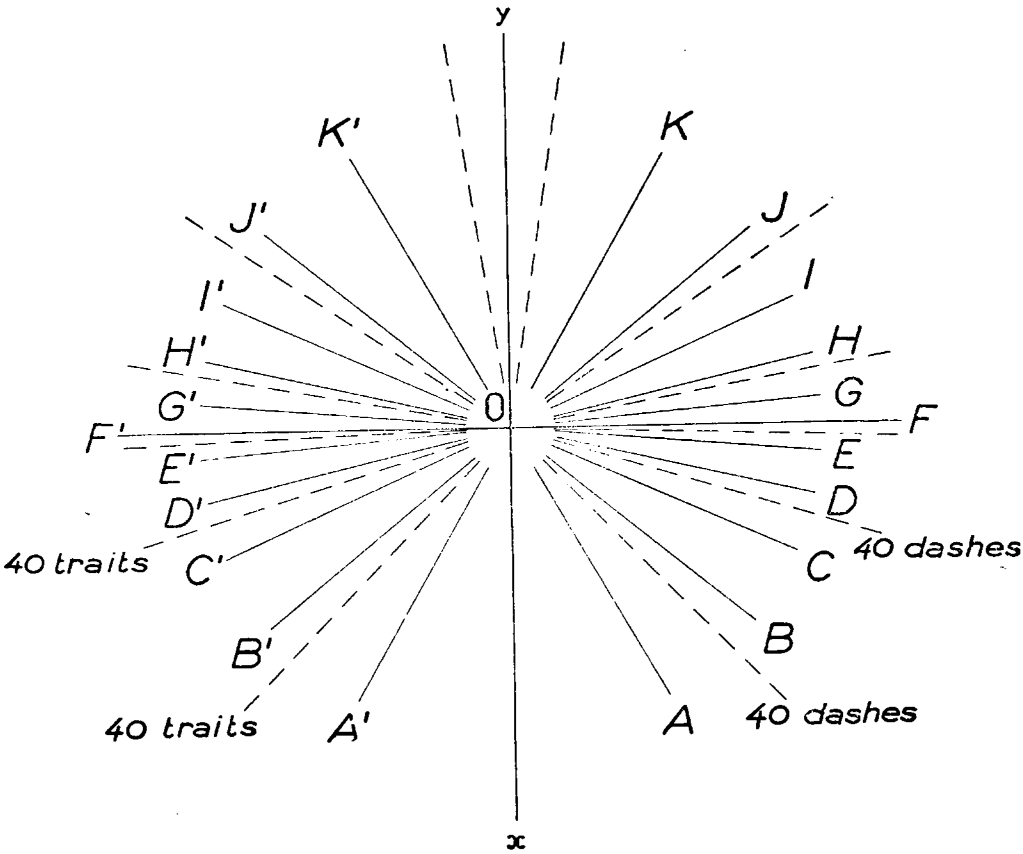

FIG. 2

Figure 2 shows the total number of sectors formed on the horizon by the radio beacon, limited by the great circle bearings corresponding to the 60-dot and 60-dash Consol signals, and an example has been given showing the great circle bearings corresponding to Consol signals with the two dot and dash types of signal for a value corresponding to a 40-dash Consol signal. Symmetry with regard to the position line of the antennae will be noticed. It can be seen from the figure, therefore, that when a definite Consol signal is picked up uncertainty exists with regard to the position line occupied by the observer, according to the sector in which he is located. This uncertainty is resolved either by taking a radiogoniometrical bearing during the omnidirectional transmission or by using the reckoning furnished by navigation, which makes it possible to resolve the uncertainty by comparison with the twelve great circle bearings shown on the sketch.

If charts are available showing the loci corresponding to each Consol signal, the position of the ship can be plotted directly on the chart at the intersection of the loci.

Tables of great circle bearing values can also be drawn up, computed from True North, and corresponding for a determinate station to each Consol signal. The value of the Consol signal is shown in the left-hand column of these tables and on the corresponding line are the twelve corresponding great circle bearings, computed in relation to True North, of all the loci where the same signal will be heard.

The complete set of tables for each station consists of two plates, one for dot signals (cycle beginning with dots) from 1 to 60 and the other for dash signals (cycle beginning with dashes) likewise from 1 to 60. The tables give emphasis to the zone of uncertainty previously mentioned for each station.

The tables are computed by means of the formula :

$$\sin \theta = \frac{k}{2n} \pm \frac{c}{120\,n}$$

θ is the angle computed from the principal radius perpendicular to the position line of the antennae. From the angle θ and the true position line Zv of the principal radius, the values of the true great circle bearing Nv are deduced, reckoned clockwise from North of the Consol station.

c is the number of characteristic signals of the station ;

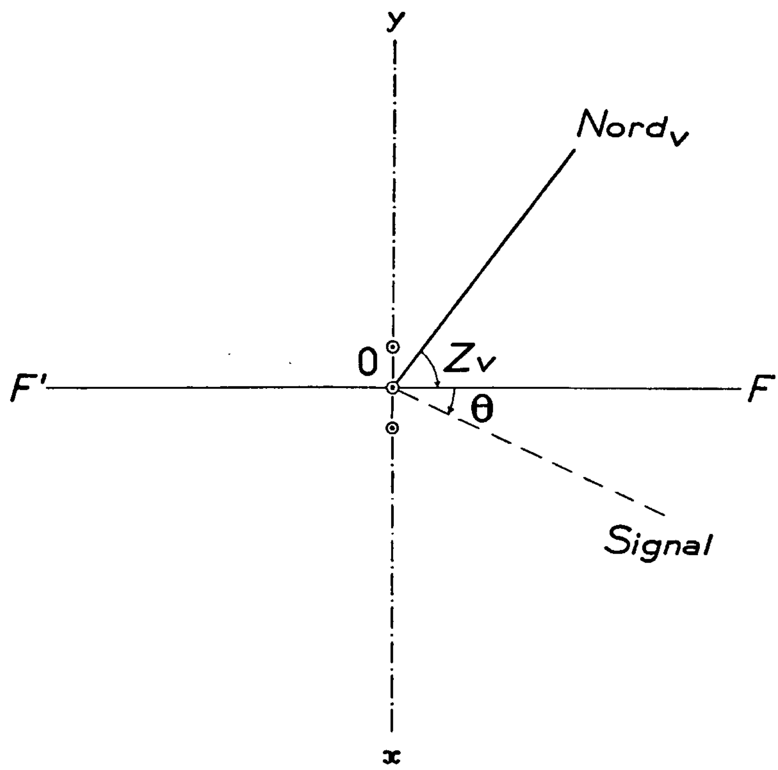

FIG. 3

n is a station constant : $n = \frac{d}{\lambda}$, d being the distance in metres of the lateral antennae with regard to the central antenna ; λ is the wave-length expressed in metres ; k is the constant characteristic of the sector which can have values of O, ± 2, ± 4, ± 6 in dot sectors and ± 1, ± 3, ± 5 in dash sectors, as shown on Fig. 4.

In the older Consol stations of the Elektra-Sonne German type, directional transmission lasts 60 seconds with dots lasting 1/6 of a second and dashes of 5/6 of a second's duration, alternating with silences of one second's duration.

In the newer stations the length of directional transmission has been reduced to 30 seconds; so that the total transmission cycle itself is reduced to one minute instead of two minutes as in the older system.

Range is variable in accordance with the radio beacon's zone of operation, time of observing, and receiver conditions. The following relevant figures may be quoted :

MAXIMUM RANGES IN MILES

Observation	Over sea				Over land
	Atlantic Lat. 55° N.	Atlantic 35° N.	0°	Mediterranean	Northern Europe
Day	1,200	1,000	700	900	1,200
Night	1,500	1,200	700	1,200	700

These ranges may decrease significantly when the degree of static noise becomes high, but the perturbing influence may be decreased by using crystal gate filters reducing the receiver band to a few hundred cycles.

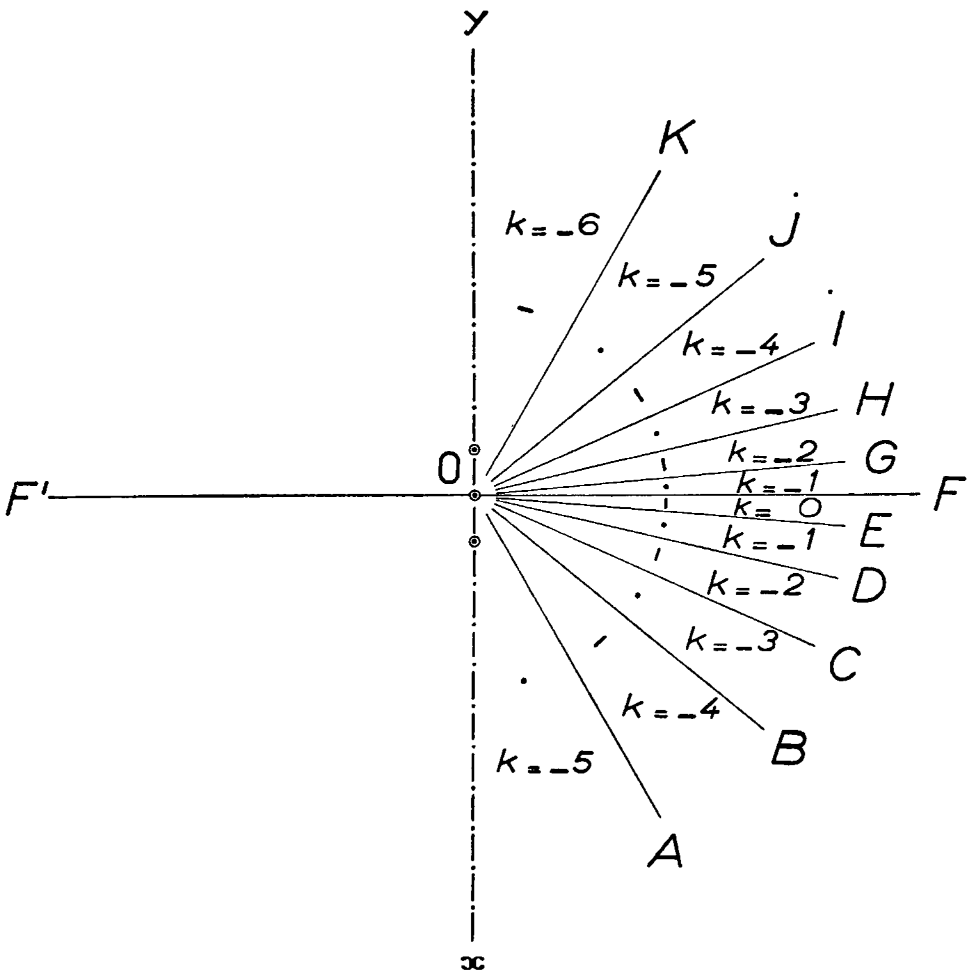

FIG. 4

With stations of the older type a position may be determined by means of two bearings at the end of a 6 to 8 minute interval, and this may be decreased from 3 to 5 minutes when the reduced cycle type of transmission becomes general, such as that of the Bush Mills station, which has been in continuous operation since 1st January 1947.

Simplification of stations is being studied by reducing the number of antennae to two and in order to eliminate idle sectors as well as ambiguity of bearings.

The construction of a signal counter and automatic recorder to facilitate use of the system is also being planned.

The Marconi Company is building a perfected Consol transmitter including 3 antennae for the French Brest station. Keying is accomplished electronically with gas-filled relay valves. Amplification of the equisignal occurs after keying. An increase in the power output is possible owing to the absence of a vacuum switch keying circuit.

In order to avoid the 60° idle sectors, it was proposed to set up two Consol stations at the same site with their base lines sited at right angles. In addition to giving 360 degree coverage, this system would make possible a method of observation resolving sector ambiguity. The count from the most favorably situated normal would be used to obtain the position line and if required the count from the other normal to resolve ambiguity.

A two-aerial system could also be applied to this dual system which then would only require three masts arranged in the form of a right-angled triangle.

In the new transmitter developed by the Marconi Company the idle time in each cycle has been reduced so that the whole operation takes place in 40 seconds instead of two minutes as on the original "Sonne" apparatus.

Audible signals are received from a monitor some distance away and these signals make it possible for the station operator to regulate manually the transmission phase in order to give the correct directional characteristics.

It should be noted that for technical reasons the relatively long base line of Consol (six wave-lengths) is a factor which contributes noticeably to the accuracy of the performance of this navigation aid, including its comparative freedom from site and polarisation errors, and that any attempt tending to reduce the length of the baseline drastically would impair the accuracy of performance. A very short baseline, in the neighbourhood of half a weve-length, would reduce the performance of the system to that of a shore-based radiogoniometrical station with Adcock aerials. Reduction of the baseline by half, such as might be done in converting from 3 aerials to 2 aerials, would possibly be of lesser importance (see : "Theoretical comparison of the Two—and Three—aerial Sonne System").

The " Instituto Hydrografico de la Marina ", Cadiz, has just issued Special Publication No. 2, "Radiofaros Consol", which describes the practical use and elementary fundamental theory of Consol transmission. The publication also analyzes the method of the navigator's using great circle bearings obtained for the purpose of having a locus. A few extracts from the theoretical section, attributable to Lieutenant Commander Don José García de Quesada, are given below.

It should be pointed out that if graphic representation on a chart of a great circle bearing corresponding to a Consol signal is theoretically a line, in reality the same signal may be heard in a zone whose axis is the same line. This is due to the fact that while the position of the observer is continuously changing, the signals vary at intervals whose characters differ. The error cannot exceed one half of the interval between two consecutive signals consisting of different characters.

As Consol position lines are loci with an equal difference of phase between transmissions of the antenna, they may be affected by abnormal operation of the transmitter.

THEORETICAL PRINCIPLES OF THE CONSOL SYSTEM

The three transmission antennae A, B and C, are sited in a straight line, central antenna B being equidistant from the outside antennae B and C. The common length between them is in practice from two to three times the wave-length.

Transmission is accomplished by means of a transmitter T, connected with the three antennae A, B and C (Fig. 5).

Phasing of the feeding-circuit of the outside antennae is subject to a double cycle consisting of an abrupt dephasing change in jumps of 180° besides a continuous dephasing increase from 0° to 180°. The abrupt dephasing is effected by a dephaser P, and the continuous increase by a dephaser D.

Frequency of transmission is the same for the three antennae ; for reasons of convenience explained further on, however, the range of the central antenna is approximately four times that of the waves radiated by the outside antennae.

The central antenna transmits the distinctive call-sign of the station as well as the

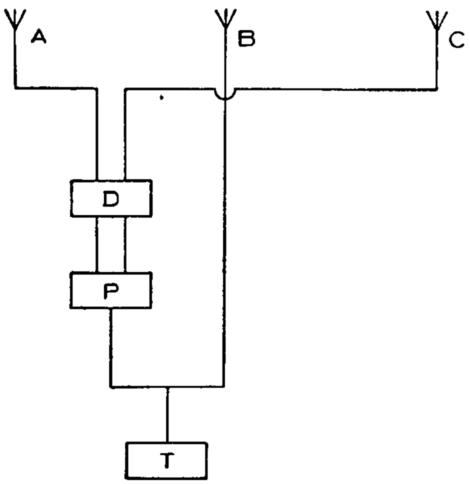

FIG. 5

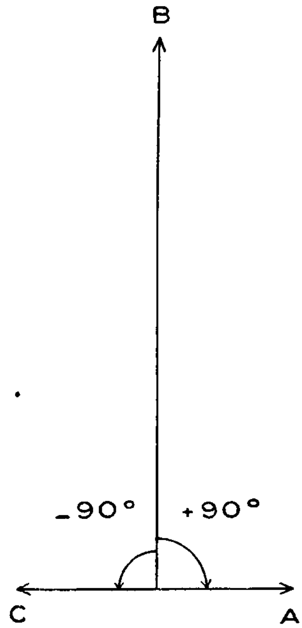

FIG. 6

omnidirectional waves. The outside antennae begin to operate at the outset of the directional transmission in conjunction with the central antenna, effecting the dot and dash transmission and rotation of the equisignal.

Directional transmission begins with an initial 90° phase change of the feeding circuit of the outside transmitters in relation to the central transmitter. The current of A moves 90° ahead of that of B, and C's current is 90° behind B's. B's feeding circuit phase does not vary, and will henceforth be used for reference.

The phases of the wave-lengths of all three antennae are therefore in a same relative position, represented by Figure 6.

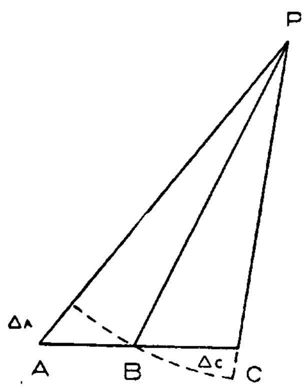

FIG. 7

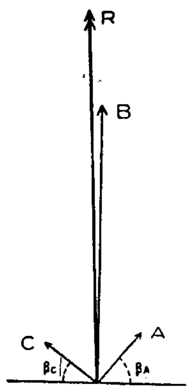

FIG. 8

The phases of the three waves as they arrive at point P may now be examined. (Fig. 7). If the signals arrived at P in the same phase relation as upon leaving the radio beacon, it is obvious that the signal received, a result of all three signals, would be equal to the signal of the central antenna B, since the other two cancel one another out, being in opposite phases.

This does not actually occur, however, since P is at different distances from A, B and C. The difference in distance has no appreciable effect on the relative signal intensity at P, but compared with the wave-length it causes variation in the phases of the signals arriving at P. Signals from A are delayed with regard to B's, while C's gain on B's. Figure 8, showing the gain and delay phase angles B_A and B_C are proportional to the differences in distance $\Delta_A = PA - PB$ and $\Delta_C = PC - PB$. If the difference in distances is equal to a wave-length, the phase gain or delay will be 360°. Vector R (Fig. 8), shows the result at P.

Were the signals to arrive at P in the same position as upon transmission, the polar or azimuthal diagram at the initial instant would of course be a circle ; actually, owing to the dephasing occurring because of the differences in distance AP, BP, CP, the loci of points receiving the same signal are lines where

$\Delta_A - \Delta_C = (PA - PB) - (PC - PB) = PA - PC =$ constant, i.e., hyperbolae of the foci A and C.

If a point remote from the station is involved, the distance AC is invariably short with regard to the distance separating the observer from the station, and the hyperbolae will consequently be virtually straight lines concurrent to the central point B (Fig. 9).

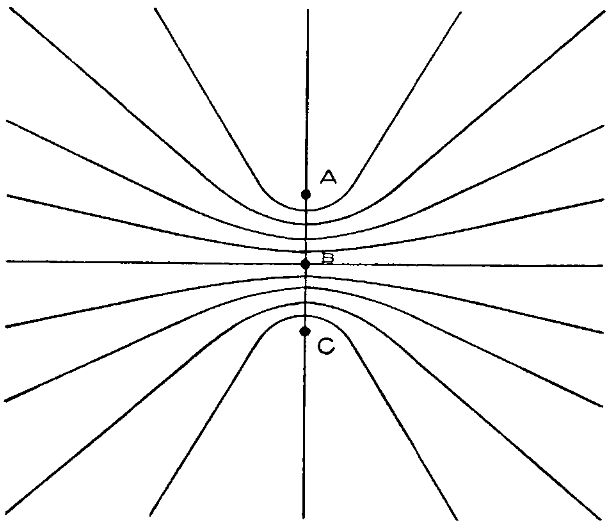

FIG. 9

It follows that, with regard to sufficiently long distances, each bearing corresponds to a single signal which remains the same along the whole length of the line, except near the antennae.

If point P is far enough away, it may be assumed that directions PA, PB and PC (Fig. 10) are parallel ; $\beta_A = \beta_C$ and $\Delta_A = \Delta_C$ (Fig. 11).

Let θ be the bearing of P reckoned from the normal at B in the direction of the antennae. Let AB = d.

$$\Delta_A = d \sin \theta$$

or :

$$\sin \theta = \frac{\Delta_A}{d} \quad \ldots\ldots (1)$$

On the other hand :

$$\frac{\Delta_A}{\beta_A} = \frac{\lambda}{2\pi}$$

Whence :

$$\Delta_A = \frac{\lambda \beta_A}{2\pi}$$

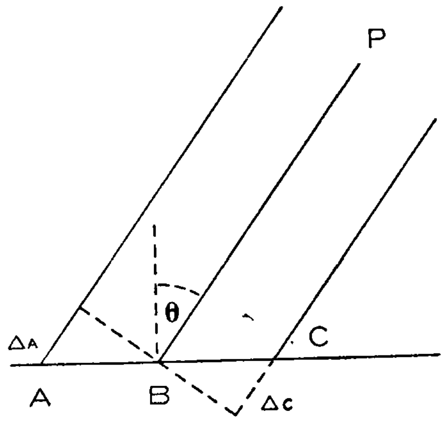

FIG. 10

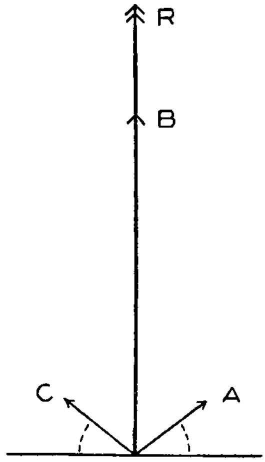

FIG. 11

AB $= n\lambda$, n representing the number of times the wave-length is contained in AB. Therefore :

$$\sin\theta = \frac{\lambda\beta_A}{2\pi} \times \frac{1}{n\lambda} = \frac{\beta_A}{2n\pi}$$

The signal received will therefore be at maximum strength when vectors A and C are superimposed on the signal from B, and at minimum strength when in opposition to it.

The former occurs when :

$$\beta_A = \beta_C = \frac{\pi}{2} + 2k\pi \quad \text{(k being an integer)}$$

and the latter when :

$$\beta_A = \beta_C = \frac{3\pi}{2} + 2k\pi \quad \text{(k being an integer)}$$

Maximum values are moreover :

$$\sin\theta = \frac{\frac{\pi}{2} + 2k\pi}{2n\pi} = \frac{1+4k}{4n}$$

Minimum values correspond to the following :

$$\sin\theta = \frac{\frac{3\pi}{2} + 2k\pi}{2n\pi} = \frac{3+4k}{4n}$$

Figure 12 represents the diagram where the value n is equal to 3.

During directional transmission the feeding circuit of the outside antennae undergoes a phase variation in sudden jumps of 180°. This takes place by means of a dephaser P. The variation rhythm is the following : signal without phase change during a quarter of a second ; sudden phase variation of 180° maintained at its new value for three quarters of a second ; another sudden variation bringing it back to its former condition, and so on (see Fig. 13).

During the whole cycle, 30 phase changes of each kind occur. In the newer stations, the complete cycle is reduced to 30 seconds, and the phases last respectively 1/8 and 3/8 seconds.

The observer at P receives at the initial instant the resultant R (Fig. 14) ; following the 180° phase variation, he receives R', and the signal at P passes from R to R' at the rhythm shown on Figure 15.

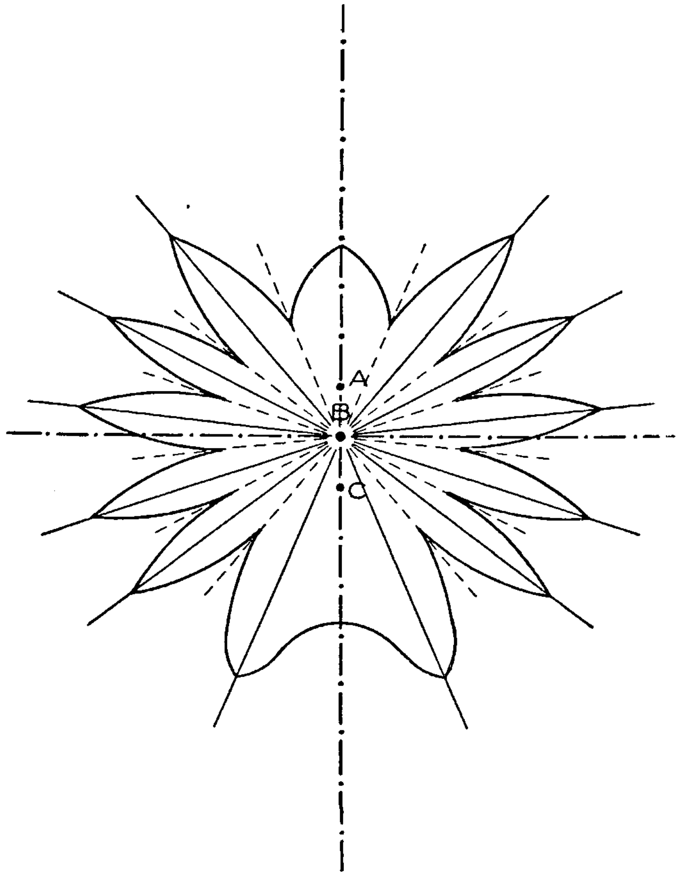

FIG. 12

If resultant R is greater than R', the observer will detect a signal lasting one quarter of a second followed by a weaker signal lasting three quarters of a second, that is, he will receive a signal made up of dots on a background of noise cause by the R' resultant. This condition of reception may be shown graphically by Figure 15. If, according to the observer's position, the R resultant is smaller than R', he will detect dash signals lasting three quarters of a second spaced on a background noise lasting 1/4 second (Fig. 16).

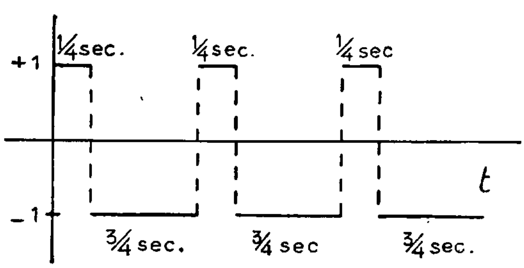

FIG. 13

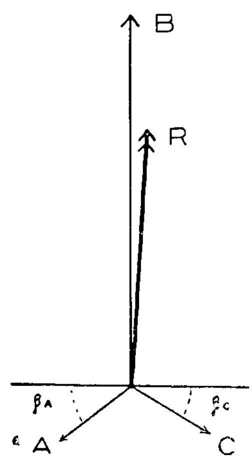

FIG. 14

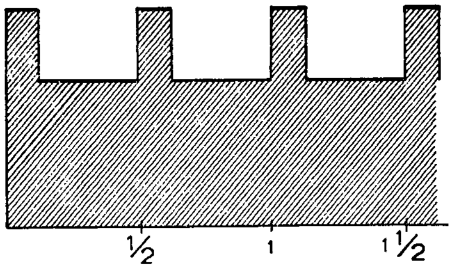

FIG. 15

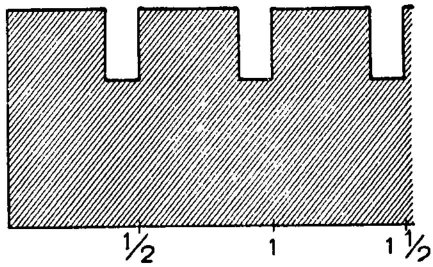

FIG. 16

After the sudden 180° phase jump, the azimuthal diagram (Fig. 12) changes into a symmetrical polar diagram, as shown by the dotted lines on Fig. 17, and the jumps from one diagram to another continue at the rhythm of Figure 15. The observer at P, for a bearing where the signal corresponding to $S = +1$ (Fig. 13) is stronger than the signal corresponding to $S = -1$, will hear dot signals, which grow clearer in proportion to the increase in the difference between the $S = +1$ and $S = -1$ signals. The observer at P' will hear dots more clearly than the observer at P. The observer at Q will hear dash signals.

For observers at M the signal will not vary at the time of the S change ; they will detect a continuous sound motivated by a balance between the aural reception of the dots and dashes. These observers are on lines called *equisignal lines.*

The *equisignal line* may be defined as that along which a continuous sound is heard, where the outside antennae cancel out owing to phase opposition. The signal received all along the equisignal line is that transmitted by the central antenna B.

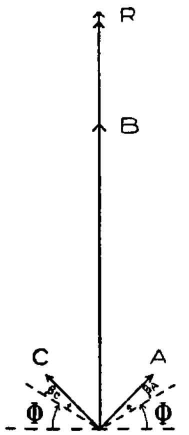

FIG. 18

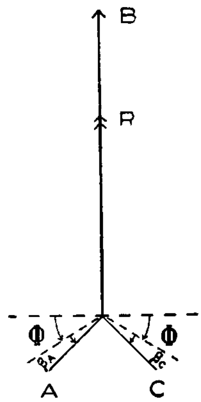

FIG. 19

This occurs at points where $\beta_A = \beta_C = 0°, 180°, 360°$, and in a more general way where $\beta = k\pi$ (k being an integer) that is :

$$\Delta_A = \frac{\lambda}{2} k$$

or :

$$\sin\theta = \frac{\lambda k}{2} \times \frac{1}{n\lambda} = \frac{k}{2n} \quad \ldots\ldots (2)$$

This equation enables the equisignal bearing to be computed from the radio beacon.

It can be seen from the formula that these bearings do not correspond to evenly spaced lines ; their interval increases when the normal is deviated from. This occurs because a constant increase of sin θ represents a greater increase of θ when the angle is closer to 90°.

Continuous dephasing. Rotation pattern.

During the whole directional cycle, the phases of the currents feeding the outside antennae are moreover subjected to a second variation consisting in a continuous and progressive dephasing increase from 0° to 180°.

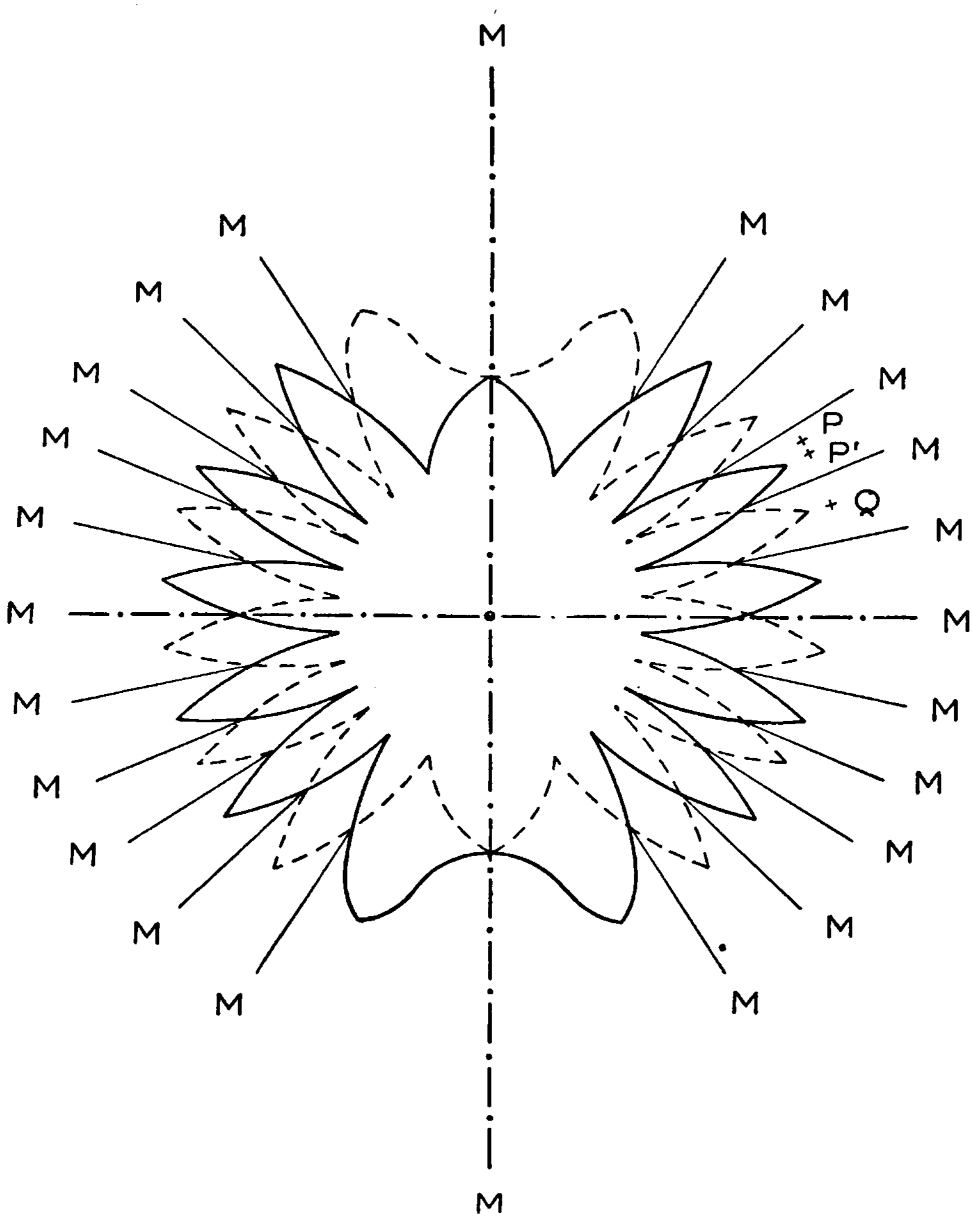

FIG. 17

Let Φ be the dephasing occurring at an instant t. Following this continuous variation, the signals received will correspond to Figure 18 for $S = +1$, and to Figure 19 for $S = -1$. The *equisignal lines* for instant t will be at loci where :

$$\beta + \Phi = k\,\pi \quad (k \text{ being an integer})$$

To the phase angle Φ corresponds the wave-length fraction $d\,\Phi$ such as :

$$\frac{d\,\Phi}{\Phi} = \frac{\lambda}{2\pi}$$

whence :

$$d\,\Phi = \frac{\lambda\,\Phi}{2\,\pi}$$

Thence, the equation (1) takes the form of :

$$\sin\theta = \frac{\Delta A - d\,\Phi}{d}$$

And by substitution the following is obtained :

$$\sin\theta = \frac{\frac{\lambda\,k}{2}}{n\,\lambda} - \frac{\frac{\lambda\,\Phi}{2\,\pi}}{n\,\lambda} = \frac{k}{2\,n} - \frac{\Phi}{2\,\pi\,n} \;\ldots\ldots(3)$$

This formula, compared with formula (2) shows that the equisignal at instant t has rotated with relation to its initial position. This rotation, depending upon the term

$$\frac{\Phi}{2\,\pi\,n}$$

is a function of phase angle Φ.

When Φ varies from 0° to π, $\sin\theta$ varies from :

$$\frac{k}{2\,n} \text{ to } \frac{k}{2\,n} + \frac{\pi}{2\,\pi\,n} = \frac{k+1}{2\,n}$$

meaning that each equisignal line has rotated in the course of the cycle so that it occupies the position of the contiguous equisignal.

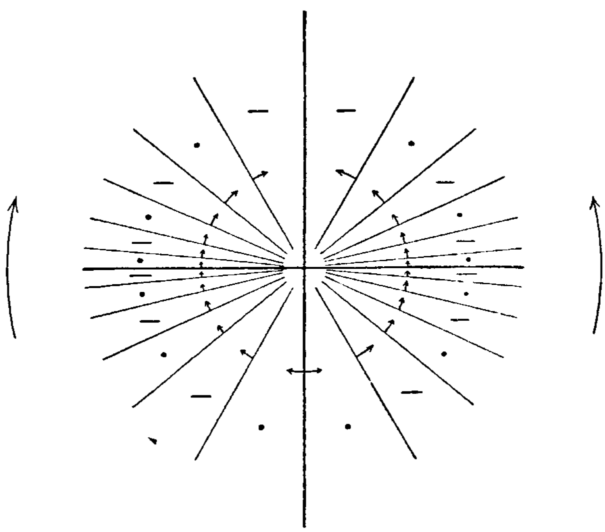

FIG. 20

It has been pointed out above that the equisignals were not equally spaced, and as the variation of angle Φ is regular, the deduction is that the rotation speeds are different in each sector, according to their various positions with relation to the normal.

At the end of the cycle, the equisignals have rotated together with the whole radiation pattern to ocupy the position of the next sector. Moreover, in sectors where dots were initially heard, dashes will be heard at the end of the cycle and vice versa.

The direction of rotation is counterclockwise when θ decreases and Φ increases ; this occurs on the right-hand side of Figure 20, which shows the direction of rotation of equisignals.

The equation (3) for the right-hand side of the pattern is :

$$\sin \theta = \frac{k}{2n} + \frac{\Phi}{2\pi n} \quad \ldots\ldots (3A).$$

During the entire directional transmission the observer will hear according to the pattern a cycle beginning with dots and ending with dashes, or vice versa ; Fig. 21 shows a dynamical diagram of it. Immediately preceding and following the equisignal, it is obviously difficult to distinguish the dots from the dashes, and in general one or two of the characters are lost, masked by the noise of the weak signal. This has the effect of a widening of the equisignal. The sharpness of the latter depends upon the ratio of the B antenna signal to the signals of A and C. The lower the ratio, the sharper the equisignal. A value of 4 is usually taken, as a lower value would not be suitable for other reasons.

By counting the number of characters (dots or dashes) reaching the observer before the equisignal, an accurate bearing inside the sector can be obtained.

Computing of bearings

The formula for computing equisignal bearings has been given above as a function of the phase angle Φ. The relation of this angle with the observer's character count until the equisignal is reached will now be examined.

If the observer were on the *equisignal line,* he would count 60 characters ; if he counts only c characters, having a uniform variation, the following ratio is obtained :

$$\frac{60}{\pi} = \frac{c}{\Phi}$$

whence :

$$\Phi = \frac{c \cdot \pi}{60}.$$

Formula (3) becomes :

$$\sin \theta = \frac{k}{2n} - \frac{c\,\pi}{2\,\pi\,n \cdot 60} = \frac{k}{2n} - \frac{c}{120\,n}$$

and formula (3A) becomes :

$$\sin \theta = \frac{k}{2n} + \frac{c}{120\,n}$$

so that in order to compute the bearing, it is first necessary to know the correct sector position, the sector being described by the integer k in the above equations.

Accuracy of bearings.

If an error of one unit is made in the character count, angle θ is affected by an error $\delta\theta$ which will be computed thus :

$$\sin(\theta + \delta\theta) = \frac{k}{2n} - \frac{c+1}{120\,n}$$

but,

$$\sin(\theta + \delta\theta) = \sin\theta \cos\delta\theta + \cos\theta \sin\delta\theta$$

that is, since $\delta\theta$ remains small :

$$\sin(\theta + \delta\theta) = \sin\theta + \delta\theta \cos\theta = \frac{k}{2n} - \frac{c+1}{120\,n}$$

whence :

$$\delta\theta \cos\theta = \frac{-1}{120\,n}$$

or :

$$\delta\theta = \frac{-1}{120\,n \cos\theta}$$

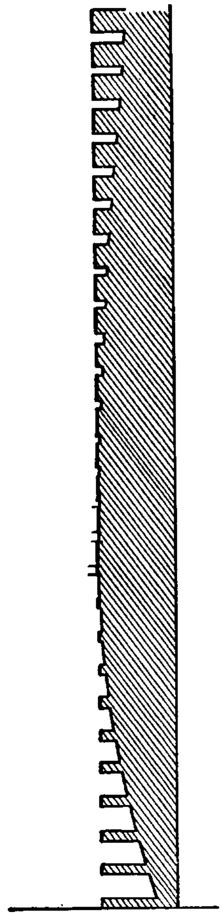

FIG. 21

The minimum value of the error occurs on the normal bearing for which $\cos\theta = 1$. When deviated from, the error increases until in the vicinity of the *antenna lines* it reaches an unacceptable value.

This error limits the number of usable sectors, and on either side of the *antenna lines* is an idle sector zone where observations cannot be guaranteed.

Besides the *equisignal lines* which follow the hyperbolic paths described above, there are other *equisignal lines* that occur during pattern rotation.

Figures 22 and 23 show that if the signals of the outside antennae reach the observer so that their resultant is at right angles with the central antenna signal, the size of the total resultant is not changed by the sudden jumps. Phase changes occur, but which are of no concern here. Under these conditions there are other loci consisting of closed curves around the central antenna and along which the resultant remains unchanged as the sudden jumps occur.

If a point near the antennae is considered, angles β_A and β_C then have a distinct value, and perpendicularity of the resultant of the outside antennae with the central antenna occurs, as in Fig. 24 in which :

$$(\beta_A + \Phi) - [\pi - \beta_C + \Phi] = 2\,k\,\pi$$

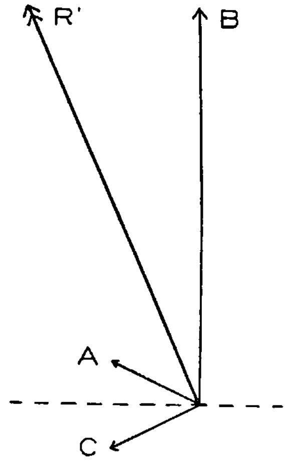

FIG. 22

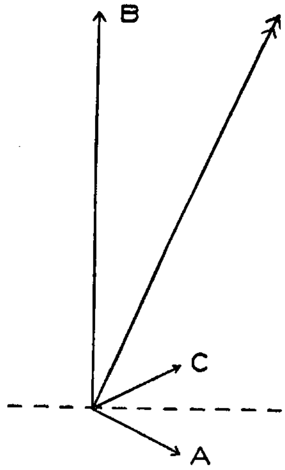

FIG. 23

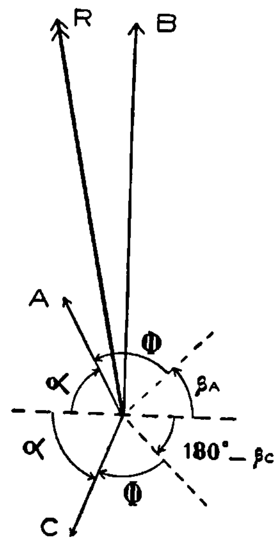

FIG. 24

or :

$$\beta_A + \beta_C = (2\,k + 1)\,\pi$$

This condition is independent of angle Φ as the loci remain stationary after the rotation cycle begins.

The condition of the latter formula requires $\Delta_A + \Delta_C$ to be constant.

Now :

$$\Delta_A + \Delta_C = (PA - PB) + (PC - PA)$$

and :

$$PA - PB \leq d \qquad \text{and} \qquad PC - PB \leq d$$

therefore it must be that :

$$\Delta_A + \Delta_C \leq 2\,d,$$

otherwise no new *equisignal lines* exist.

On the other hand the ratio

$$\frac{\Delta_A + \Delta_C}{\beta_A + \beta_C} = \frac{\lambda}{2\pi}$$

added to the above-mentioned condition gives :

$$\Delta_A + \Delta_C = (2\,k + 1)\,\frac{\lambda}{2}$$

From which it is deduced that *equisignal lines* exist only for integral values of k, verifying the formula

$$(2\,k + 1)\,\frac{\lambda}{2} \leq 2\,d$$

limiting the lines in the vicinity of the antennae. This is why a 25-mile minimum has been chosen as the usable distance from the antennae.

On either side of the lines shown on Fig. 25 by broken lines, dots and dashes are heard ; signals heard between two hyperbolic *equisignal lines,* however, are not always the same in the vicinity of the antennae.

In February 1949 tests made with a view to determining the practical possibilities of Consol beacons on the Spanish coast were made by the *Hydrographic Service of the French Navy* on board two despatch vessels.

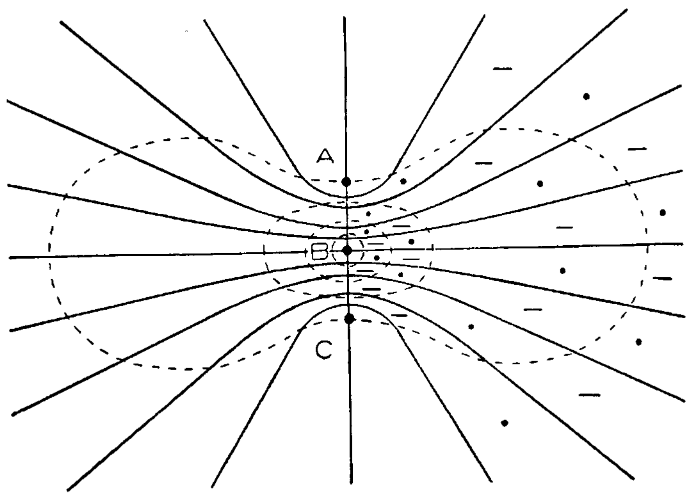

FIG. 25

A report of the tests was published in the *International Hydrographic Bulletin,* No. VI of June 1948 from pages 126 to 129 and the following conclusions were reached :

With the present receiving equipment the Consol system gives, for average observation of distances obtained, a bearing of greater accuracy than that obtained by ordinary mean frequency radio compass. The intercept error attains the accuracy of astronomical navigation when the horizon is poor.

Final measurement should be adjusted for two systematic causes of error which should be investigated and determined accurately. These causes of error consist of the following :

1. Influence of coastal refraction ;
2. Influence of electro-magnetism of the hull.

The investigations mentioned above should be carried out only at certain well determined positions by swinging the ship systematically and eventually recording a deflection curve for the second influence.

So far as the utilisation of the bearing is concerned, the plotting of far distant bearings could be facilitated on certain landfall charts by tracing several true bearings at one degree intervals, as for instance on Chart No. 3007 bis published by the *French Hydrographic Office.*

However, as other radio beacons, Consol does not provide sufficient accuracy for the mariner at great distances. For instance, at a point 530 miles from the Sevilla Consol station and 510 miles from the Lugo station, a Consol position was obtained by means of two bearings which bisected one another at 40 degrees (therefore under favorable conditions) but which showed a degree of inaccuracy that often reaches ten miles ; a long-distance Consol position is therefore inferior to an astronomical position.

On the other hand, for short distances, excellent results are obtained, as tests undertaken last December by M. Hugon, Head Professor of Hydrography, revealed ; these tests were carried out on board a surveying tender while checking a Consol chart he had drawn up of the landfall area on the Spanish, Portuguese and Casablanca coasts. (Extract from the "Revue Maritime", Paris, July 1949, pages 893 and 894 ; August 1949, pages 1014 to 1018 ; September 1949, page 1160).

When the Consol radio beacon at Quimper goes into operation, it will be extremely useful ; and a *Brest Landfall Chart* may be imagined that would be a Consol chart in Mercator projection showing in addition to Consol bearings, several characteristic lines of sounding, positions of sea and air radio beacons in the area, and reference marks to coastal radar-screen views.

GENERAL CONCLUSIONS.

A comparison at present of the merits of performance of the three tested radio-electrical aids : Consol, Decca, and Loran, is out of the question, if only for the reason that all three systems at their present stage of development cannot be used for identical purposes.

While "Consol" may provide in many instances a crude determination of position on the order of a poor astronomical observation during periods sometimes long prior to landfall, " Loran " reaches its maximum efficiency in open sea areas and as its name indicates, at a great distance from the transmitters, ensuring a slightly higher rate of accuracy than an astronomical determination.

"Decca", on the contrary, is the ideal system for landfall and coastal navigation between an area of very wide coverage and the close-lying radar area, with an accuracy surpassing even the determination of position by bearings.

In order to reach a complete evaluation, account would of course have to be taken of the following factors : extension of the chains, facility of handling and of reading the system, which limits or extends its use to different categories of mariners, and the very uneven cost of installation ; one would moreover have to anticipate improvements which gradually no doubt will, continuing along the lines already followed, cause the weaknesses of each to disappear.