COMPUTATION OF LATTICE CHARTS FOR THE DECCA NAVIGATOR SYSTEM IN THE GAUSS CONFORMAL PROJECTION

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Geodetic data. Co-ordinate System.

Since 1912 Swedish hydrographical charts have been constructed in the Gauss Conformal or Traverse Mercator projection with co-ordinate systems originating on the meridians :

0	2°5	5°0	7°5	W. Stockholm
		5°0		E. Stockholm

The Bessel Ellipsoid is used.

It is mainly for the South of Sweden that these new charts have been drawn and as surveys are done area by area the use of different co-ordinate systems has not been inconvenient. Now, however, because of the Decca Navigator System, the Hydrographic Office is obliged to use a common system throughout, namely 2°5 W. This change causes no great inconvenience. It is necessary to compute some conversion factors from the old systems to the 2°5 W System, but most of this has already been done by the Geodetic division of the Geographic Survey Office of Sweden.

The Scale error in this common system for N Gotland wiht a difference of longitude $\lambda = 3°2$ is only 2 metres in 10,000 metres and the West Coast with a difference of longitude $\lambda = 5°8$, 10 metres in 10,000.

As the co-ordinates are transferred the neighbouring grid is rotated clockwise by 0.00005 radians or 31".83 about a point in the meridian 5° W from Stockholm and 6,300 kms from the equator, and the distances from that point increased by 1,00002, that is 40 cms per 20,000 metres. The Y-co-ordinates have the initial value 1,500,000 for the meridian 2°5 W of Stockholm.

Errors in distances (Chart-Ellipsoid).

The extensive computations begin with distances in the plane between the transmitters and a number of scattered points in the lattice covering the area. The reduction to the sphere is carried out using the following formula that shows the difference in distance on the ellipsoid (s) and in the plane (s') according to the expression.

$$s' = s + \frac{s}{2}\left[\frac{y_1 + y_2}{2}\right]^2 \frac{1}{R^2} + \frac{s}{24}\frac{(y_1 - y_2)^2}{R^2} + \text{terms of higher degree}$$

(F.R. Helmert: Die Math. u. Phy. Theorien der Höheren Geodäsie, Leipzig 1880, s. 481).

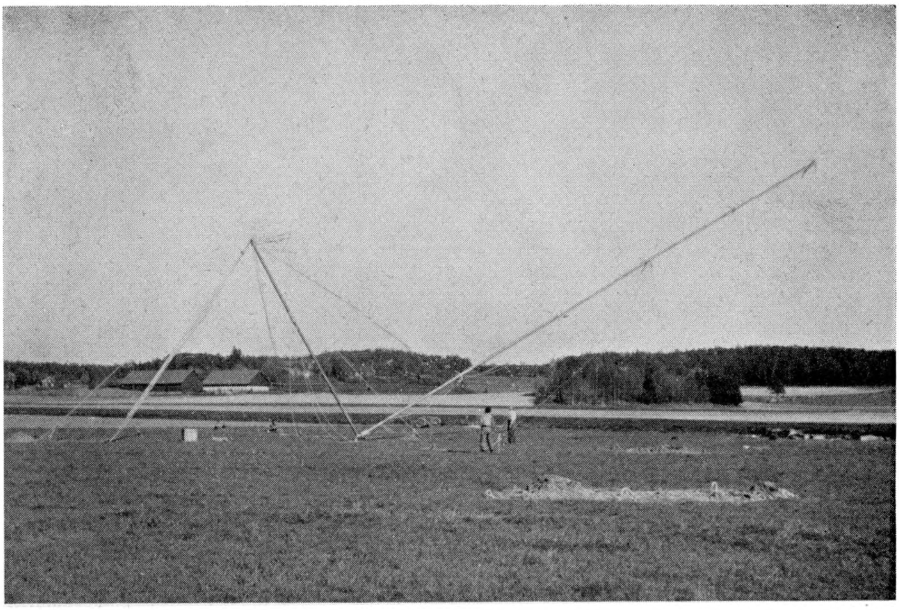

FIG. 1
Green Slave at Tystberga. Erection of mast.

FIG. 2
Green Slave at Tystberga. Mast anchorage.

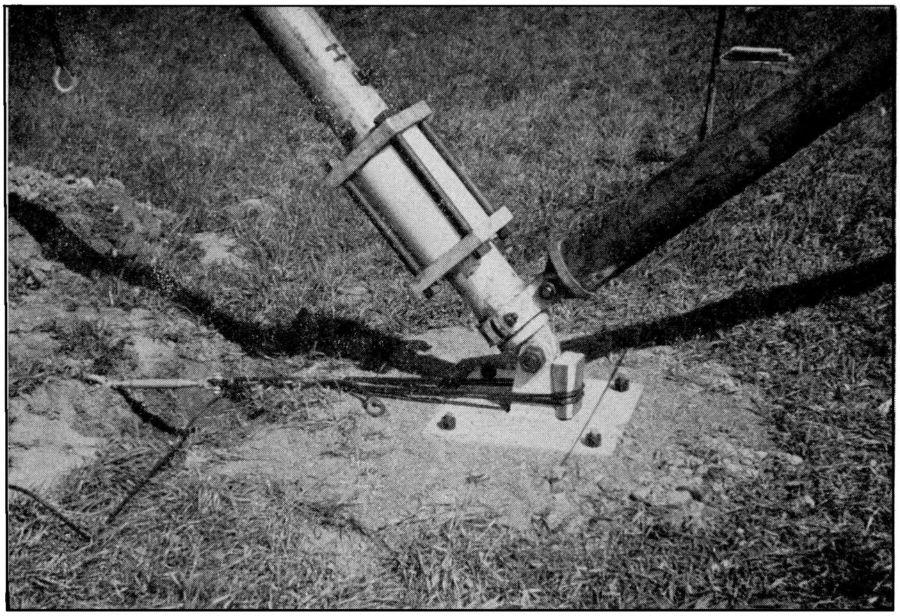

FIG. 3
Green Slave at Tystberga. Detail of mast anchoring.

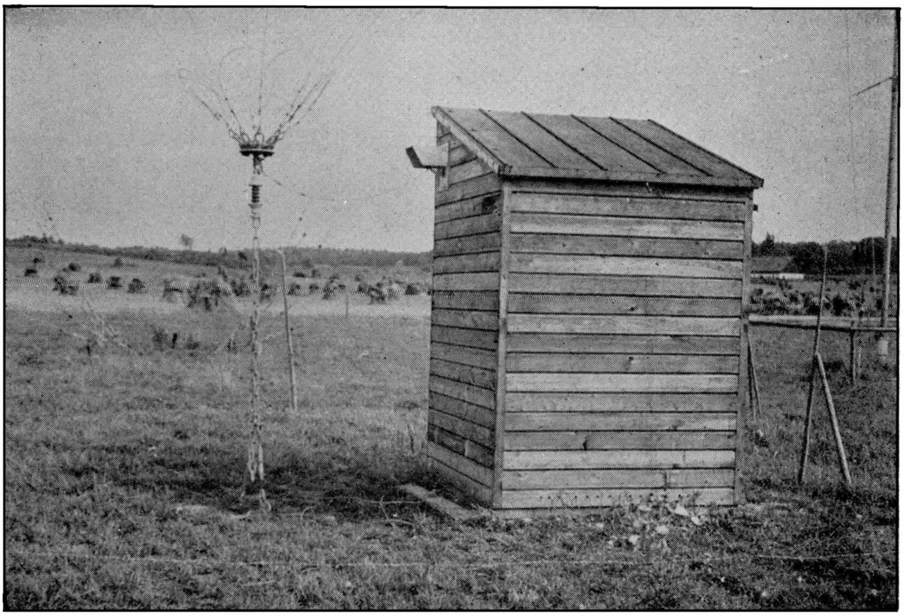

FIG. 4
Green Slave at Tystberga. Coil House showing junction of antenna feed lines.

where y_1 and y_2 are the Y co-ordinates of the points between which the distance is to be measured and R the mean radius of curvature in the area. Below are some tables computed in 1947 illustrating the magnitude of such differences.

$$\log s' - \log s = b_1 + b_2$$

b_1 and b_2 are expressed to the 7th decimal place.

$y_1 + y_2$ km $y_1 - y_2$	Increase in cm. pr. 10 kms	
	b_1	b_2
0 km.	0 cm.	0 cm.
100 —	30.6 —	10.2 —
200 —	122.6 —	40.9 —
300 —	275.8 —	91.9 —
400 —	490.4 —	163.5 —

If the y-co-ordinates are not greater than 100 km. the expression gives values for the increment not greater than 1 mm. for distance up to 80 km. If the y-co-ordinates are in the order of 400 km. and the distance 80 km. the uncertainty in measuring a distance is about 1 dm. if terms of the 4th order are not taken into account and about 1 mm. if they are included. Below are some examples giving the distances from Tystberga, the North Slave station, to some places in the Baltic.

Gauss' Conformal Projection

PLACE	s' — s	$\Delta\ \alpha$
Visby	24 metres	42"
Windau	192 —	1' — 37"
Brüsterort	190 —	3' — 41"

The table is computed for a mean latitude of 58°00' and the effect of change of latitude for distances in the order of 200 km. is illustrated by the following.

$y_1 + y_2$	LAT.	b_1	s : 10.000	s' — s
250 km.	60°	831.5	191.3 cm.	3.826 cm.
	55°	832.3	191.7 cm.	3.834 cm.
				8 cm.

So for a change of latitude of 1° the effect on a distance of 200 km. is 1.6 cm. which may be disregarded.

The next step in the computation is the determination of the Lane numbers L using the expression :

$$L = \frac{f}{c} (S_{AB} + S_{AP} - S_{BP})$$

where f = comparison frequency of the stations A + B.
c = propagation speed.
S_{AB} = distance between master and slave.
S_{AP}, S_{BP} = distances of point P from staions A & B.

Interpolation.

After the hyperbolic co-ordinates have been calculated for the intersections in the rectangular grid, the distance between grid parallels being 10,000 metres, interpolation can be done in two steps, namely :

1. Inverse interpolation for computing every 10th lane.
2. Common interpolation for every varied number.

The interpolation table is arranged to give the best cut between the pattern-lines and the co-ordinates of the rectangular grid. For a pattern direction tending to be East and West intercepts are made on Y-axis : for patterns tending to run North and South the intercepts are sectioned off the X-axis.

The method of calculating distances over the surface of the earth described above is quite practical for the moderate distances of 100-300 kilometres in the Baltic survey.

For areas near to the base line or to the transmitting stations interpolation becomes difficult, almost impossible. In this case a new grid is used, with the base line as the X-axis and either one of the stations, or the point midway between them, whichever is the most convenient, as the origin. The Scale errors in the Gauss Conformal Projection are small as the following table shows for the station of Skedshult.

DISTANCES EAST-WEST TO SKEDSHULT	PROJECTION FACTOR
20-30 km.	1 metre
50 km.	3 metres
100 km.	11 metres

At short distances one can neglect the projection correction and this simplifies the problem of constructing hyperbolic patterns near the stations. Using a small (2 dm.) co-ordinatograph aligned along the base line as X-axis, the computed co-ordinates are readily plotted from the basic expressions :

$$\begin{cases} \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \\ c^2 = a^2 + b^2 \end{cases}$$

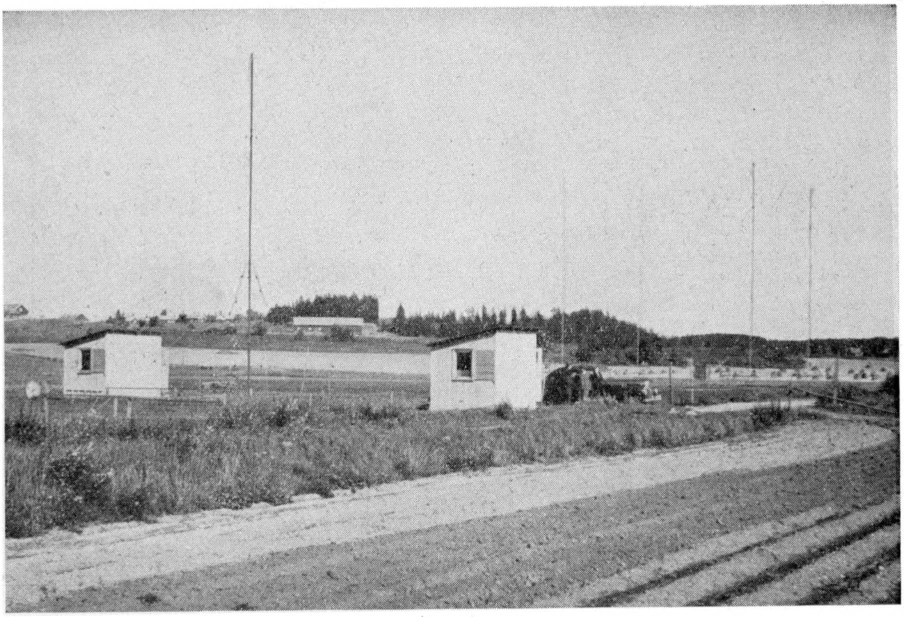

FIG. 5

General view of Green Slave, showing transmitter housing in centre, transmitting masts to the right, single receiving antenna mast for receiving phase control signal and power-house on the left.

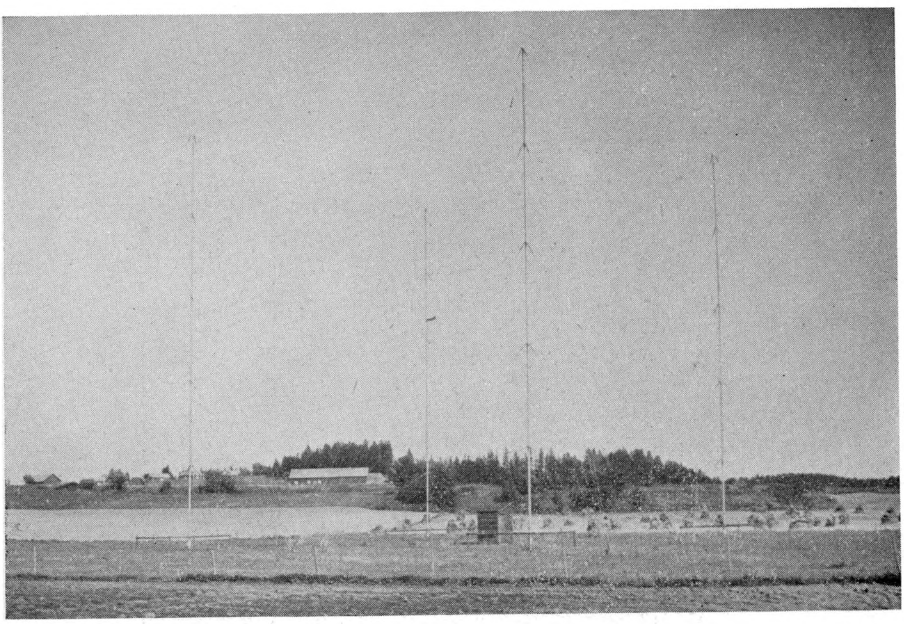

FIG. 6

Green Slave at Tystberga. Transmitting antenna,

FIG. 7
Green Slave at Tystberga. Transmitting house interior, showing phase control receivers with the transmitters just showing on the right. The apparatus under the bench is test equipment.

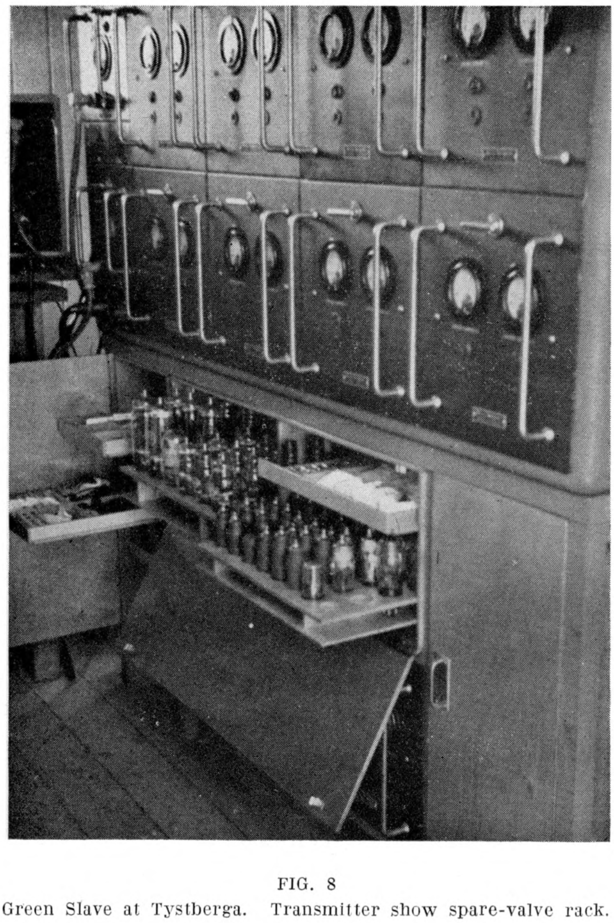

FIG. 8
Green Slave at Tystberga. Transmitter show spare-valve rack.

Displacement of the co-ordinates in Hatt's Azimuthal Equidistant Projection.

The patterns have two curvatures : The one just computed from two expressions above, the other depending upon the curvature of the earth. Investigating the projection errors in Hatt's projection with a transmitting station as centre we have :

$$x^1 = (1 + \frac{y^2}{3r^2})\,x \qquad r = \sqrt{R.\,M.}$$

$$x = s.\cos\alpha$$

$$y^1 = (1 - \frac{x^2}{6r^2})\,y \qquad y = s.\sin\alpha$$

where s and α are the radial distance and azimuth of a point in the pattern. Using the equations for plane hyperbolic patterns we get co-ordinates x and y in Hatt's Projection instead of spherical co-ordinates. The errors in x and y become $x^1 - x = dx$ and $y^1 - y = dy$. For x = 50 kms and y = 100-200 kms we draw the following table :

x \ y		100 km.	150 km.	200 km.
10 kms....	x =	1 m.	2 m.	4 m.
	y =	0.05 m.	0.075 m	0.1 m.
100 kms....	x =	10 m.	20 m.	40 m.
	y =	5 m.	7.5 m.	10 m.

[Hatt : Des co-ordonnées rectangulaires. Paris 1887.]

Survey Area.

The new Decca Navigator System is in use in an area between Gotland and the East Coast of Sweden, limited by the following parallels and meridians :

Lat. ...... 57° 00' N. and 58° 45' N.
Long....... 16° 10' E. and 19° 40' E.

Throughout the entire area grid co-ordinates have been computed for every 5' in latitude and 10' in longitude using tables drawn up by the Geographical Survey Office of Sweden.

Geodetic positions for the transmitting stations.

	Lat.	Long.
1. Fårbo (Red Slave)........	57° 22' 39"9	16° 28' 44"9
2. Skedshult (Master)	58° 04' 51"1	16° 30' 07"5
3. Tystberga (Green Slave)..	58° 50' 09"9	17° 14' 23"1

Rectangular Co-ordinates in Gauss' Conformal Projection.

	x	y
1.	6361 516.2	1540 351.2
2.	6439 828.0	1540 929.0
3.	6524 613.6	1582 649.5

Natural distances between the stations.

	Projection corr.	Natural distances (s)
Red pair 1-2.	— 1.58 m.	78 312.3 m.
Green pair 1-3.	— 4.59 m.	94 489.8 m.

Trials with the Swedish Survey vessel "Gustaf af Klint".

Effect of variation in propagation speed.

The following is the data used for the lattice survey chart used in the summer of 1948 :

Propagation speed.	c	= 299 350 km./sec.
Green comparison frequency. . . .	f	= 265,548
Red — —	f	= 354,065
Wave length green.	λ	= 1127.29 metres
Red wave length	λ	= 845.47 metres
No. of lanes. Green.	l	= 167.640
Red	l	= 185.252

Observations were conducted in the "Gustaf af Klint" at 50 stations. Differences in observed readings and computed co-ordinates were recorded and the results are appended in three tables. It was obvious from these results that the speed used for computation was too low and needed to be increased by 300 km./sec. to 299.650 km./sec. The survey charts which are to be used in 1949 will be computed with this speed. The effect of variation of speed on the lane number l is as follows :

$$\frac{dc}{c} = -\frac{dl}{l}$$

Where d c and d l are the variations in speed and lane number.

The definitive data for computing the lattice-charts is therefore :

Propagation speed.		= 299,650 km./sec.
Wave length Green	λ	= 1128.42 m.
Wave length Red		= 846.31 m.
No. of lanes Green.	l	= 167.473
No. of lanes Red		= 185.065

Correction for passage of signal over land.

Mr. H. Larsson of the Defence Research Institute has deduced an expression to show the influence of stretches of land on the pattern (See his report on the operation of the "Gustaf af Klint" in the summer of 1948).

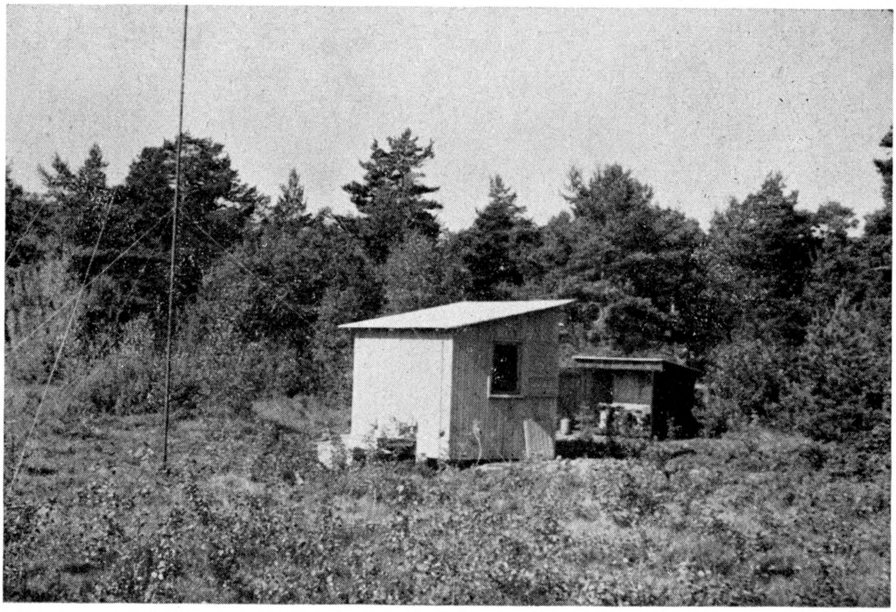

FIG. 9
Monitor Station at Gryt.

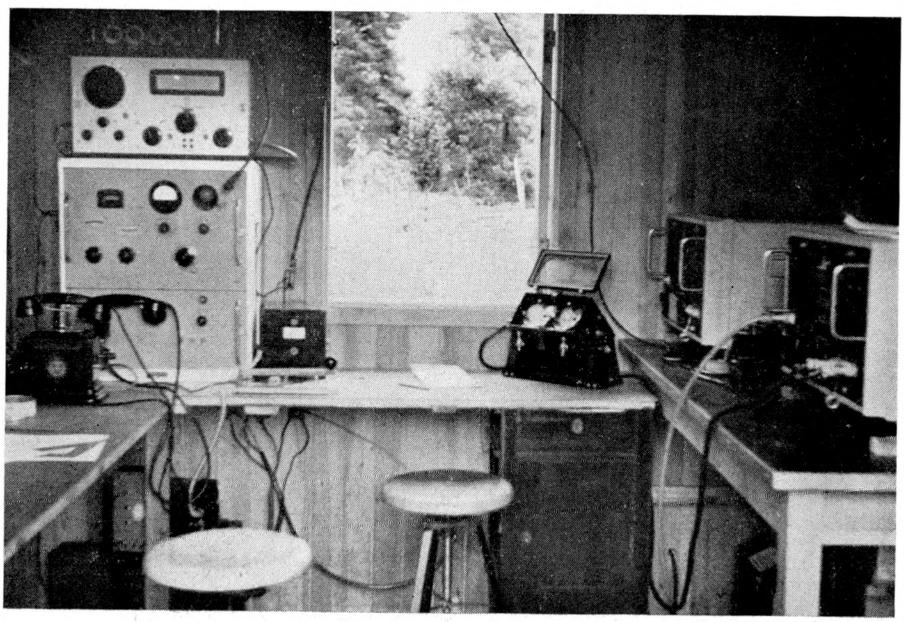

FIG. 10
Interior of Monitor at Gryt.

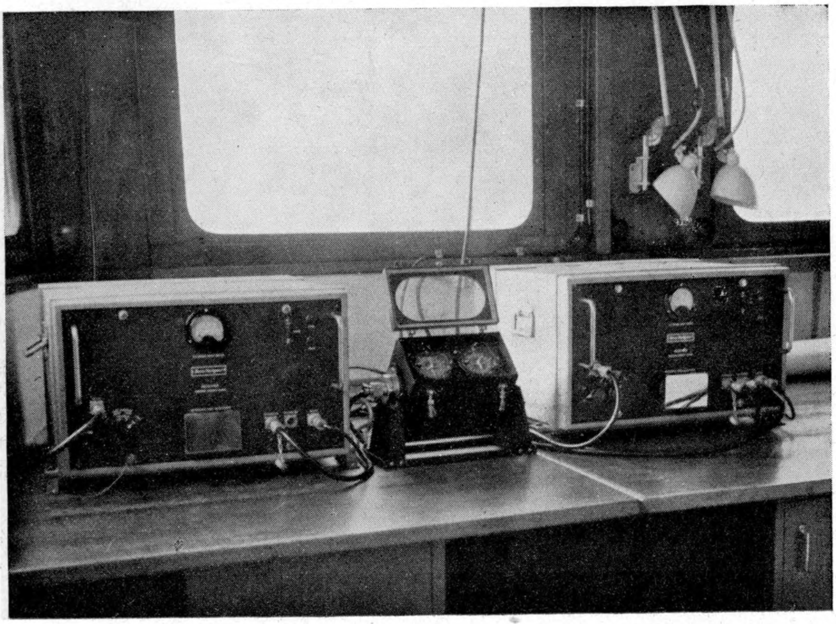

FIG. 11
« Gustaf af Klint ».
Two receivers and one set of Deccometers mounted in the draughting-room.

FIG. 12
Deccometers on bridge of « Gustaf af Klint ».

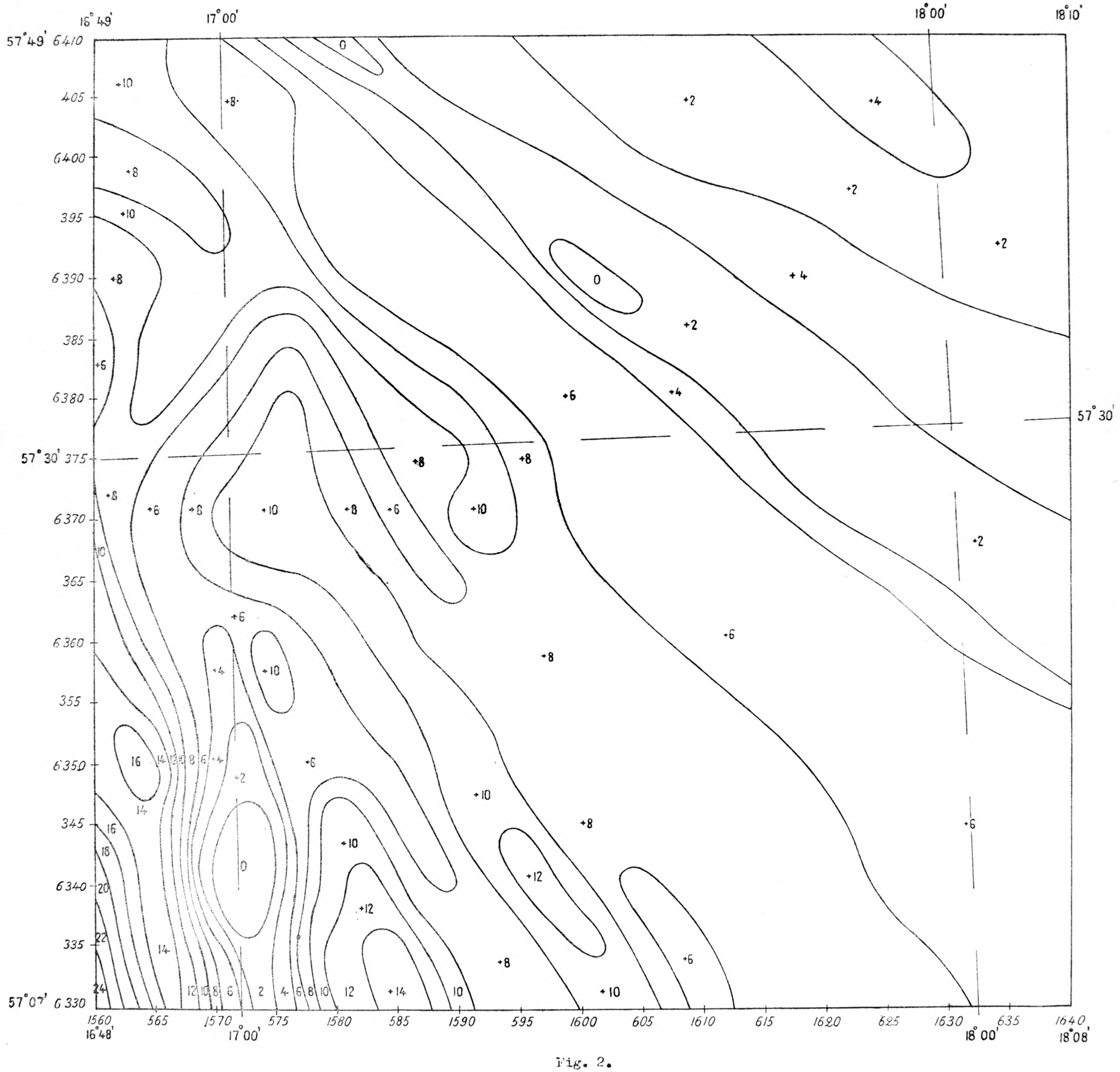
16°49'
17°00'
18°00'
18°10'
57°49' 6410
405
6400
395
6390
385
6380
57°30' 375
6370
365
6360
355
6350
345
6340
335
57°07' 6330
57°30'
1560
16°48'
565
1570
17°00'
575
1580
585
1590
595
1600
605
1610
615
1620
625
1630
18°00'
635
1640
18°08'

Fig. 2.

When the correction has been applied, the residual errors, unpredictable and random, have a mean value of 3.5 hundredth of a lane, considerably more than the standard accuracy of 1.5 hundredth of a lane for the deccometer readings. The following table shows the mean radial error e_r for different accuracies.

	d l = 0.01 lane	internat. value d l = 0.015 lane	d l = 0.03	d l = 0.045
Häradskär fyr..	e_r = 8 m.	e_r = 12 m.	e_r = 24 m.	e_r = 36 m.
Olands n. udde.	16	24	48	72
Visby	18	27	54	81
Färö	29	42	84	126
Gotlands s. udde	36	54	108	162

A chart to the scale of 1 : 500,000 was used to compute the corrections for the distance travelled by the signals over land. In the Oland—Gotland Survey area the corrections have been computed for each intersection of the x and y ordinates in a rectangular grid, according to Mr. Larsson's formula. Insufficient information was forthcoming when a grid of ordinate distance of 10 km. was used so the ordinates were increased in number to form a grid with sides of 5 km. and it is apparent from this that the corrections, though showing considerable variation, have uniform trends. These corrections have been plotted in contours of equal correction as shown in fig. 2.