If we use the difference in velocity $\Delta c = c_w - c_l$, we get

$$\Delta \phi = f\left(\frac{l_A - l_B}{c_w - \Delta c} - \frac{l_A - l_B}{c_w}\right)$$

The expression $l_A - l_B$ is the difference in the paths over land, which we call $\Delta l$, then

$$\Delta \phi = \frac{f}{c_w (c_w - \Delta c)} (c_w . \Delta l - c_w . \Delta l + \Delta c . \Delta l)$$

$$\Delta \phi = \frac{f . \Delta c . \Delta l}{c_w (c_w - \Delta c)} \cong \frac{f}{c_w{}^2} . \Delta c . \Delta l \cong \frac{\Delta c}{c_w} . \frac{\Delta l}{\lambda}$$

If the speed over a certain land area is constant, $\Delta c$ is constant.

The expression $\frac{f}{c_w{}^2} . \Delta c$ is therefore a constant, which we call k, and the error will be $\Delta \phi \cong K . \Delta l$ where $k = \frac{\Delta c . f}{c^2}$.

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## APPENDIX II

### A new technique for the construction of hyperbolic charts.

The Hydrographic Office of Sweden uses for its survey charts a rectangular grid on a Gauss Projection. In this projection, where distances are truly represented in every direction, it is possible to construct hyperbolic lattice charts from first principles, using graphical methods. A survey of the waters of the Southern Baltic is being conducted by the Hydrographic Office, using the Decca Navigator System, and since the portable transmitters are moved to different positions from time to time, the construction of charts by the normal method of computation represents a big expenditure of time and labour. In order to cut down this expenditure of hitherto necessary effort, the technique described below was evolved by the author. The method in addition to cutting down the time for producing a chart from two or three months to a matter of a few days, and maintaining the accuracy of a computed chart, has two incidental advantages.

The first is that, owing to the large numbers of intersection points the hyperbola is much more easy to construct and, secondly, with this method it is possible to introduce corrections for varying velocities of propagation.

The method involves the use of a graduated beam-compass, described and illustrated later in this article. The pivot of the beam-compass is taken to be one of the pair of transmitters generating the hyperbolae, and a field-board, representing the area to be surveyed, is carefully placed in position with respect to the pivot. To effect this positioning, four random points near the corners of the area A, B, C, and D in fig. 1 have distances and azimuths from the transmitter computed. The points are then plotted in the required scale on the field-board and the azimuths are drawn in. Using the azimuths as a rough guide for alignment, one of the four points (A) is located accurately under the pencil of the beam compass, which is set to the scaled distance of the point from the transmitter. The field-board is then pivoted about this point until one of the other points (B) is located under the beam-compass pencil, re-set to give the distance of the new point from the transmitter. The board is then fixed and the position is checked from the two remaining points (C and D).

With the board thus set up, arcs of circles are drawn, having as radii successive multiples of a wave-length in the comparison frequency, the wave-length varying with the propagation speed over different types of terrain. These radii also take into account the curvature of the earth, the correction being made from standard tables prepared by the Ordnance Survey Office of Sweden for general use with the Gauss Conformal Projection. When this operation is complete, it is repeated for the second pair of transmitters, comprising the master and second slave station. The circles now on the field-board are identified by means of a numerical system to facilitate plotting the hyperbolae and numbering them. Figure 2 demonstrates the principle of the numerical system used. The circles are numbered from the master station towards the slave station, the numbers being given a positive sign for the circles emanating from the master and a negative sign for those from the slave. If there is, on the base line between the two stations, a distance equal to N wave-lengths plus a fraction of a wave-length equal to X, then the numerical characteristic of the first circle emanating from the slave is —N and its radius is X. This circle cuts the base line between the stations in the same point as the circle + N from the master station. The hyperbola originating in this point has as locus all the points of intersection of the circles whose numerical sum is +N—(—N) i.e. 2 N. Similarly the identity of any hyperbola running through the intersection of any two circles can be immediately determined by subtracting the number of the slave circle from the number of the master circle. Figure 2 will illustrate that each hyperbola has its own constant numerical characteristic throughout its length.

The hyperbolic pattern is completed from the above construction and the process repeated for the second pair of stations with their appropriate comparison wave-length. In practice, since there is a common station, the master, in each pair, the plotting of the points in both patterns is done in one operation.

The first beam compass made by the Hydrographic Office is 7 metres in length (3.6 metres is the largest radius required in the present survey with a scale of 1 : 50.000) made of T-section duralumin with 41 metre scales on the upper surface. It carries a drawing-head fitted with vernier scale, to read to 1 mm. and is supported by two movable trolleys or bogies, one near the drawing-head and one between the drawing-head and the pivot. Sheets of glass are placed under the trolleys. The beam is in 1 metre sections so that for short radii only a short compass is used. When all the sections of the beam are in use, the beam is braced with piano-wire guys for stiffness.

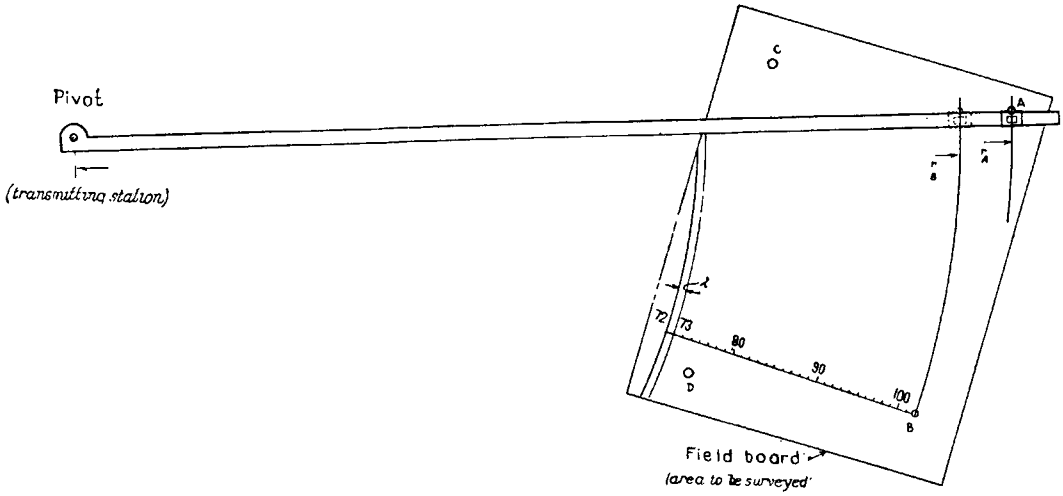


FIG. 1

The correct positioning of the fixed board with respect to the pivot (the transmitting station) is done by means of 4 pre-computed points A, B, C, D. Two are used for the actual positioning, the other two for checking. The azimuth through B towards the station has wave-length graduations, corrected for the curvature of the earth, which are a very useful check, when the circles are drawn by means of the beam compass.

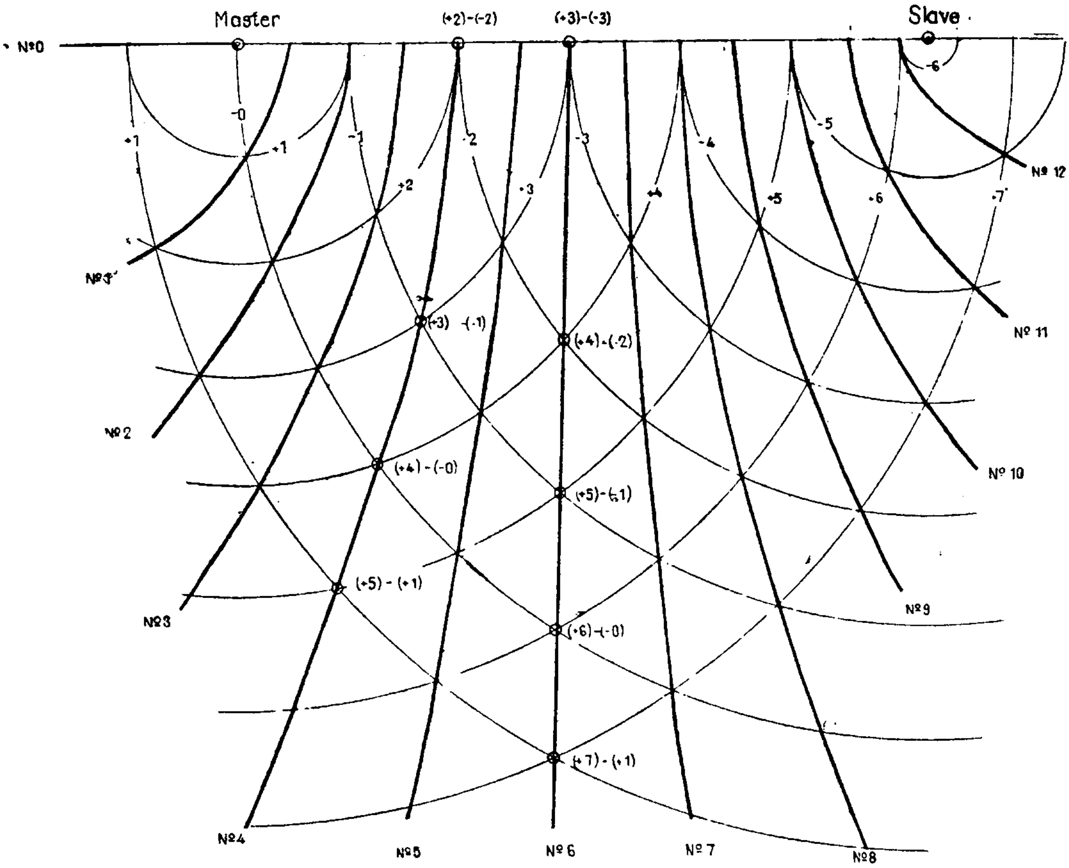


FIG. 2

Diagram to show that if the generating circles are numbered in the manner described in the article, the hyperbola identity is readily determined by the algebraic difference of the generating circle numbers.

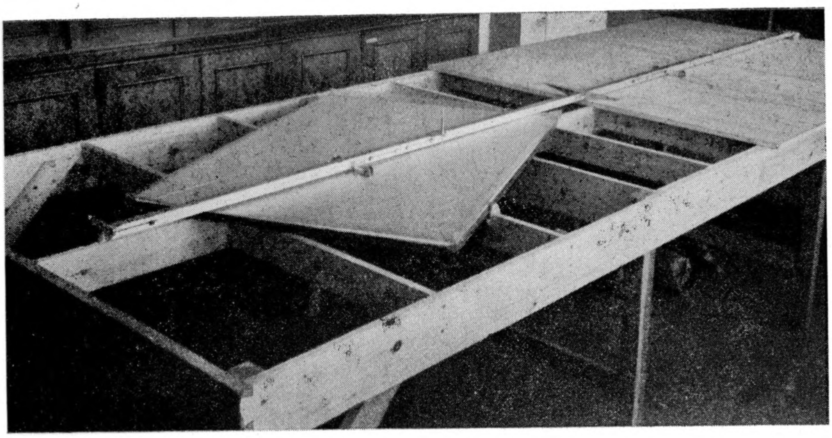

FIG. 3
Overall view of beam compass.

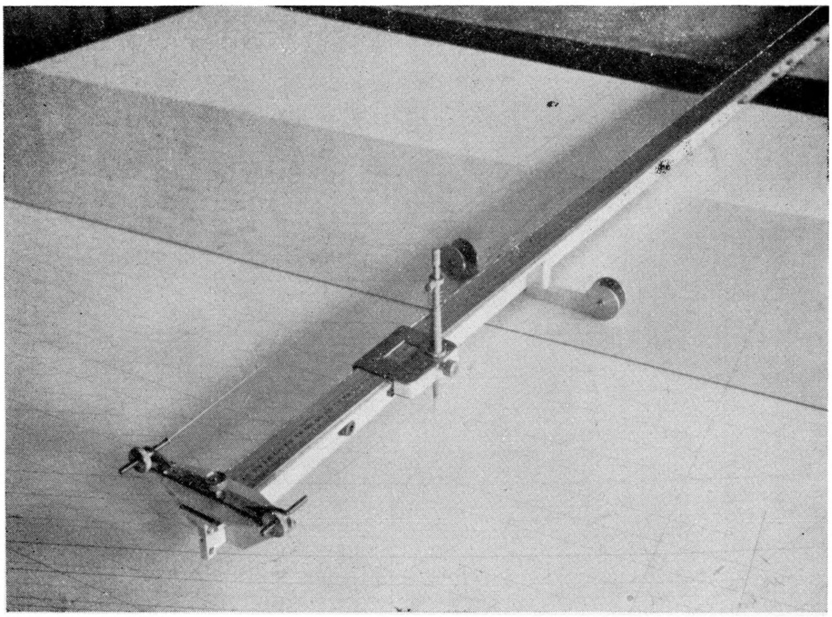

FIG. 4
Close-up of drawing head.

Future models will be constructed of square-section tubing to overcome the objectionable flexibility that exists, when working at extreme radius without the guys. The pencil is a fine ball-point pen maintained by a light spring : it has proved to be very satisfactory. The beam compass is permanently fitted to a special table, the top of which is removable in sections to permit the clamping of the field-board, as shown in the photograph.

An outline of the procedure in force at the Hydrographic Office for the construction of lattice charts is as follows : Two sheets of paper are pasted to the field-board, with white of egg, one above the other. The fixing points are plotted to scale and the bearings of the transmitting stations drawn in. These direction-lines have wave-length graduations, corrected for curvature of the earth, extrapolated from the computed point. Originally these graduations were intended to be means of determining the radii of the circles to be drawn, but in practice it was found difficult to place the pencil on the marks and very much more satisfactory to use the vernier scale. These graduations, however, are a very useful check to the value set on the vernier and are therefore still set off on the azimuth lines as may be seen in fig. 1. The board is set up as previously explained and the circles drawn in different colours. Intersections of the circles are pricked through to the lower sheet of paper and some of the hyperbolae constructed and drawn with sufficient pressure to lightly indent the second sheet. The top sheet is now removed and the indentations of those hyperbolae which were drawn in on the top sheet are used as a guide for identification and completing the remaining lines of the pattern through the pricked intersections. This latter operation is greatly facilitated through the large number of points at which the splices or battens can be placed.