THE ADMIRALTY TIDE TABLES, PART III, AND THE PREDICTION OF THE HEIGHT OF TIDE BY A NEW METHOD.

by :

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The Division of Tidal Work of the Hydrographic Department of the British Admiralty has published in June 1936 a very important work which marks a real progress in the question of tidal prediction, always of much interest to the navigator.

This work, which is the result of the collaboration of Commander H.D. WARBURG, R.N., Superintendent of Tidal Work of the Hydrographic Department, and A.T. DOODSON, D.Sc., F.R.S., Associate Director of the Liverpool Observatory and Tidal Institute, is entitled : *The Admiralty Tide Tables, Part III, containing Instructions for Predicting Tides and Tidal Streams and for Analysing Observations.* It is accompanied by tables to assist in the prediction and analysis of the tides. Its purpose is to set forth an entirely new method for the prediction of tides and tidal currents, which has now been adopted as the standard method of the British Admiralty beginning with the year 1937.

This method, which is entirely new, combines the advantages of the older non-harmonic method which utilised the Establishment of the Port, and the method using purely the harmonic constants. The first method, although in use from the beginning for the prediction of tides, has been found in general to be less accurate, while the harmonic method, on the other hand, involves long calculations which tend to make it less practicable.

Thus the reception accorded this new publication by the maritime world vouches for the deep interest taken in these methods which permit the work of prediction to be considerably reduced, while at the same time rendering the results more accurate. Numerous articles have already been published on the subject of these new tables and we mention in particular, those of Commandant Mario ZITO, published in the " Annali of the R. Istituto Superiore Navale ", Napoli 1938, p. 145; that of M. van ROON, in " De Zee ", den Helder N° 3, May 1938, p. 131; that of Lieutenant E. FERREIRA DE ALMEIDA in the " Anais do Club Militar Naval " of Lisbon, April and July 1939, p. 247 and 279.

This Third Part of the Admiralty Tide Tables constitutes a truly complete manual of Tidal Prediction, and, as such, may be regarded from two points of view; that of the navigator, principally interested in the sections I and II of the work devoted to the prediction of tides (heights of tide) and that devoted to the tidal streams; while the hydrographic surveyor will be primarily interested more in section III reserved for the analysis of the tides and the determination of the local constants.

This last part, as well as the tables and the examples of the calculation which it contains, are of very special interest to the hydrographic engineers. They contain an indispensable practical supplement to the theoretical developments which have been expounded in several classical works and notably in the publications on the subject of harmonic analysis made in the course of the last few years by the technicians of the Tidal Institute of Liverpool.

The analysis of tidal observations, the method of treatment, constitute an apanage especially reserved for hydrographic engineers who, above all, have the necessary time to devote to the elaboration of the meticulous calculations arising from their observations. In this connection, the treatise and the tables to assist prediction in Section III of the Admiralty Manual supply to-day a very appreciable contribution.

The pages and those tables concerning the prediction, with the developments relative to the expounding of the method, appear to us to be more especially destined to the navigator, the mariner, with a view towards a more rapid utilisation, as compared with that of the analysis, and therefore the technical developments in this part should rather be limited to the indispensable minimum permitting him to understand the logical sequence of the operations to be effected.

This has been accomplished in a very complete manner in the first two chapters of the new part III of the Admiralty Tide Tables.

We shall give below a brief summary of it, accompanied by several considerations of a general nature with the idea of bringing this very interesting method to the attention of foreign readers. The new edition in 1938 of the Admiralty Tide Tables, Part II, does furnish in fact, with its practical data listed for more than 6000 ports or stations, the best means for the determination of the state of the tide at present at the disposal of the mariner, with a view to insuring navigation in the estuaries or shallow channels, besides the 450 principal ports for which detailed tidal predictions are furnished for the year 1940 of the daily times of high water and low water or the hourly heights of the tide, as published by the various Hydrographic Offices.

The new Admiralty Method is not, it is explained, strictly speaking a harmonic method, but it is based, however, on the use of some of the principal harmonic constituents of the tide.

Among these, the most generally known, as a result of the analysis of the observations, which in general do not exceed one lunar month of 29 days (except in exceptional circumstances) are the following, using the Darwinian notation :

1°) *Semi-diurnal Group.*

M_2 mean lunar principal, period 12 h. 25 m, related to the moon's transit of the meridian.

S_2 mean solar principal, period 12 h. 00 m, related to the sun's transit of the meridian.

N_2 larger lunar elliptic, period 12 h. 39 m, depending on the moon's parallax.

K_2 luni-solar declinational, period 11 h. 58 m, merging practically with S_2 of the same period, into one wave ($S_2 + K_2$) of period 12 h. 00 m, the amplitude of K_2 being, on an average, equal to 0.272, that of S_2.

2°) *Diurnal Group.*

K_1 luni-solar declinational (0.18 for the moon; 0.08 for the sun, on a total influence of 0.26 for both), period 23 h. 56 m.

O_1 principal lunar declinational, period 25 h. 49 m.

P_1 principal solar declinational, period 24 h. 04 m, that is in the vicinity of K_1 with which it merges in the analysis of observations ($K_1 + P_1$) period 24 h. 00 m, the amplitude of P_1 being, on an average, equal to 0.331 that of K_1.

3°) *Quarter-diurnal Group.*

M_4 lunar quarter-diurnal, period 6 h. 13 m.

MS_4 compound quarter diurnal constituent from $M_2 + S_2$, period 6 h. 06 m.

these two waves being taken into account only when it is presumed that the influence of shallow water justifies their intervention in the synthesis.

In the harmonic method, these 9 tidal waves, or constituents, amply suffice in general to make all the predictions required by navigation. Permanent or annual tables, which are in existence, enable one to make all the calculations relating thereto; we cite in particular the following :

Major A.W. BAIRD. — A manual for tidal observations and their reduction by the method of harmonic analysis — 1886.

Dr. J.P. van der STOK. — Wind and Weather, Currents, Tides and Tidal Streams in the East Indian Archipelago, Batavia, 1897.

P. HAVERKAMP. — Zeevaartkundige tafels — 2nd part — s' Gravenhage, 1918.

PAUL SCHUREMAN. — U.S. Coast and Geodetic Survey — Special Publication N° 98. A Manual of the Harmonic Analysis and Prediction of Tides, Washington, 1924.

H. BENCKER. — Tables perpétuelles pour le calcul des marées par les Constantes Harmoniques, Monaco, 1934.

INTERNATIONAL HYDROGRAPHIC BUREAU. —

Tables for the calculation of Tides by means of Harmonic Constants (Tables pour le calcul des Marées par les Constantes Harmoniques). Supplément à la Publication Spéciale N° 12 du Bureau Hydrographique International, Monaco, 1926.

In order to avoid some of the elaborate calculations for the contribution of each of the constituent waves to the total tide, it has been proposed to group together the waves of the same parity or those having closely neighbouring periods.

This has been followed out in the new method now adopted as standard by the British Admiralty. In addition, this method eliminates the use of certain constituents by means of the various artifices which we shall describe later.

In the pure harmonic method, the calculation is usually limited to that required for the determination of the contribution made by each constituent to the total height by means of permanent tables; for instance, for each round hour of the particular day. In the British Admiralty method, on the contrary, the determination is made for each constituent or group of constituents of the time of high water for that group on the day in question.

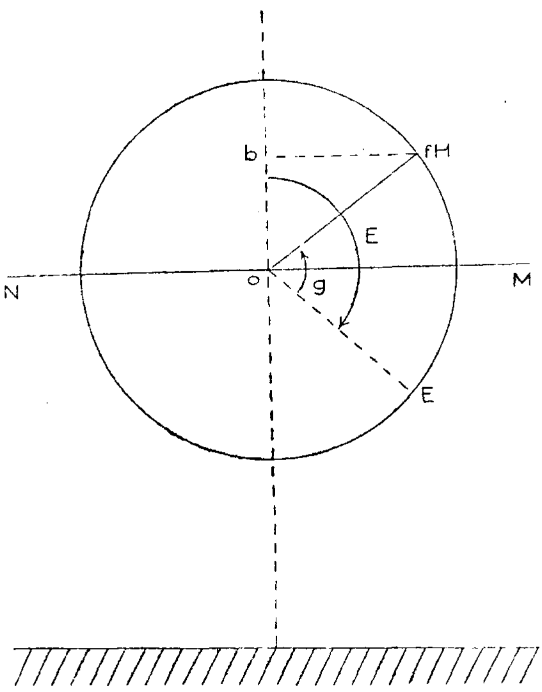

Fig 1

We know that with the usual notation :—

E = the astronomical argument or phase of a static constituent (Equilibrium tide) at any given hour;

g = the phase-lag of the corresponding constituent at the place (situation or establishment);

H = mean semi-amplitude of the corresponding constituent at that place, the height above the mean level o b, due to this constituent at this time and at this place will be given by f H cos (E-g) (fig. 1) and the height of the tide is the sum of all the values of fH cos (E-g) for all the constituents at that place. In general, the calculation is limited to the nine constituents mentioned above.

In the new method of the British Admiralty, the number of factors is limited to four combined mean constituents :

1° — a lunar semi-diurnal constituent M_2

2° — a combined semi-diurnal constituent $(S_2 + K_2)$

3° — a combined diurnal constituent $(K_1 + P_1)$

4° — a diurnal lunar constituent O_1

In the course of the luni-solar cycle of 18.6 years (Saros), the mean value of each of these constituents, which varies with the changes in the declination or the parallax of the sun and moon, is exactly represented by the value of the pure harmonic constants M_2 S_2 O_1 K_1. These latter are used as a basis of the British Admiralty method.

The tables 1 and 2, pages 86 to 96 of the Admiralty Tide Tables, Part III, supply the corrections which have to be applied to the pure harmonic constants of these four waves, in order to take into account the astronomical conditions mentioned above, and this is done in an original manner by applying the corrections to the high water.

The astronomical argument, or phase E, of a harmonic constituent for a given day and hour t, may be written in the following manner, calling *n* the speed or increment in angle per unit of time of the wave.

$$E = E_o + nt.$$

Its high water occur at the time T for which :

$$\cos (E + nT - g) = 1, \quad \text{that is :}$$

nT (High water angle) see fig. 1. $= g - E$.

The amplitude of this high water is equal to fH.

In the British method, the high water angle (that is the time of high tide expressed for convenience in angular measure), and the amplitude of each of the major constituents is predicted in the following manner :

$$\begin{cases} \text{HWA (High Water Angle)} = g + b + c \\ \text{amplitude} = H x B x C \end{cases}$$

g and H represent the harmonic constants of the waves as they are usually found in the Lists of Constants and in particular in the Admiralty Tide Tables, Part II, 1938.

For each of the four fundamental waves contemplated by this method, the corrections b and B are supplied by Table I of the Admiralty Tide Tables, Part III, 1936, as a function of the year and date; the correction *c* is derived from Table 2, of the A.T.T. as a function of the time of transit of the moon at the local meridian; the correction *C* is furnished by Table 2 (b) of the A.T.T. as a function of the parallax of the moon.

Let us examine successively the corrections in detail :

1° — *Correction C for the parallax of the moon* — This correction for amplitude, which of course does not affect the solar constituent S_2 is applied to the lunar constituents M_2 K_1 and O_1 . Its purpose is to take into account the effect of the constituents N_2 and Q_1, major elliptics, which are thus eliminated from the summation.

The mean value of ϖ ☽ is 57' and its extreme values at apogee and perigee are respectively 62' and 52'.

For M_2 and O_1, the parallax intervenes for each in the same manner and in the ratio of the cube of the distance :

$\frac{tg^3 \varpi ☽}{tg^3 57'}$ is the ratio furnished by Table 2 B; and it varies from 0.76 to 1.28 depending on the value of the parallax.

For the luni-solar declinational constituent K_1, the ratio of the action of the moon (ratio of the astronomic co-efficients), is 0.18 as compared with 0.08 for the sun, or 18/26 of the total effect. In this last ratio, there is reduced for K_1 the correctional values of M_2 and O_1, given by Table 2 B; they vary therefore from 0.84 to 1.19 depending upon the values of the parallax.

2° — *Correction c for the transit of the moon at the local meridian* — This correction for the phase is applicable only to the lunar constituents M_2 and O_1, which contain in the expression of their astronomic arguments the difference 2 (h-s); the difference in mean longitudes *h* and *s* of the sun and the moon. The astronomic argument of M_2 at the moment of static high water of this wave is none other than the time of transit of the moon at the local meridian.

Table c of the A.T.T., Part III, gives this time transformed into convenient angles of 360° for 12 h. 25 m, the period of M_2, the principal lunar constituent.

The residual correction for adapting the phase of M_2 to the astronomic conditions of the luni-solar cycle is given by the factor bM_2 of Table I of the A.T.T., as a function of the year and date. This is none other than — *u* of M_2, *u* designating the slowly varying part of the astronomical argument which retains the same mean value for each year of the cycle: + 2° in 1935, 0° in 1941, 2° in 1944, 0° in 1950 etc.

The accessory correction for adapting in the same way the phase of O_1

to the astronomic conditions of the luni-solar cycle, is given by the factor b C_1 of Table I, and represents, on the whole, the difference between the arguments $E_{M2} - E_{O1}$, for the year and date; or also the argument of K_1; because by putting :

$$\begin{array}{lcl} E_{M2} & = & 2t + 2h - 2s \\ E_{O1} & = & t + h - 2s \\ \hline E_{M2} - E_{O1} & = & t + h \end{array}$$

which is equivalent to E_{K1}; this argument of K_1 being increased, on the other hand, by $+ u_{M2}$.

3° — *Amplitude Correction of the constituents M_2 and O_1, during* the course of the luni-solar cycle. These are given under B of the Table I of the A.T.T., and correspond respectively to the usual factors for the reduction of these astronomic constituents,

$$\text{or} \quad f\, M_2 = 0.92 \frac{1}{\cos^4 \frac{1}{2} I}$$

$$f\, O_1 = 0.38 \frac{1}{\sin I \cos^2 \frac{1}{2} I}$$

as a function, for instance, of the inclination I of the lunar orbit.

4° — *Amplitude and Phase Corrections for the declinationals P_1 and K_2.* — These two waves are very close with regard to speeds to K_1 and S_2 respectively and it is logical to combine them with these latter, the phases being practically the same and the theoretical amplitudes being respectively :—

$$\text{ampl. } P_1 = 0.331 \text{ ampl. } K_1$$
$$\text{ampl. } K_2 = 0.272 \text{ ampl. } S_2$$

In this connection we might recall here the manner in which this combination can be made which the reader will find given in detail in the classic work of Paul SCHUREMAN, cited above, pages 94 to 97.

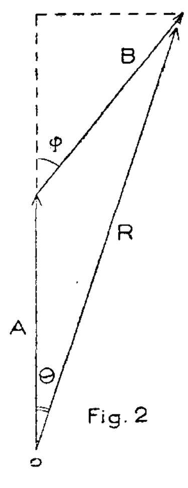

Fig. 2

Let A and B, be two components of which the difference in phase is φ (Fig. 2); let R be their resultant,

We have :

$$B \sin \varphi = R \sin \theta$$

$$R \cos \theta = A + B \cos \varphi \qquad \text{from which:}$$

$$\operatorname{tg} \theta = \frac{B \sin \varphi}{A + B \cos \varphi} \qquad \theta = \arc \tan \frac{\sin \varphi}{\frac{A}{B} + \cos \varphi} \qquad (1)$$

$$R^2 = A^2 + B^2 + 2\ AB \cos \varphi \qquad (2)$$

$$R = A \sqrt{1 + \frac{B^2}{A^2} + 2 \frac{B}{A} \cos \varphi}$$

These formulae (1) and (2) furnish the acceleration θ of the principal A due to the secondary B and its augmenting coefficient. This being true for the mean values, account should also be taken, in the course of the luni-solar cycle, of the astronomic coefficients of A and B, in such a manner that we shall obtain the following developped formulae :

$$\left. \begin{aligned} \theta &= \frac{f B}{f A} \arc \tan \frac{\sin \varphi}{\frac{A}{B} + \cos \varphi} \\ R &= A \times \left[1 + \frac{f B}{f A} \left(\sqrt{1 + \frac{B^2}{A^2} + 2 \frac{B}{A} \cos \varphi} - 1 \right) \right] \end{aligned} \right\} (3)$$

θ gives the phase correction which it is necessary to apply to that of A.

The parenthesis gives the amplitude correction factor of A. These two constructive elements are furnished, in an indirect manner, and for their mean value during the course of the luni-solar cycle, by Table 18 of the Admiralty Tide Tables, Part III.

We shall not elaborate the point further here.

$$\text{For } P_1 \text{ and } K_1 \left\{ \begin{aligned} &\frac{A}{B} \text{ theoretical} = 3.039\,;\ \frac{B}{A} = 0.331\,;\ \frac{f B}{f A} = \frac{1}{f K_1} \\ &\varphi = (\text{phase } P_1 - \text{phase } K_1) \end{aligned} \right.$$

$$\text{For } K_2 \text{ and } S_2 \left\{ \begin{aligned} &\frac{A}{B} \text{ theoretical} = 3.665\,;\ \frac{B}{A} = 0.272\,;\ \frac{f B}{f A} = f K_2 \\ &\varphi = (\text{phase } K_2) \end{aligned} \right.$$

The Table I of the Admiralty Tide Tables furnishes for K_1 and S_2, in the columns b and B, the respective values of θ and of the brackets in formula (3).

Further, in the column B of S_2, account is taken of the effect of the variation in parallax of the sun in the course of the year, by an additional variant factor of from + 0.05 to — 0.05 from 1st January to 30th June etc. (Tide T2). We know in fact that the solar tide at perihelion has for theoretical coefficient 1.05, and 0.95 at aphelion; while it is 1.18 and 0.85 for the lunar tide at perigee and apogee.

Having thus adjusted the basic constituents M_2, S_2 K_1 O_1 to the astronomic conditions of the moment, at least in so far as concerns their High Waters, the Admiralty method combines, on the one hand, the high waters of M_2 and S_2 and, on the other hand, the high waters of K_1 and O_1, in such a manner as to obtain a single semi-diurnal High Water on the day under consideration, and then a single diurnal High Water.

The combination of the constituents of the same species is accomplished practically by means of tables (Table 3, pages 97 to 101 of the Admiralty Tide Tables, Part III) as a function of the angular differences of the vectors which represent them; that is, the differences between the hours of High Water (d) (Fig. 3) and of the ratio of their amplitudes $D = \frac{M}{S}$. Table 3 furnishes the angle e (indicated on Fig. 3) and the factor $E = \frac{R}{M}$, on the assumption that the speeds of the constituents to be combined are similar; that is 30° per hour for the semi-diurnal and 15° per hour for the diurnal.

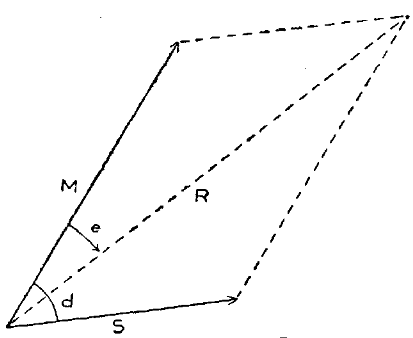

Fig 3

On the same assumption concerning speeds Table 4 (pages 102 to 110 of the Admiralty Tide Tables, Part III) permits the combination of the semi-diurnal and the diurnal by an analogous method for obtaining the times and the heights of water of the High Tide and Low Tide of the day, referred to the mean level.

For the rest, Table 5, of the Admiralty Tide Tables, Part III, allows of the correction, with the aid of the above parameters, that is of d and D, either for the semi-diurnal tide S_2, or the diurnal tide K_1, of the error in speeds for the intermediate hours.

Finally, Table 6 gives, for any constituent whatever, the height of water due to the constituent above or below the Mean Level as a function of its amplitude and its phase.

The use of these tables is greatly facilitated if one employs the printed calculation forms brought out under the auspices of the instructors at H.B.M. Navigation School at Portsmouth, or the forms for calculation given in the Admiralty Tide Tables, Part III, under the types A and B on pages 26 and 27 of the book. (Forms H.D. 288 A and B).

The Part III of the Admiralty Tide Tables also furnishes in its preface some general information regarding tides in shallow waters (M_4 , MS_4 etc.) and on the simple corrections to the times and heights of High Water which it is necessary to apply in certain localities, on this account, conforming to the indications furnished on this subject by the supplementary Tables of the Admiralty Tide Tables, Part II, (shallow water corrections) either as mean corrections, or exceptionally as a function of the time of the moon's transit at the local meridian.

Also the Admiralty Tide Tables, part II, furnish for certain localities, the seasonal variations of height (seasonal corrections) when they are known and are sufficiently great. They are then furnished for each month and allow the correction for the change in mean level Z_0.

Finally, we cannot close without mention of the fact that the Admiralty Tide Tables, Part III, devotes an important chapter to the discussion of the various tidal levels adopted for marine charts (chart datum) either by the different nations or in different waters. These data are of primordial importance for the calculations and to insure the safety of navigation in avoiding strandings.

Even though by international agreement, it has been decided to adopt for use on marine charts a datum " so low... that the tide will but seldom fall below it " the datums in use, although fulfilling these conditions, are widely different. The following, should be noted together with their approximate harmonic notation :

1. Lowest possible Low Water : $Z_0 - 1.2\,(H_{M_2} + H_{S_2} + H_{K_1})$
2. Mean Level of the Lowest Low Waters in each Calendar Month : (harmonic equivalent depending on the type of the tide).
3. Mean Low Low Water Springs :— » » »
4. Indian Spring Low Water : $Z_0 - (H_{M_2} + H_{S_2} + H_{K_1} + H_{O_1})$
5. Mean Low Water Springs : $Z_0 - (H_{M_2} + H_{S_2})$
6. Mean Lower Low Water : $Z_0 - [H_{M_2} + (H_{K_1} + H_{O_1}) \cos 45°]$
7. Mean Low Water : $Z_0 - H_{M_2}$

8. Mean Sea Level : Z_0

The special data on the reference planes for the principal ports are given in a special table in the Admiralty Tide Tables, Part I.

The new method of the British Admiralty applies also to the prediction of tidal streams when their harmonic components are known. Some data of this kind are given in Section II of the Admiralty Tide Tables, 1938, where for several stations we find the harmonic constants of the four principal components of the current. At the stations where the currents are rotary two series of constants are given: one for the North component (+ direction 000°) and the other for the East component (+ direction 090°) and it is necessary to make the prediction separately for each of the hourly rates of the two components which are then finally recombined by utilising, for instance, the Traverse tables. The same type of calculation is applicable to the speed in knots (Kn) by substituing this for the amplitudes (ft).

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