USE OF AIRCRAFT IN SURVEYING

Conjugate Photographs - Dangerous Surfaces

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I.

It is well known that, being given two photographs of the same terrain taken by aircraft from two different viewpoints, it is possible to determine the reciprocal orientation of the photographs, that is the relative positions of the two viewpoints and of the two pencils of homologous radius vectors which issue therefrom, without knowing the exact position of any single point on the ground. We can explain this in the following general manner :

Consider three rectangular axes having as origin the first view-point O and controlled with respect to the photograph. The luminous ray issuing from the origin and passing through a point on the photograph may be defined by the angle α which it makes with the X-axis and the angle γ made with the Z-axis by its projection on the plane YOZ. The photograph will supply us with the magnitude of these angles according to the axes chosen.

We shall define the second station S with respect to the same axes, by its distance Δ from the origin, and the angles u and w which the straight line OS makes with the X-axis and the projection of OS on the plane YOZ with the Z-axis. Let us also establish, with regard to the second photograph, a second system of rectangular axes of origin S. The homologous ray of the former, i.e. passing through the same point, identified on the second photograph, will be defined with respect to the second axes by the angles α' and γ'.

But this ray, having to pass through the same point on the ground as the first, should intersect it. This will therefore not occur if we assume that the second system of axes, which is chosen arbitrarily, is parallel to the first. It will become necessary to displace, without distorting, the second pencil of rays in the second system of axes by a triple rotation about its origin, which may be defined by the angles of Euler : φ ψ and θ. The cosine directors of the ray considered, which were :

$$\cos \alpha' \qquad \sin \alpha' \sin \gamma', \qquad \sin \alpha' \cos \gamma',$$

then become :

$$m = a \cos \alpha' + a' \sin \alpha' \sin \gamma' + a'' \sin \alpha' \cos \gamma'$$

$$n = b \cos \alpha' + b' \sin \alpha' \sin \gamma' + b'' \sin \alpha' \cos \gamma'$$

$$p = c \cos \alpha' + c' \sin \alpha' \sin \gamma' + c'' \sin \alpha' \cos \gamma'$$

The values of the quantities a, a', a''; b, b', b''; c, c', c'', with respect to the angles of Euler, are the following :

$$\begin{cases} a = \cos\varphi\cos\psi - \sin\varphi\sin\psi\cos\theta \\ a' = -\sin\varphi\cos\psi - \cos\varphi\sin\psi\cos\theta \\ a'' = \sin\psi\sin\theta \end{cases} \qquad \begin{cases} c = \sin\varphi\sin\theta \\ c' = \cos\varphi\sin\theta \\ c'' = \cos\theta \end{cases}$$

$$\begin{cases} b = \cos\varphi\sin\psi + \sin\varphi\cos\psi\cos\theta \\ b' = -\sin\varphi\sin\psi + \cos\varphi\cos\psi\cos\theta \\ b'' = -\cos\psi\sin\theta \end{cases}$$

In order that the two homologous rays issuing from S and O should meet, it is necessary that they should lie in the same plane with the straight line OS, and consequently that the determinant D should become zero.

$$(1) \qquad D = \begin{vmatrix} \cos u & \sin u \sin w & \sin u \cos w \\ \cos\alpha & \sin\alpha\sin\gamma & \sin\alpha\cos\gamma \\ m & n & p \end{vmatrix} = 0$$

In this equation, the quantities $\alpha, \gamma, \alpha', \gamma'$ are deduced from the photographs; the quantities u, w, φ, ψ and θ are the five unknowns. They may be determined, at least theoretically, if we can write five equations of this kind for the five different homologous points identified on the photographs; and this without knowing any of the positions of the points on the ground. The unknown Δ, which determines the scale, does not appear in the equations such as (1) and cannot be determined without the knowledge of one point on the ground. We can only determine the directions of the straight lines such as OS; and for this reason the line OS is called the base.

In practice we work by successive approximations. We adopt the nearest values of the five unknowns, values which are provided with great facility by mechanical means. We then introduce these values in D, which then takes, in place of zero, the values $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4, \varepsilon_5$ which we shall consider as infinitely small magnitudes of the first order.

In seeking which are the increments, equal to the 1st order, which it is necessary to give to the approximate values of the unknowns to bring the value of D to zero, we shall have five linear equations with five unknown quantities, the solution of which is generally possible. (1)

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(1) The unknown quantities which we have just employed, allow a rapid demonstration of the results which we hope to show here. But, in the practical adjustment of the restitution apparatus, there is an advantage in using other unknown quantities which are easier to join up to the points identified in the photographs. We have shown in the Hydrographic Review, Vol. VIII, page 109 and seq., a special method which often permits the number of unknown quantities to be reduced to three.

II.

The solution of the five linear equations will only be possible, however, if the determinant of the co-efficients of the unknown quantities is not identically zero. If it was zero, the method of calculation should be modified; and it would be necessary to conclude that the solution of the five equations (1) involves a double root. This root, which corresponds to the second position of the airplane, is not indeterminate, but its calculation may become more intricate. This is furthermore a case which is very unfavourable to the accuracy of restitution. It is comparable to the case where, in the search for the position of the view-point with respect to three known points on the ground, identified on the photograph, it is found on the right circular cylinder circumscribing the triangle formed by the three points, known as the dangerous cylinder. (See Hydrographic Review, Vol. IV, N° 2, pages 85 and 92; Vol. VII, N° 1, pages 106 and seq.; and Vol. VIII, N° 2, pages 30 and 31).

In order to find out in which case there will be a double root, let us assume that the direction OS has been determined by means of equations (1), and let us take this direction as the axis of X. Let us place the pencil passing through S correctly with respect to its homologue issuing from O, i.e. in such a manner that each ray meets its homologue at the corresponding point on the ground. Each of the rays issuing from S will be defined by the angle α' which it makes with the axis of X and by the angle γ which its projection on the plane XOY makes with the Z-axis. If ρ and ρ' are the distances of the point on the ground from O and from S, and XYZ are the coordinates of this point, we shall have the relations :

$$(2) \qquad \begin{array}{lll} \cos\alpha = \dfrac{X}{\rho}, \quad \sin^2\alpha = \dfrac{Y^2+Z^2}{\rho^2}, & & \sin\alpha\sin\gamma = \dfrac{Y}{\rho}, \quad \sin\alpha\cos\gamma = \dfrac{Z}{\rho}\cdot \\ & \sin^2\gamma = \dfrac{Y^2}{Y^2+Z^2}, & \\ \cos\alpha' = \dfrac{X-\Delta}{\rho'}, \quad \sin^2\alpha' = \dfrac{Y^2+Z^2}{\rho'^2}, & & \sin\alpha\sin\gamma = \dfrac{Y}{\rho'}, \quad \sin\alpha'\cos\gamma = \dfrac{Z}{\rho'}\cdot \end{array}$$

If a point infinitely close to the point S, having coordinates $\Delta + \xi$, η and ζ can also satisfy the equation, then equation (1) becomes :

$$(3) \qquad \begin{vmatrix} \Delta + \xi & \eta & \zeta \\ \cos\alpha & \sin\alpha\sin\gamma & \sin\alpha\cos\gamma \\ m + dm & n + dn & p + dp \end{vmatrix} = 0$$

with :

$$m = \cos\alpha', \qquad n = \sin\alpha'\sin\gamma, \qquad p = \sin\alpha'\cos\gamma$$

The quantities dm, dn and dp derive from the three rotations to which it is necessary to submit the pencil, which first issued from S, in order that it can be made to pass through the five points on the ground. This displacement will be very small and of the same order as the quantities ξ, η and ζ, and therefore θ which indicates the displacement with respect to the Z-axis will be very small. By neglecting the second order of θ, we have :

$$da = -2\sin^2\frac{\varphi+\psi}{2}, \qquad da' = -\sin(\varphi+\psi), \qquad da'' = \theta\sin\psi$$

$$db = \sin(\varphi+\psi), \qquad db' = -2\sin^2\frac{\varphi+\psi}{2}, \qquad db'' = -\theta\cos\psi$$

$$dc = \theta\sin\varphi, \qquad dc' = \theta\cos\psi, \qquad dc'' = o$$

The quantities da, da' having to be very small, we see that $\varphi+\psi$ should be an infinitely small magnitude of the 1st order, without it being essential that φ and ψ respectively should be very small. We then have, by neglecting everywhere the second order :

$$da = o\,, \qquad da' = -(\varphi+\psi), \qquad da'' = \theta\sin\psi$$

$$db = (\varphi+\psi) \qquad db' = o\,, \qquad db'' = -\theta\cos\psi$$

$$dc = -\theta\sin\psi, \qquad dc' = \theta\cos\psi, \qquad dc'' = o$$

and consequently :

$$dm = -(\varphi+\psi)\sin\alpha'\sin\gamma + \theta\sin\psi\sin\alpha'\cos\gamma,$$

$$dn = (\varphi+\psi)\cos\alpha' - \theta\cos\psi\sin\alpha'\cos\gamma,$$

$$dp = -\theta\sin\psi\cos\alpha' + \theta\cos\psi\sin\alpha'\sin\gamma.$$

The calculation of the determinant (3) then gives us :

$$\begin{aligned}\Delta\sin\alpha\,[-\theta\cos\alpha'\sin\gamma\sin\psi + \theta\sin\alpha'\cos\psi - (\varphi+\psi)\cos\alpha'\cos\gamma]\\ +\eta\,(\sin\alpha\cos\alpha'\cos\gamma - \cos\alpha\sin\alpha'\cos\gamma)\\ +\zeta\,(\cos\alpha\sin\alpha'\sin\gamma - \sin\alpha\cos\alpha'\sin\gamma) = o\end{aligned}$$

which then becomes, by taking into account the equations (2):

$$\begin{aligned}(Y^2+Z^2)\,\theta\cos\psi - (X-\Delta)\,Z\,(\varphi+\psi) - (X-\Delta)\,Y\,\theta\sin\psi\\ +\eta\,Z - \zeta\,Y = o\end{aligned}$$

This equation is linear and homogeneous with respect to the 5 unknown quantities, which are infinitely small magnitudes of the 1st order : $\theta\cos\psi$, $\theta\sin\psi$, $\varphi+\psi$, η and ζ. We can then write five similar to them, by replacing X, Y and Z by the coordinates of the five points on the ground. But these five equations cannot exist at the same time, unless the determinant of the co-efficients of the five unknown quantities is equal to zero; that is if we have :

$$\begin{vmatrix} Y_1^2+Z_1^2 & (X_1-\Delta)\,Z_1 & (X_1-\Delta)\,Y_1 & Z_1 & Y_1 \\ Y_2^2+Z_2^2 & (X_2-\Delta)\,Z_2 & (X_2-\Delta)\,Y_2 & Z_2 & Y_2 \\ Y_3^2+Z_3^2 & (X_3-\Delta)\,Z_3 & (X_3-\Delta)\,Y_3 & Z_3 & Y_3 \\ Y_4^2+Z_4^2 & (X_4-\Delta)\,Z_4 & (X_4-\Delta)\,Y_4 & Z_4 & Y_4 \\ Y_5^2+Z_5^2 & (X_5-\Delta)\,Z_5 & (X_5-\Delta)\,Y_5 & Z_5 & Y_5 \end{vmatrix} = o$$

This determinant may be simplified and written :

$$(4) \qquad \begin{vmatrix} Y_1^2 + Z_1^2 & X_1\,Z_1 & X_1\,Y_1 & Z_1 & Y_1 \\ Y_2^2 + Z_2^2 & X_2\,Z_2 & X_2\,Y_2 & Z_2 & Y_2 \\ Y_3^2 + Z_3^2 & X_3\,Z_3 & X_3\,Y_3 & Z_3 & Y_3 \\ Y_4^2 + Z_4^2 & X_4\,Z_4 & X_4\,Y_4 & Z_4 & Y_4 \\ Y_5^2 + Z_5^2 & X_5\,Z_5 & X_5\,Y_5 & Z_5 & Y_5 \end{vmatrix} = 0$$

If we suppress the index 1 of the letters X, Y and Z of the first horizontal line, the equation (4) represents a ruled quadric, generally hyperboloïd of one sheet, which contains the straight line OS, passes through the four points numbered 2, 3, 4, 5 and has circular sections perpendicular to OS (and consequently its real major axis parallel to the plane ZOY), conditions which suffice for its determination. And we shall see that the direction OS corresponds to a double root if the fifth point on the ground, numbered 1, also lies on this quadric. Should that occur, it will suffice in general to choose another point on the two photographs corresponding to a point on the ground which has not this property, in order that this relative indetermination may be avoided. Meanwhile, if all the points on the ground are located on this quadric, no group of five points will allow the use of this method of calculation, and we shall also find ourselves in another unfavourable case in so far as concerns the accuracy of adjustment of the restitution apparatus; whence the name *dangerous surface* which has been given to this quadric.

It is certain that no ground will ever have exactly the shape of this ruled quadric; but if the shape approaches it sufficiently, the conditions of restitution will become mediocre and it would be better to choose an altitude or direction of flight which avoids this drawback. The dangerous surface may be a right circular cylinder if the minors of the determinant (4), coefficients of the terms in XZ and XY are zero. Certain valleys may have a shape which differs little from a section of a cylinder; in that case it would be better that the base, line of flight, should not be parallel to its axis, or that it should be located at a sufficient altitude so that it does not lie on the cylinder.

The dangerous quadric may be decomposed into two planes, of which one lies along the base and the other perpendicular to the base, if the minor, co-efficient of $Y^2 + Z^2$, is zero, and if the ratio of the minor co-efficients of XZ and Z is equal to that of the minor co-efficient of XY and Y.

But it does not seem likely that this case should be encountered on the ground, any more than those in which the quadric becomes a cone or a hyperbolic paraboloïd. In practice, a ground flown over which is practically flat, need not give rise to a dangerous surface.

Even though the terrain has not the shape of the dangerous surface, it is nevertheless necessary to avoid taking the fifth point on this surface.

When choosing the five homologous points on the pair of photographs, one is often tempted to take four of them in the vicinity of the four angles of the part common to the photographs; two straight lines joining up the points, may then be parallel to the base line or line of flight (approximately known), and define with this line a right circular cylinder which will contain the four points and the base. Care should be exercised to choose the fifth point clearly outside this cylinder to avoid finding oneself in the case of the double solution and consequently in the state of relative indetermination; at least where the terrain has not exactly the shape of that surface. Similarly, if two of the chosen points are, with the base, in the same plane and the other two in a plane perpendicular to the base, care must be taken in choosing the fifth point that it does not lie in either of these two planes.

III.

The equations (1) may provide us, aside from the solution of OS, which is really the direction of the second point of view, other solutions for which the homologous rays corresponding to the five chosen points on the photographs shall meet. Let OS_1 be one of the solutions of which we carry over the directions u and w to the same system of axes employed in chapter II, where we have adopted the straight line OS as the axis of X.

If we consider any two homologous rays whatever, issuing from O and S, that which proceeds from O being characterized by the angles α and γ, while that issuing from S by the angles α' and γ, these angles are related by the equations (2) to the coordinates of the corresponding point on the ground.

Let us move the pencil proceeding from S parallel to itself, to a point on OS. Then, by the rotations φ, ψ and θ which give rise to the equations (1) we arrive at the condition where the five rays under consideration come to rest against the five homologous rays issuing from O. Their cosine directors will be m, n and p such as those given in Chapter I, except for the fact that γ' is here equal to γ.

The equations (1) may therefore be written :

$$\begin{vmatrix} \cos u_1 & \sin u_1 \sin w_1 & \sin u_1 \cos w_1 \\ \cos \alpha & \sin \alpha \sin \gamma & \sin \alpha \cos \gamma \\ m & n & p \end{vmatrix} = 0$$

Replacing the trigonometrical lines α, α' and γ, by their values (2), the determinant then becomes :

$$\begin{vmatrix} \cos u_1 & \sin u_1 \sin w_1 & \sin u_1 \cos w_1 \\ X & Y & Z \\ a(X-\Delta) + a'Y + a''Z & b(X-\Delta) + b'Y + b''Z & c(X-\Delta) + c'Y + c''Z \end{vmatrix} = 0$$

We shall give it the form :

$$(5)(X-\Delta)\begin{vmatrix} \cos u_1 & \sin u_1 \sin w_1 & \sin u_1 \cos w_1 \\ X & Y & Z \\ a & b & c \end{vmatrix} + Y\begin{vmatrix} \cos u_1 & \sin u_1 \sin w_1 & \sin u_1 \cos w_1 \\ X & Y & Z \\ a' & b' & c' \end{vmatrix}$$

$$+ Z\begin{vmatrix} \cos u_1 & \sin u_1 \sin w_1 & \sin u_1 \cos w_1 \\ X & Y & Z \\ a'' & b'' & c'' \end{vmatrix} = 0$$

This equation may be verified if we substitute in it, for X, Y, Z, the coordinates of any of the five points on the ground, and for a, b, c, a', b', c', a'', b'', c'', their values given in I, where φ, ψ and θ should then have the values which correspond to the solution OS_1. But equation (5) cannot generally be verified if X, Y and Z are the coordinates of another point on the ground. They can only become so for this point, if it lies on a surface of the second degree, the ruled quadric, represented by equation (5).

The stereoscopic examination of the two photographs taken from the viewpoints O and S will give an exact image of the terrain after the proper orientation has been effected, if one examines them from the two relative positions corresponding to O and S. But if the two relative positions correspond to the two viewpoints O and a point on OS_1, one cannot generally find a suitable orientation except for the five homologous rays chosen; the other homologous rays on the two photographs will not meet and consequently there will be no impression of relief. It will not suffice that one point on the ground, other than the five points considered, should be on the quadric (5); it is necessary, in order to obtain the impression of ground relief over the entire terrain observed, that all points on the terrain photographed should lie on this quadric. This can never rigorously occur in practice, but, thanks to the faculty of accommodation of the eye, which allows small differences to pass unnoticed, it will suffice if the shape of the terrain is a close enough approximation of this shape to produce the impression of ground relief. This shape of terrain will be found the more easily if the common parts in the two photographs represent a strip which is not very wide, or if the airplane in its flight follows very nearly a course parallel to the thalweg of a valley.

Let us call P, Q and R the three determinants of equation (5), which may then be written :

$$(5 \text{ bis}) \quad (X-\Delta)\,P + YQ + ZR = 0$$

We see that this quadric, which contains the five control points on the ground, also passes through the point S and contains the straight line OS_1; — this suffices to define the quadric. The three planes which are obtained from the equations by equating to zero the three determinants P, Q and R, pass through OS_1. The plane $P = O$ is the plane tangent to the quadric at the origin.

If all the points on the terrain are on this quadric, the stereoscopic examination effected from the two relative positions corresponding to O and a point on OS_1, will furnish a plastic image which will be false. The homologous rays will still meet in pairs, but not at homologous points on the terrain.

Let ξ, η and ζ be the coordinates of this point S_1 taken on OS_1. The image will be furnished by the intersection of each of these pairs of homologous straight lines :

$$\frac{X}{\cos\alpha} = \frac{Y}{\sin\alpha\sin\gamma} = \frac{Z}{\sin\alpha\cos\gamma} \quad \text{et} \quad \frac{X-\xi}{m} = \frac{Y-\eta}{n} = \frac{X-\zeta}{p}.$$

In m, n, and p we have $\gamma' = \gamma$, and according to the equations (2) :

$$\cos\alpha' = \frac{\rho\cos\alpha - \Delta}{\rho'}, \quad \sin\alpha'\sin\gamma = \frac{\rho}{\rho'}\sin\alpha\sin\gamma.$$

Substituting these values in the equations for the homologous straight lines, we see that the locus of their points of intersection will be furnished by the equation :

$$(6) \quad \begin{vmatrix} X-\xi & aX+a'Y+a''Z & a \\ Y-\eta & bX+b'Y+b''Z & b \\ Z-\zeta & cX+c'Y+c''Z & c \end{vmatrix} = 0$$

This is the equation of a quadric which will be the plastic pseudo image of the ground when it is represented by the equation (5). It is a ruled quadric, which passes through the point S_1 and contains the straight line OS in its entirety. Its equation may then be written, by taking into consideration the known relations between the co-efficients a, b, c, a', b', c', a", b", c" :

$$(6\text{ bis}) \quad Y\left[a''(X-\xi)+b''(Y-\eta)+c''(Z-\zeta)\right]$$
$$= Z\left[a'(X-\xi)+b'(Y-\eta)+c'(Z-\zeta)\right].$$

This ruled quadric (generally hyperboloïd ~~of~~ one sheet) passes through the point S_1 and contains the straight line OS.

Its tangent plane at O has the equation :

$$Y\left(a''\xi+b''\eta+c''\zeta\right) = Z\left(a'\xi+b'\eta+c'\zeta\right).$$

The intersection of this plane with the plane $P = O$, tangent at O to the quadric (5) has as equation, adopting the notations :

$$A = a\xi+b\eta+c\zeta\,, \quad A' = a'\xi+b'\eta+c'\zeta\,, \quad A'' = a''\xi+b''\eta+c''\zeta\,,$$

$$\frac{X\,(b\zeta-c\eta)}{(b\xi-a\eta)\,A''+(a\zeta-c\xi)\,A'} = \frac{Y}{A'} = \frac{Z}{A''}.$$

One can easily prove that this straight line lies entirely and at the same time on the quadric (5) and on the quadric (6); it is common to the two quadrics. We shall call it OT. The plane tangent at O to the quadric (5) intersects the surface following the two lines OS_1 and OT. The two quadrics, aside from the common straight line OT, intersect mutually following a twisted curve of the third order. For all the points of this straight line and for this curve, the impression of ground relief, as a consequence of examining the points stereoscopically from the view-points O and S, will be accurate; it will be false, however, for all the other points lying on the surface (5). The two surfaces are tangent at the two points where the twisted curve intersects line OT.

In order that equations (5) and (6) should be identical, it is necessary and it suffices that :

$$\frac{\xi}{a} = \frac{\eta}{b} = \frac{\zeta}{c} \, .$$

The quadric will then be reduced to a cone :

$$Y\,(a''X + b''Y + c''Z) = Z\,(a'X + b'Y + c'Z) \, .$$

In that case, the two straight lines OS_1 and OT will merge into one.

But this equation no longer defines the distance of the points on the ground from the origin. It appears further that this cone cannot correspond to any case which occurs in reality.

The quadric (5) well merits the name " dangerous surface " because, in spite of a very real impression of relief, the restitution made from the two view-points O and S_1 will be false. This case has occurred in practice and this has led Mr. G. Poivilliers, the inventor of that excellent restitution apparatus, the Stereotopograph, to warn of the existence of this dangerous surface, which differs radically from the dangerous surface studied in Chapter II defined by equation (4). Mr. Poivilliers has given an elegant geometrical demonstration to which we refer the reader.

The ruled quadrics (4) and (5) have in common the five points on the ground, as well as the points O and S. We know that, in that case, they have an eighth point in common. But the quadric (5) generally does not contain the straight line OS in its entirety, such as is the case on the contrary on the quadric (4), and it generally does not have the circular sections perpendicular to OS. Further, these two " dangerous surfaces " correspond to two very different cases; the first to the case of a double solution, which is however correct; and the second to a simple solution, but false.

It is only very exceptionally that these two quadrics can coincide.

The problem posed in Chapter I may therefore be summarized as follows :

Given the 5 points on the ground and their images viewed from the two points O and S, the solution of the equations (1) furnishes, besides the correct direction OS, a certain number of directions OS_1, $OS_2 \ldots\ldots OS_n$, determined and independent of the general shape of the terrain. Through five points on the ground, and through O and S, pass an infinity of quadrics, on one of which only there is the straight line OS_1, to the exclusion (in general), of the other directions OS_2, $OS_3 \ldots\ldots OS_n$.

If the ground merges with the shape of this ruled quadric, well defined by the five points, the points O, S and the straight line OS_1, every point of view taken from OS_1 will yield, together with the view-point O, a false plastic image. And, in that case, nothing will be gained by choosing five other points on the ground or on the photographs; they would still give the solution OS_1 (as well as OS), because the rays issuing from every point of OS_1 may all rest against the homologous rays issuing from O (but these five points no longer give the same solutions OS_2, $OS_3 \ldots\ldots OS_n$). The other real solutions of the equation (1), such as OS_p, cannot give rise to any plastic image, at least in so far as they are not located on the quadric (5) which we have assumed represents the shape of the terrain. There is only the straight line OT, which responds to this condition; but it is not in general a solution of the equations (1). In any case there can never be more than two directions giving rise to false plastic images.

BIBLIOGRAPHY

Georges POIVILLIERS. — *Conférence* — Bulletin de Photogrammétrie, 1933, N° 6, page 126.

» » — *Comptes rendus de l'Académie des Sciences,* 23 Mai 1934, page 1845.

» » — *Propriété perspective de certaines surfaces et son application aux levés photographiques aériens* — Archives Internationales de Photogrammétrie, Tome VIII, fascicule 2, pages 244-246.

Sébastian FINSTERWALDER. — *Die geometrischen Grundlagen der Photogrammetrie* — Jahresbericht der deutschen Mathematikvereinigung, 1899.

» » — *Eine Grundaufgabe der Photogrammetrie und ihre Anwendung auf Ballonaufnahmen* — Abhdlg. d. k. Bayer. Akad. d. Wiss. II. Kl. XXII. Bd. II. Abt. (1903), pages 225/60.

» » — *Die Hauptaufgabe der Photogrammetrie* — Sitzungsberichte der Bayerischen Akademie der Wissenschaften — 7 Mai 1932.

Richard FINSTERWALDER. — *Der unregelmässige Fehler der räumlichen Doppelpunkteinschaltung* — Allgemeine Vermessungs-Nachrichten 1932, pages 641, 644, 657/669, 673/680.

» » *Der unregelmässige und systematische Fehler der räumlichen Doppelpunkteinschaltung und Aerotriangulation* — Bildmessung und Luftbildwesen 1933, pages 55/68.

» » *Der gefährliche Zylinder beim Normalfall der räumlichen Doppelpunkteinschaltung* — Zeitschrift für Vermessungswesen, 67, 1938, pages 433/41.

» » *Der gefährliche Ort der photogrammetrischen Hauptaufgabe und seine Bedeutung besonders bei der Auswertung von Luftaufnahmen im Gebirge* — Bildmessung und Luftbildwesen, 13, 1938, pages 103/109.

O. von GRUBER. — *Einfache und Doppelpunkteinschaltung im Raum.* G. Fischer, Jena, 1924.

» » *Ferienkurs in Photogrammetrie,* pages 47/52 — K. Wittwer, Stuttgart, 1930.

P. GAST. — *Das Einschneiden aus dem gefährlichen Ort in der Aerophotogrammetrie* — Zeitschrift für Vermessungswesen, 1929, page 614.

R. BOSSHARDT. — *Über den Einfluss der Gelände — Höhenunterschiede beim optisch-mechanischen Einpassen von Luftaufnahmen* — Suisse — Zeitschrift für Vermessungswesen und Kulturtechnik, 1933, cahiers 5 et 6.

Chr. NEUMANN. — *Zur aüsseren Orientierung geneigter Bildpaare* — Bildmessung und Luftbildwesen, 1932, pages 168/180.

SCHWIDEFSKY. — *Einführung in die Luft— und Erdbildmessung,* pages 84/87 — Teubner, Leipzig, 1936.

Emilio WOLF. — *Orientaçao de um par de photographias aéreas no estereographo* — Annaes Hydrographicos — Rio de Janeiro — Tome IV, 1936, pages 43/46.