NOTES ON THE USE OF DEFLECTORS

by

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INTRODUCTION.

The use of the deflector has fallen somewhat into desuetude since its invention, because at first the instrument was devised for use on the cards of the dry compasses, of the Thomson or similar type, having a very small magnetic moment (about 200 cgs for the 10 inch card). The introduction in current navigation some years ago of the liquid compass with a magnetic system showing strong magnetic moments (about 3900 gauss for the Bamberg compass), the inductive reactions produced by their use on the soft iron correctors, such as the spheres, lateral cylinders and transverse bars, appeared to prohibit the further accumulation of magnets of great strength, provoking new complex field perturbations in the immediate vicinity of the compass card and its correctors, for the purpose of effecting measurements which should, above all, remain simple [1].

Nevertheless, the modern tendency to revert in the liquid compasses to cards with reduced magnetic moments (of the order of 1500 cgs in the Doignon compass and 500 cgs in the Chetwynd compass) and the use of shorter needles, permits to a certain extent the reversion to an instrument of a very practical nature, especially when its use is restricted to several elementary operations in which an approximation is sought rather than great accuracy.

For this purpose the deflector is a very useful aid in the hands of the officer charged with the supervision of the compasses, especially on board large ships with several magnetic compasses, the functioning of which must be checked. In addition to the standard and the steering compass, there is often a portable compass available, on which one may experiment and undertake useful studies with regard to the magnetism on board ship.

Finally the deflector is a very attractive instrument for giving instructions in compass compensation and training the students. In the following pages we have grouped a few notes on the general principles governing the various uses to which the different types of deflectors may lend themselves in this connection.

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[1] See : N. OGLOBLINSKY " *La Revue Maritime* ", Paris, Février 1928, pages 145-171.

I

DESCRIPTION.

1. *Definition.* — As is generally known, the deflector is an instrument which serves to deflect (deviate) the magnetic needle of the compass from its normal direction for the purpose of determining the value of the directive force towards the North magnetic Pole. The effect which it thus produces in a given position is called the deflection or the deviation.

2. *Principle.* — The principle on which the deflector is based is that of the action of a magnetized bar on a magnetized pole ($+$ 1) placed at a distance R normal to the centre of the bar (fig. 1).

We know that the effect f thus produced (or the field of the magnet) is, in this case, parallel to the direction of the bar, directed in the conventional direction (red-blue) and that its value is given by the expression :

$$f = \frac{\mathfrak{M}}{R^3}$$

in which $\mathfrak{M}$, the magnetic moment of the bar magnet has a value equal to the product of the magnetic mass of each pole $\pm m$, multiplied by the intercalary distance l, $\mathfrak{M} = ml$ (l being in general about 5/6 of the geometrical length L of the bar magnet).
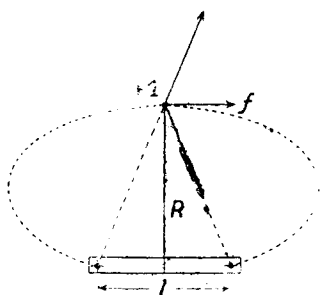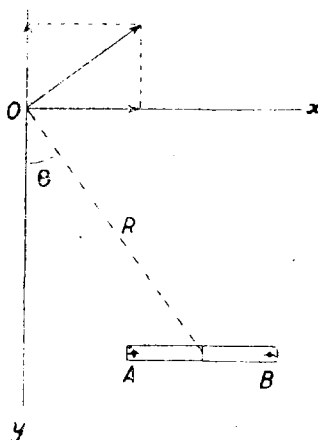
(Fig. 1)   (Fig. 1 bis).

Note. — There is shown in Fig. 1 the scheme usually employed in the binnacle of the magnetic compasses, in the disposition of the compensating magnets, and the law $f = \dfrac{\mathfrak{M}}{R^3}$ which is that governing the compensation for the longitudinal and transverse components of the semi-circular magnetism acting on the compass card. In a general way the action of a magnet AB on an isolated pole O, resolved along the two rectangular axes

Ox and Oy, of which one is parallel to the magnet (fig. 1 a) is given by the formulae :

$$\left\{ \begin{array}{l} 1/2\,\dfrac{\mathfrak{M}}{R^3}\left(1 - 3\cos 2\,\theta\right) \\[2ex] 1/2\,\dfrac{\mathfrak{M}}{R^3}\,3\sin 2\,\theta \end{array} \right.$$

3. *First Arrangement.* — Let us place on a horizontal table the needle which, in repose, will orient itself in the direction Nm of the magnetic North.

Let us place, as indicated in fig. 2, a transverse magnet at a distance R from the centre of the compass.

If the size of the compass needle is small with respect to distance R, so that the actions $+ f$ and $+ f$ on the poles may be considered as equal and parallel, we shall obtain the position of equilibrium δ (deflection or deviation of the needle with respect to the magnetic meridian) by the equation $F = H \tan \delta$ or $f = H \delta$ if δ is small. This approximation is valid in practice if the distance R is greater than, or equal to, 4 times the radius of the compass (double the total length of the needle). It is easy to determine the value of the deflection angle δ if the compass is provided with a graduated arc or if the compass needle (or the needle system) is provided with a card graduated in degrees or otherwise.
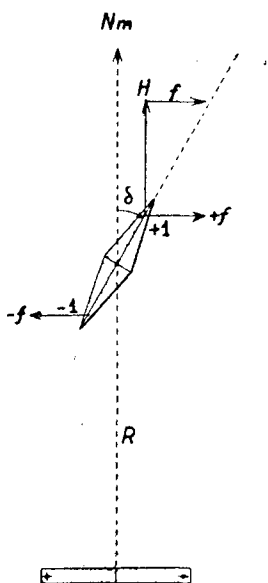
(Fig. 2)
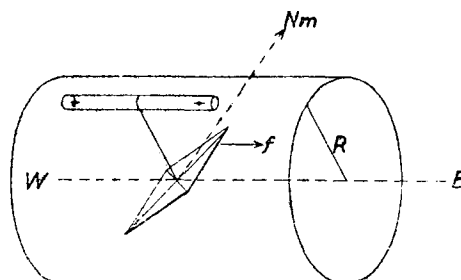
(Fig. 3)

4. *Object.* — The formulae for the deflector, $f = \dfrac{\mathfrak{M}}{R^3} = H \tan \delta$

in which R and δ are directly measured, can lend themselves to several

combinations permitting direct or relative measurements of either the magnetic moments $\mathfrak{M}$ or the magnetic field H.

5. *Example.* — For example, to compare one with the other, two magnets placed at the same distance from the compass, in the condition shown in fig. 2, give :

$$\mathfrak{M}_1 = R^3 H \tan \delta_1 \quad \text{or} \quad R^3 H \delta_1$$
$$\mathfrak{M}_2 = R^3 H \tan \delta_2 \quad \text{or} \quad R^3 H \delta^2 \qquad \text{or} \qquad \frac{\mathfrak{M}_1}{\mathfrak{M}_2} = \frac{\delta_1}{\delta_2}.$$

This formula and the arrangement shown in fig. 2 is used at the French Hydrographic Office for *the rapid calibration of magnets,* knowing, as a result of previous direct measurement the value of the magnetic moment $\mathfrak{M}_1$, of one of them (*), say 1680 cgs for instance. We proceed in the following manner : we move the magnet $\mathfrak{M}_1$ towards the compass until the needle indicates 16° 8 on the graduated limb. Thereupon the position of the magnet $\mathfrak{M}_1$ is marked by a chalk line on the table. The magnet $\mathfrak{M}_1$ is thereupon removed and replaced in the *identical position* by the magnet $\mathfrak{M}_2$; the position of equilibrium δ_2 thereupon taken by the compass needle evaluated in degrees and multiplied by 100 then gives the magnetic moment $\mathfrak{M}_2$ in cgs units.

6. *Second arrangement.* — The effect of the magnet will remain the same whether this magnet, retaining the same orientation, is placed at the same distance R above the axis of the compass or below it, or in general if it remains on the surface of a cylinder with the axis, East-West magnetic passing through the centre of the compass and having a radius R (fig. 3). Thus let us place above the liquid compass a magnet of any convenient
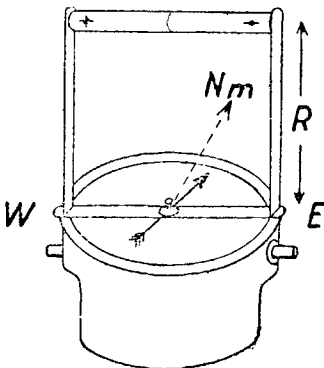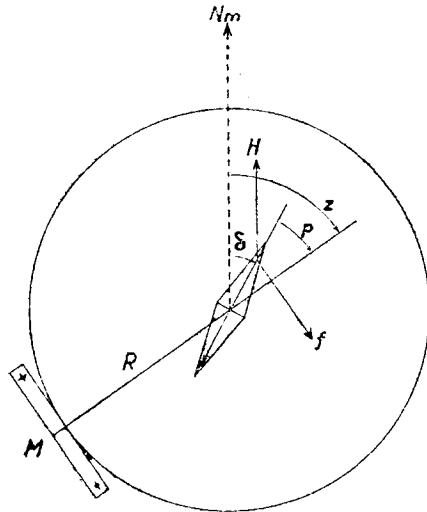
(Fig. 4)   (Fig. 5)

(*) See : *Hydrographic Review,* Vol. V, N° 1, Monaco, May 1928, page 185.

strength, by afixing it at a specified distance R above the plane of the needles of the card on the upper transverse of the sighting alidade (fig. 4).

This arrangement permits us, with the ship held steady on any given heading, to orient the magnet in the West-East magnetic plane in order that the modified position shown in fig. 2 may be realised.

The deflection δ_1, thus produced, is related to the directive force H_1 on this heading by the equation $\dfrac{\mathcal{M}}{R^3} = \text{constant} = H_1 \tan \delta_1 = H_2 \tan \delta_2 = H_3 \tan \delta_3 \ldots\ldots$ etc. whence $H_1 = K \cotan \delta_1$, $H_2 = K \cotan \delta_2$ $\ldots\ldots$ etc. for other headings of the vessel.

And it is clear that this simple disposition of the deflector, which is easily carried out with the means available aboard ship, permits the evaluation in terms of the cotangents of the deflections, of the relative directive forces on all headings of the vessel; i.e. it permits the tracing of the elliptic dygogram of forces (*). For this purpose one can make convenient use of the tables giving the natural cotangents of the arcs. (See also the following par. 8 a).

7. *Other Arrangements.* — If, while remaining perpendicular at its centre to the radius CM; the magnet describes about the centre C of the compass a circle of radius R, the intensity of its action f on the $(+ 1)$ pole of the needle, will remain the same, but the direction of f varies like that of the magnet (fig. 5).

The equilibrium position of the compass needle (deflection) δ as defined for a given position of the magnet ($Z = z + 90°$) with respect to the magnetic meridian, or $P = p + 90°$ with respect to the deviated needle, is given by the equation :

$$H \sin \delta = f \cos p$$

which is obtained by writing that the projection of the acting forces (H and f) in the direction perpendicular to the deviated needle is zero, or also that the sum of the moments of forces H and f with respect to the pivot, is zero (fig. 6).

8. *Maximum deviation (third disposition).* — The maximum deviation (deflection) for a given magnet at a given distance R is realised when $\sin \delta = \dfrac{f}{H} \cos p$. is a maximum, that is when $p = o$.
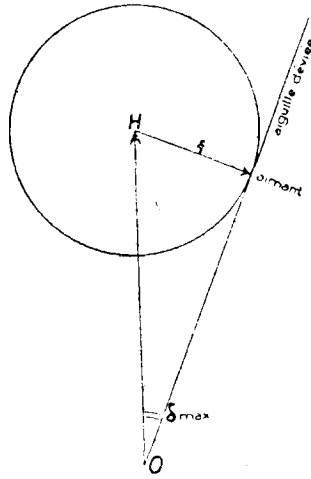

(*) See: F.J. Evans — Admiralty Manual for the Deviations of the Compass — London, 1893, page 160.

E. Guyou — Manuel des Instruments nautiques, Service Hydrographique, Paris 1921, page 108.

Then the orientation of the magnet is " perpendicular to the direction of the deviated needle " (fig. 7).
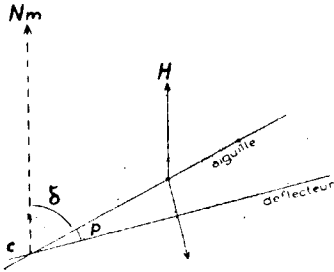
(Fig. 6)　　　　　　　　　　　　　　　(Fig. 7)

The equilibrium formula then reduces to :

$$\sin \delta = \frac{\mathrm{I}}{\mathrm{H}} \frac{\mathfrak{M}}{R^3}$$

$$\text{or } \mathrm{H} = k \frac{\mathrm{I}}{\sin \delta}$$

Note. — For the rest one might examine the maximum value of the deflection attained, depending on the relative values of f and H. From this we see that in order to realise the conditions shown in Fig. 7 and in order to obtain a convenient deflection, it is necessary to employ a deflector having an appropriate action f.

8 a. — *Determination of the values of H' on different headings* — *Plotting the elliptic dygogram.* — Starting from any given heading, the North for instance, first steady on the course then place the deflector in position and adjust it to obtain the desired angle of 90° and then note the position of equilibrium δ of the compass card and inscribe this opposite the heading under consideration. Removing the deflector, then head on the next course, whereupon the above described operation is repeated.

Once the complete swing around the horizon has been completed the values of H' are calculated by compiling the table as follows :

| $N°$	Compass heading	δ	$H' = \dfrac{\mathrm{I}}{\sin \delta}$
0	North	30°	2.00
I	N 30° E	18°	3.20
2	....	...	....
	etc.	etc.	etc.

The dygogram is then traced with the aid of this table on the rectangular axes OX and OY parallel to the longitudinal and transversal axes of the ship. On the various vectors o, 1, 2 etc... lay off to a convenient scale, for instance 3 mm. per unit, the values of H'. By plotting the smooth curve through the extremities of these vectors, we obtain the ellipse representing the dygogram of directive forces on the different headings.

By means of well known graphic processes we then determine its centre Ω, its major axis a and its minor axis b, the angle γ which the major axis makes with OX, the direction of the vessel's head and the projection ω of Ω on OX.

From this the various practical coefficients are obtained.

$$\text{o } \Omega = F_2 \qquad\qquad B = \omega \text{ o}$$

$$a = F_1 + F_3 \qquad\qquad C = \omega \Omega$$

$$b = F_1 - F_3 \qquad\qquad D \text{ or } \sqrt{D^2 + E^2} = \frac{a - b}{a + b}$$

$$\frac{E}{D} = \text{tg } 2 \gamma$$

$$2 \lambda H = a + b$$

9. *Minimum deviation.* — In the same conditions the minimum deviation (deflection zero) is evidently realised when $\cos p = O$, that is when p equals 90° or 270° or when P equals 180° or 360° $= Z$. The magnet is then parallel to the magnetic meridian, either in the red-blue or blue-red direction (figs. 8 & 9).
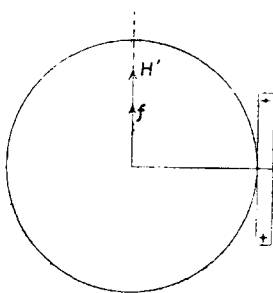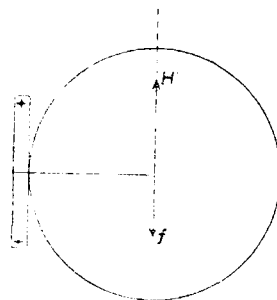
(Fig. 8)   (Fig. 9)

In the latter case the action of the magnet reinforces the directive force. In the first case it diminishes it, and :

If $H' > f$ the needle points to the North.

If $H' < f$ the needle is reversed and points to the South.

10. *Special case. (Fourth disposition).* — If $H = f$, the compass is indifferent. In this special case the formula reduces to :

$$H = \frac{\mathfrak{M}}{R^3}$$

The total field which surrounds the compass in its immediate vicinity becomes zero and in order that the compass needle may be brought out of this condition of indifference and made to take a decisive direction, it is necessary to bring near the compass a small directive magnet φ (sometimes called the " calming magnet " because it serves to prevent the interminable oscillations of the needle in a weak field) (fig. 10).

11. *Practical disposition.* — It is not absolutely necessary that the deflector magnet should be maintained in the horizontal plane of the compass, because all that has been stated subsequent to paragrpah 7 remains true provided only that the magnet is maintained at its distance R and in the proper orientation. In particular it will be more convenient, especially aboard ship, to place the deflector or magnet above the centre of the compass card at a vertical distance R from the plane of the latter, which will be fixed by the dimensions of its own support with an even greater accuracy than a lateral distance.
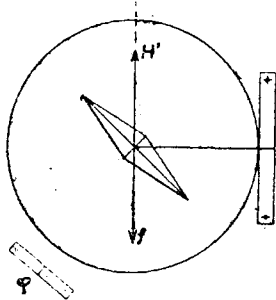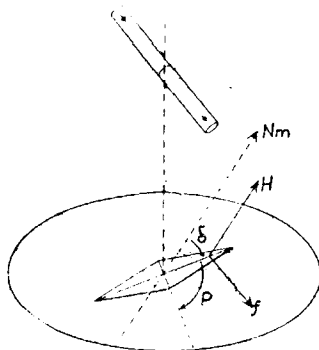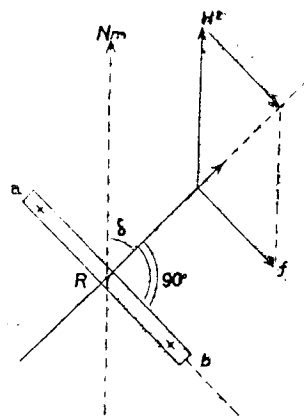
(Fig. 10)   (Fig. 11)   (Fig. 12)

This arrangement possesses the further advantage of permitting an easy evaluation of the deflection δ and also of the angle P called the " angle of position " of the deflector, the value of which is represented by the graduation on the card marked by the blue pole of the magnet or by its projection on to the plane of the compass card. (Fig. 11). Thus the deflector is oriented about the vertical passing through the pivot of the magnetic needle or of the complex card, in such a manner that it may exercise its deflecting action in the requisite azimuth for constituting those geometric figures which lend themselves readily to manipulations or calculations, either in combination with the direction of the directive field or else with the position of equilibrium of the needle.

In particular, fig. 12 represents, in the projection on the plane of the card, the state of equilibrium of the compass needle and of the deflector at the maximum deviation. H' is the horizontal component of the magnetic field for any heading whatever of the vessel.

12. *Approximate Formulae.* — All the formulae for the deflectors which we have considered above are deduced from the general formula :

$$H \sin \delta = f \cos p$$

(α)   those considered in paragraphs 3 and 6 derive from the preceding, when $p = -\delta$, or $H = f$ cotan δ (case of easterly deflection).

(β)   those considered in par. 8 derive from it when p equals zero or $H = f \dfrac{1}{\sin \delta}$ (case of maximum deflection).

(γ)   those considered in par. 10 derive from it when $\cos p = O$ and $\delta = O$, and $H = f$ (case of indifference).

In all these formulae, $f = \dfrac{\mathfrak{M}}{R^3}$ and $\mathfrak{M} = ml$.

13. *Use of Deflectors.* — Deflectors are instruments which permit in particular the measurement by one of the above mentioned formulae of the directive force acting upon the needles of the compass on the different headings, by means of the deviation which they impose on the compass card, on being placed in a specified position above it.

They especially permit the equalization between them of the directive forces on the different headings, so that in this manner one can proceed with the compensation of the compass without recourse to external observations. This method, which necessitates only an approximate knowledge of the direction of the magnetic meridian, may therefore by applied on the high seas when the sky is so completely overcast that not a single star is visible.

14. *Historical.* — The principle of the use of the deflector was established about 1866 by Edward SABINE, later improved and made of practical service by Sir William THOMPSON in 1878, who produced a deflector arranged in such a manner that one can vary its action f on the compass card for the same angle of position. The formula $f = \dfrac{\mathfrak{M}}{R^3} = \dfrac{m\,l}{R^3}$ shows that one can, in fact, act on the one or other of the two parameters l or R in order to cause a variation in the action on the card of a magnet of a given magnetic mass m, the special magnetic constant of the deflector.

Since then, WAGHORN, COLONGUE, FOURNIER, CLAUSENS, FLORIAN, OGLOBLINSKY, BALLVÉ and the CARNEGIE INSTITUTION of Washington have developed different types of deflectors by utilizing one or the other of the above formulae (α), (β), (γ), depending upon the case.

15. *Different Types of Deflectors.* — 1°) The simplest type of deflector consists of an ordinary magnet mounted on the sighting vanes of the alidade as shown in fig. 13. This arrangement can easily be carried out with the means available aboard ship. The two wooden wedges N and S fitting into the rabbits of the adjusted vanes by means of brass butterfly nuts permit of an easy control of the action f of the magnet by adjusting its distance above the plane of the card. By changing the magnet, the value of $\mathfrak{M}$ can also be altered. This arrangement is not susceptible of great accuracy and applies primarily to the case of measurement by means of " easterly deflection " (para. 6 and formula α).
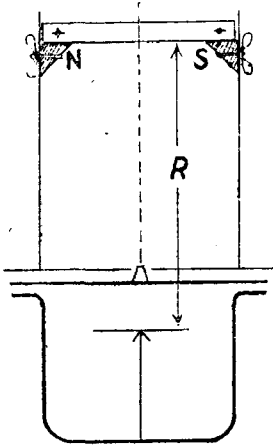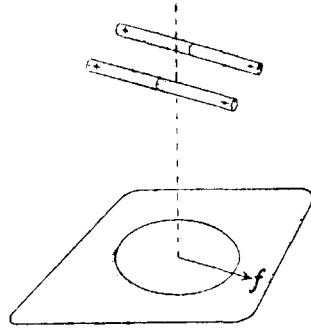

(Fig. 13)　　　　　　　　　　　(Fig. 14)

2°) To render the deflecting action f more uniform over a large region surrounding the centre of the card, it has been proposed to utilise two magnets of equivalent magnetic moment in parallel and tandem as shown in fig. 14. Their combined action is equivalent to that of one single magnet having the same orientation.

3°) Fig. 15 shows the type of deflector in which the height of the magnet R above the plane of the card may be varied by means of a spiral screw V. A pointer which moves over a graduated scale permits the accurate determination of position of the magnet in altitude and consequently the value of its action $f = \dfrac{\mathfrak{M}}{R^3}$.

The successive actions $f_1, f_2 ....$ etc. corresponding to successive divisions on the scale are in the inverse ratio of the cubes of the distances $\dfrac{1}{R_1^3}, \dfrac{1}{R_2^3} \cdots$ etc. and the scale is divided sometimes according to this law, and sometimes into equally spaced graduations, the latter being useful when it is simply desired to check the position of the magnet with regard to its height above the compass card.

4°) A deflector somewhat analogous to the above consists of two magnets in tandem (as shown in fig. 16) both of which move up and down a graduated vertical column by means of a rack and pinion C. This arran-
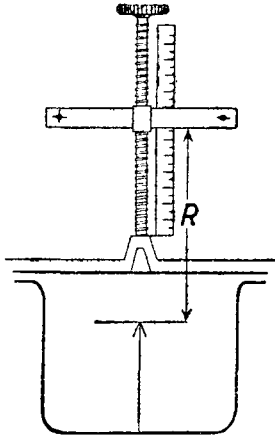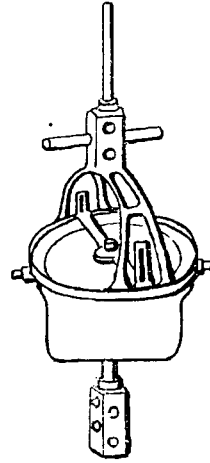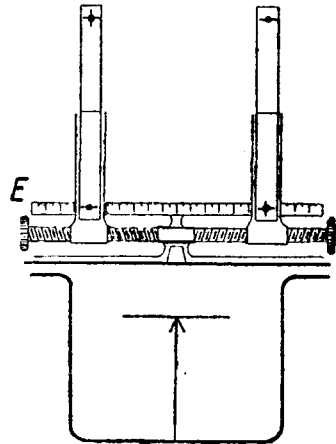

(Fig. 15) (Fig. 15 bis) *Carnegie Sea Deflector.*

gement lends itself readily to the direct evaluation of the directive force $H = \dfrac{\mathfrak{M}}{R^3}$ by means of formula (γ) and of direct readings of the scale; provided the magnetic moment of the group of deflecting magnets remains unchanged and provided also that the plane of the compass card corresponds exactly with the origin of the graduations.
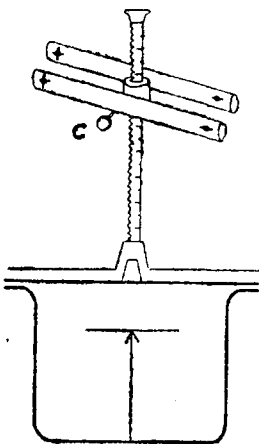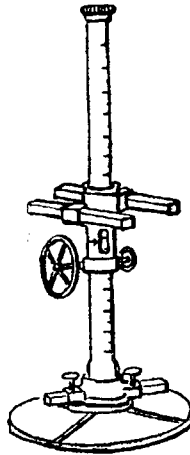
(Fig. 16) *Déflecteur Ogloblinsky.* (Fig. 17)

5°) Another form of deflector consists in an arrangement whereby the active poles of two vertical magnets are situated on the same diameter above the compass card. (Fig. 17). These magnets are contained in two vertical sleeves mounted on the same worm shaft with oppositely turned

threads in such a manner that the intercalary distance separating the two ·
poles acting on the compass card may be varied at will by rotating this
shaft. This distance, l may be read off from the position of the pointer
moving over the graduated scale EE. The action of the deflector on the
compass card is proportional to $f = \dfrac{m\,l}{R^3}$. In this case, l is the variable
quantity which permits the action of the deflector to be controlled. In fact,
it is the four poles of the two deflecting magnets which act upon the card.
Their mean action is proportional to the quantity $K \left[\dfrac{1}{R^3} - \dfrac{1}{(R + \lambda)^3} \right]$
in which R equals the distance of the nearest poles from the compass card
and λ the vertical polar distance common to each of the magnets.
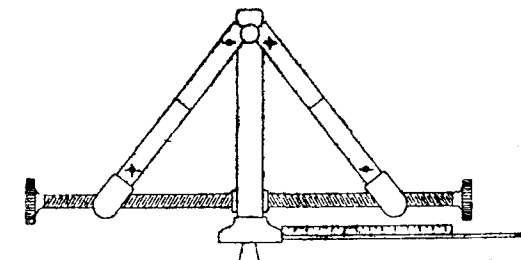
Practically $R \geqslant \dfrac{\lambda}{2}$ so that the mean action is proportional to

$$K \left[\frac{1}{R^3} - \frac{1}{3,\,4\ R^3} \right] \text{i.e., approximately : } 0.73\ K\ \frac{1}{R^3},$$

equivalent to that of a somewhat weaker magnet $0.73\ K$ placed at the same
distance R, or of the same magnet placed at a greater distance $1.32\ R$.
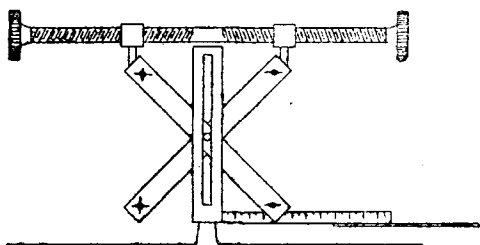
(Fig. 18).   (Fig. 19).

6°)   The Thomson (Lord Kelvin) deflector is composed of four crossed
magnets arranged two by two in tandem (Fig. 18) and operates on an
analogous principle, the variable distance of the nearest most active poles
being measured along a graduated scale. Each time the pointer returns to
the same division on the scale, the action of the deflector resumes its
identical value. In addition, a micrometer drum gives the divisions in tenths.

7°)   Fig. 19 shows a variation of the Thomson deflector (new type).
The two poles of the juncture are practically without effect on the compass
card; the scale allows the separation of the two others to be read off pro-
portionally to the deflecting force.

If the angle made by the positive directions of the two magnets is
designated as α (fig. 20) and f is the normal effect of each of the magnets
on the pole ($+ 1$) placed at the centre of the card, then the mean action F
of the two magnets is given approximately by the following expression :

$$F = 2\,f \cos \frac{\alpha}{2}.$$

Let e represent the reading of the scale, and we have $e = \lambda \cos \dfrac{\alpha}{2}$.

Therefore $F = 2 f \dfrac{1}{\lambda} \times e$ (approximately).

(Fig. 20).
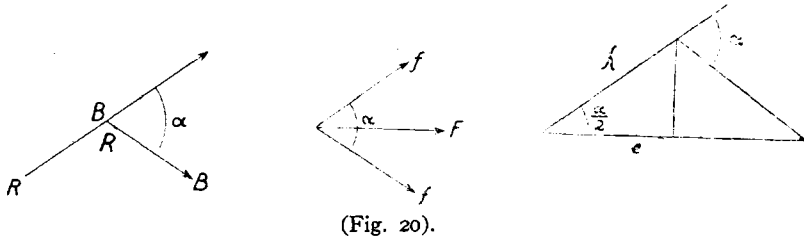
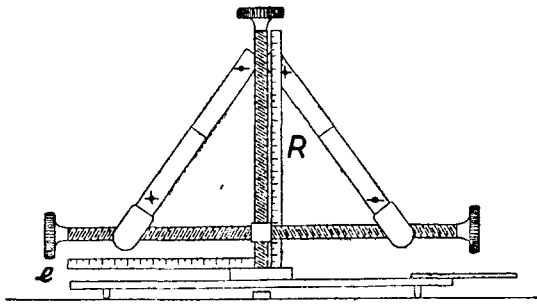8°) The deflector of this type which is shown in fig. 21 possesses the dual quality of having the distance separating the active poles made variable and also of being raised to a variable height above the plane of the compass card, so that the effect $f = \dfrac{m\,l}{R^3}$ may be regulated by means of the two parameters l and R, to each of which corresponds a scale and pointer.

(Fig. 21).

All these arrangements permit a variation in the action of the deflector on the compass needles for one and the same angle of position of the deflector.

Note 1 : The worm shaft with which a certain number of deflectors are provided permits an adjustment of their magnetic strength by means of the variation, within limits, of its distance above the plane of the compass card (fig. 22).

(Fig. 22).

(Fig. 23)

Note 2 : The strength of the magnets may also be varied by the addition of small lateral supplementary magnets, as in the Thomson deflector. These are disposed as need be in the same direction as the principal magnets, if one desires to augment their strength and with oppositely placed poles in the contrary sense. They may also be used in pairs or singly (fig. 23).

II

THE USE OF DEFLECTORS.

16. *Principal Properties.* — The use of the deflector is based on the several properties hereinafter described which were formulated by Sir William Thomson in the Journal of the Royal United Service Institution, N° XXXII, 1878, page 103.

We give below their proof in accordance with an explanation given on 28 October 1889 by Commander Guyou of the Instrument Section of the French Hydrographic Service.

Theorem I. — *If the directive force acting on the compass needles is constant on all headings, the compass has no deviation or else has a constant deviation.*

In this case we see that all the points on the elliptic dygogram, showing the combined forces F_2 (constant in the vessel) F_1 (fix on the horizon) F_3 (turning) (fig. 24) are at an equal distance from the point O, which necessitates $F_3 = O$; that means, employing the well-known notations : $\mathcal{D} = \mathcal{E} = O$. $F_2 = O$, since O is at the centre, that is $\mathcal{B} = O$, $\mathcal{C} = O$. If, in addition $\mathcal{A} = O$, as in the case in practice in general, then the compass has no deviation.

(Fig. 24).   (Fig. 25).

THEOREM II. — *If the directive force is the same on five different headings, it will remain the same on all headings.*

In this case the elliptical dygogram will have five points at the same distance from O, and as the ellipse cannot be intersected by the circle in more than four points, the dygogram will reduce to a circle of which the point O will be the centre.

THEOREM III. — *The compass having been partially corrected in such a manner that the deviations do not exceed 10°* :

1°) *if the directive forces on two opposite headings are equal, the semi-circular deviation will be zero on two headings in quadrature with the preceding.*

2°) *if they are unequal, there exists a semi-circular deviation on two headings in quadrature, of which its ratio to 53°7 is equal to the ratio of the difference of the forces to their sum* (fig. 25).

By projecting the 6 components $1, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}$, on F and the perpendiculars to F (fig. 25), we have :

$$\begin{cases} F = \cos\delta + \mathcal{A}\sin\delta + \mathcal{B}\cos\zeta' - \mathcal{C}\sin\zeta' + \mathcal{D}\cos(2\zeta'+\delta) - \mathcal{E}\sin(2\zeta'+\delta) \\ \sin\delta = \mathcal{A}\cos\delta + \mathcal{B}\sin\zeta' + \mathcal{C}\cos\zeta' + \mathcal{D}\sin(2\zeta'+\delta) - \mathcal{E}\cos(2\zeta'+\delta) \end{cases}$$

If $\delta \leqslant 10°$ and if $\mathcal{A}$ is negligeable, then :

$$\begin{cases} F = 1 + \mathcal{B}\cos\zeta' - \mathcal{C}\sin\zeta' + \mathcal{D}\cos 2\zeta' - \mathcal{E}\sin 2\zeta' \\ \sin\delta = \mathcal{B}\sin\zeta' + \mathcal{C}\cos\zeta' + \mathcal{D}\sin 2\zeta' + \mathcal{E}\cos 2\zeta' \end{cases}$$

Let us have the values of the directive forces on two opposite headings ζ' and $\zeta' + 180°$

$$F_1 = 1 + \mathcal{B}\cos\zeta_{,} - \mathcal{C}\sin\zeta_{,} + \mathcal{D}\cos 2\zeta_{,} - \mathcal{E}\sin 2\zeta_{,}$$
$$F_2 = 1 - \mathcal{B}\cos\zeta_{,} + \mathcal{C}\sin\zeta_{,} + \mathcal{D}\cos 2\zeta_{,} - \mathcal{E}\sin 2\zeta_{,}$$
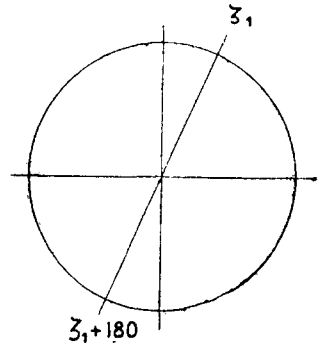
consequently :

$$\frac{F_1 - F_2}{F_1 + F_2} = \frac{\mathcal{B}\cos\zeta_{,} - \mathcal{C}\sin\zeta_{,}}{1 + \mathcal{D}\cos 2\zeta_{,} - \mathcal{E}\sin 2\zeta_{,}} = \mathcal{B}\cos\zeta_{,} - \mathcal{C}\sin\zeta_{,}$$

since the complementary terms in the denominator may be neglected in relation to 1. On the other hand, on heading $\zeta_{,} + 90°$, the semi-circular part of the deviation expressed in degrees is :

$$\frac{\delta}{53,7} = \mathcal{B}\cos\zeta_{,} - \mathcal{C}\sin\zeta_{,}$$

which proves the proposition (fig. 26).

(Fig. 26).

THEOREM *IV*. — *If* F_1, F_2, F_3, F_4 *are the directive forces on 4 consecutive headings equidistant by* $90°$, *and* δ *is the quadrantal part of the deviation on headings* $45°$ *from the preceding, we have :*

$$\frac{\delta}{53,7} = \frac{(F_1 + F_3) - (F_2 + F_4)}{F_1 + F_2 + F_3 + F_4}.$$

Let us write the values of the directive force on headings ζ, $\zeta + 90$, $\zeta + 180$ and $\zeta + 270$

$$F_1 = 1 + \mathcal{B} \cos \zeta - \mathcal{C} \sin \zeta + \mathcal{D} \cos 2\zeta - \mathcal{E} \sin 2\zeta$$
$$F_2 = 1 - \mathcal{B} \sin \zeta + \mathcal{C} \cos \zeta - \mathcal{D} \cos 2\zeta + \mathcal{E} \sin 2\zeta$$
$$F_3 = 1 - \mathcal{B} \cos \zeta + \mathcal{C} \sin \zeta + \mathcal{D} \cos 2\zeta - \mathcal{E} \sin 2\zeta$$
$$F_4 = 1 + \mathcal{B} \sin \zeta - \mathcal{C} \cos \zeta - \mathcal{D} \cos 2\zeta + \mathcal{E} \sin 2\zeta$$

consequently :

$$\frac{(F_1 + F_3) - (F_2 + F_4)}{F_1 + F_2 + F_3 + F_4} = \frac{\mathcal{D} \cos 2\zeta - \mathcal{E} \sin 2\zeta}{1}.$$

On the other hand, on heading $\zeta + 45°$, the quadrantal part of the deviation is expressed by

$$\frac{\delta}{53°7} = \mathcal{D} \cos 2\zeta - \mathcal{E} \sin 2\zeta$$

which proves the proposition.

17. *Comparison of two or more magnetic fields with the aid of the deflector*. — This operation is performed in practice when it is desired to evaluate the respective directive forces acting upon the compass, when the compass is taken ashore and when it is aboard ship; or on different headings aboard ship. For this purpose the deflector is placed in each one of the fields in such a manner that it may act upon the needles of the compass card; that is, that it may act on the magnetic system with the same " angle of position " in all the fields which it is desired to compare. (See above N° 11).

For the equilibrium deflection, if M_p is the moment produced by the deflector on the card for a constant position angle P and ρ the magnetic moment of the card, then :

$$M_p = \rho \, H \sin \delta = \rho \, H' \sin \delta' = \rho \, H'' \sin \delta'' \dots \text{. etc.}$$

from which : $\dfrac{H}{H'} = \dfrac{\sin \delta'}{\sin \delta}$ ..... etc.

or $\qquad H' = H \sin \delta \times \dfrac{1}{\sin \delta'}$

$$H'' = H \sin \delta \times \frac{1}{\sin \delta''} \dots \text{. etc.}$$

This formula $H' = \dfrac{K}{\sin \delta'}$ lends itself to the plotting of the elliptical dygogram of the forces acting on the compass on the different headings and in consequence, to its compensation. It will suffice to utilise for the graphic plotting of the dygogram a table of cosecants of the angles δ from $1°$ to $90°$, taking as the scale of construction $K = 1$. If we desire to know the exact values of the fields H', H'' etc., we may determine the value of $K = H \sin \delta$ ashore at a place where the exact value of the horizontal component H is known.

The apparatus which is used here to determine the elements of the compensation is composed, as in the case of the method of cotangents (para. 6) of a simple alidade to which the deflecting magnet is securely attached.

Note. — In order to attenuate the influence of the errors of observation, it is advisable to choose the value of P as close as possible to $90°$ (say $78° 45'$ for the E. by N. for instance, or roughly $80°$) because this renders the action of the deflector a maximum and in this position a small error in the position P is without practical importance.

Plotting of the dygogram by the Method of Oscillations. — The period of oscillation of a needle or a system of needles is given by the formula :

$$T = 2\pi \sqrt{\dfrac{I}{H. M}}$$

in which I represents the moment of inertia of the needle or the system of needles with respect to the pivot, M the magnetic moment of the system and H the horizontal component of the terrestrial magnetic force. For any magnetic heading whatever, we have :

$$T' = 2\pi \sqrt{\dfrac{I}{H' M}}$$

from whence : $\dfrac{T^2}{T'^2} = \dfrac{H'}{H}$ ou $H' = \dfrac{1}{T'^2} \times T^2 H$.

This formula permits the plotting of the polar diagram of H' as a function of the periods of oscillation T' on the various headings. For this operation one may conveniently make use of a small oscillating needle mounted in a box-wood case.

18. *Handling of the Deflector.* — The effect of the deflector is easy to understand : the card is made to move in the same direction as the displacement of the deflector, attracted by the blue pole. This attraction reaches a maximum when the direction of the " pointer " is perpendicular to the line N-S of the deviated card. (Fig. 27 and fig. 12 above) but the more the North of the card is deviated from the magnetic meridian, the more the terrestrial force tends to pull it back. If then we continue to turn slowly the index pointer of the deflector by maintaining it always in the immediate vicinity of the East of the deviated compass card, and if we

regulate as need be the strength of the deflector by acting on the worm shaft on which the magnets are mounted, it is realised that the card, reaching a point 90° from its initial position, will remain in equilibrium between the attraction of the terrestrial force H and that of the deflecting magnets $\dfrac{\mathfrak{M}}{R^3}$ which, in such position, is almost diametrically opposed to it (fig. 28).

$$H = \frac{\mathfrak{M}}{R^3}$$

19. *Normal Deflection.* — This special geometrical disposition in which we have $\delta = 90°$ and P·very close to 90° (an arrangement which is essential for the realisation of the rational use of the deflector) is called the " normal deflection ".
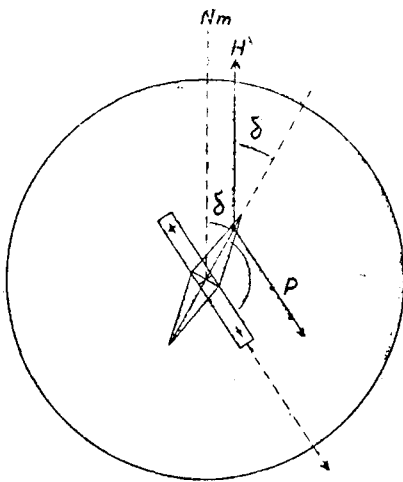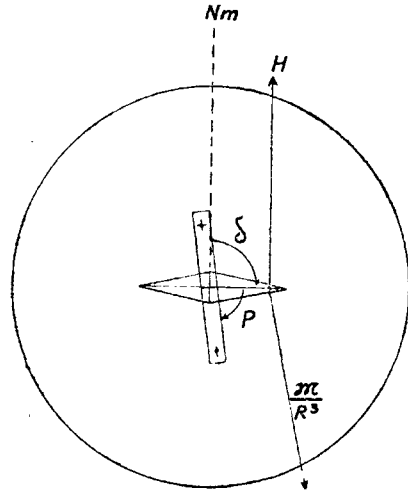
(Fig. 27)   (Fig. 28).

Note. — Instead of putting the index pointer on the East of the deviated card, it is recommended to place it finally above the E by N or at the graduation 80° for the following reason : with the adopted deflection of 90°, the deflector pointing towards the deviated East will be exactly in opposition with the terrestrial force F, if is action were not exactly equal to the latter, the compass card will tend to turn either in one direction or the other, and it will be very difficult, if not impossible, for it to attain a position of stable equilibrium; such is not the case when the two forces which are acting form an angle slightly less than 180°, because in such a case, they give rise to a resultant f, small as it may be, and situated in the interior of that angle (as shown in fig. 29).

In certain deflectors where the magnets are oriented North-South magnetic and where the distance R is variable, the action $\dfrac{\mathfrak{M}}{R^3}$ is always directly opposed to H; and when H is equal to $\dfrac{\mathfrak{M}}{R^3}$ the needles will be indifferent. Thus opposed to H; and when H is equal to the needles will be indifferent. Thus
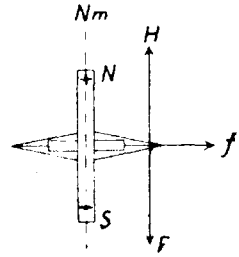
care should be taken to provide such deflectors (of the new N. Ogloblinsky type 1922) with a small transverse auxiliary magnet, the effect f of which is to stabilise the compass card when the two forces H and $\dfrac{\partial \mathfrak{M}}{R^3}$ equalize (fig. 30 and 30 a).
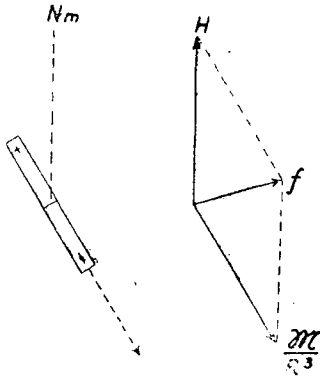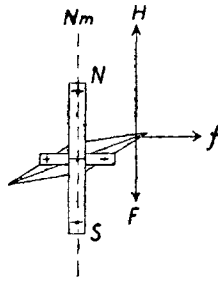
(Fig. 29).   (Fig. 30).   (Fig. 30 bis).

We have seen, on the other hand, that the deflection of 90° was chosen because it reduced the influence of the unavoidable errors of observation to a minimum. Supposing that the measured deflection equals 87° instead of 90°, and that the vessel yaws through 2 or 3 degrees, for instance, during the several minutes necessary for the observation, the measurement of the directive force will not be adversely effected by a relative error of more than about 1/2 of one hundredth, which corresponds to an error of about 1/3 degree in angular measure. This accuracy is scarcely less than that obtainable with the method of bearings.

20. *Preliminary Exercises.* — In order to handle the deflector properly and to master it completely, it is well to practice the manipulation of the instrument before hand, either aboard ship or preferably ashore, by taking the compass bowl and the compass card to some locality free from the presence of iron.

One first seeks to maintain the equilibrium of the compass card in the position of " normal deflection ". This operation, which is rather delicate for the dry compass, becomes much easier with the liquid compass in which the oscillations are strongly damped and which is less sensitive to the accidental magnetic impulses which may be communicated to it.

Having transported the compass bowl ashore, the lubbers line is then oriented in the magnetic meridian, as indicated in fig. 31.

Without touching the bowl the deflector is then placed well-centered with its spring lock engaged in the cavity prepared for it in the centre of the glass over the compass card, the " pointer " of the deflector being placed on the North of the card. Then open the pointer progressively to the right

or the left of the magnetic meridian sufficiently slowly so that no untimely acceleration is communicated to the compass card and that the position of equilibrium remains relatively regular (fig. 32).
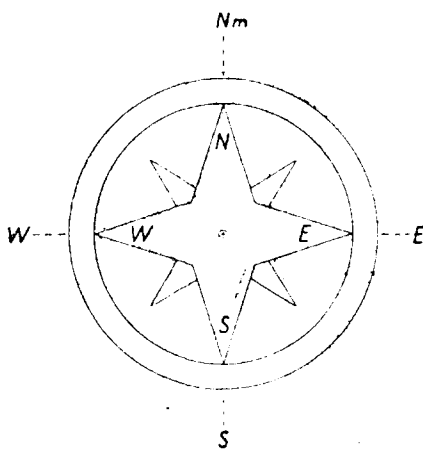

(Fig. 31)
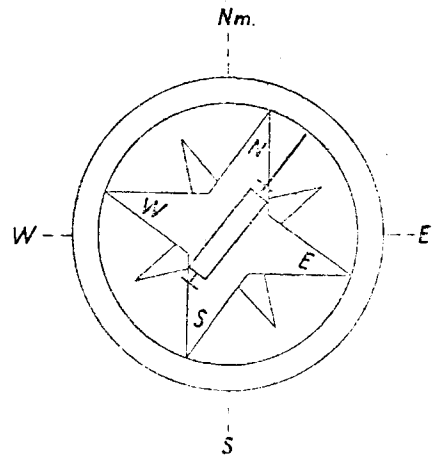
(Fig. 32)

In this movement, the North of the compass card follows the pointer and maintains itself approximately in the angle formed by the direction of the pointer and that of the magnetic North. One then adjusts the spiral of the deflector towards the centre of its travel, and if necessary, one adds the small lateral supplementary magnets (in the case of the new Thomson deflector) in order to impart to the north of the compass card a deflection (deviation) of exactly 90° from the North magnetic, while the pointer itself will be over the division 80° or 280° of the compass card (fig. 33).
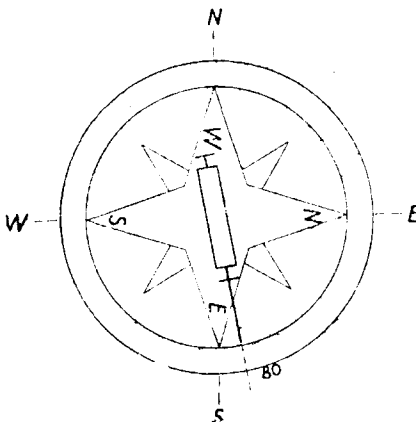
(Fig. 33)
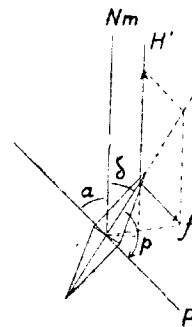
(Fig. 33 bis).

It is well to try to obtain the " normal deflection " ashore two or three times in order to become accustomed to the manipulation of the deflector. One begins the operations with the terrestrial field alone at first and then

one causes perturbations in the terrestrial field, for instance by means of magnets placed at random in the vicinity of the bowl. We then pass on to the manipulations aboard ship.

With a little dexterity one can determine in less than two minutes the position of the deflector which will produce the normal deflection. The compass card can then be restored to its initial position in a few seconds. Two consecutive observations made under the same conditions should give rise to the same reading to within 0.2 or 0.3 turns (or divisions), if the pivot is in good condition and if check has been made to see that the card returns to its point of departure (ashore) to within about 1/4 degree. If the two readings differ by more than 1/2 turn, the reading is poor or the pivot defective.

The only difficult point in the manipulation of the apparatus is to know how to obtain rapidly and accurately the position of normal equilibrium. In the vicinity of this position, if the compass card regains its original movement of rotation, from the North towards the East, it is because the deflector is too strong and one must then reduce the separation of the magnets. If, on the other hand, it tends to move backwards then the magnets are too weak and it is necessary to increase the separation of the active poles. Finally, if the compass card remains perfectly steady and the deviation obtained is 90° to within 2 or 3 degrees, the separation of the magnets is about right and the corresponding reading should be noted on the graduated micrometer or scale. It is clearly evident that if the compass card remains in equilibrium with a deflection of less than 86 or 87°, there is need to augment the effect of the deflector. When it approaches its position of definite equilibrium, it is necessary to manoeuvre the worm one turn at a time or even by half or quarter turns and to tap lightly on the bowl to overcome the inertia of the compass card.

Once the operation has been completed, and before beginning a new observation, one must allow the compass card to return to its initial position to make certain that the heading has not changed. For this it will suffice to place the pointer on the North of the deviated compass card and thereupon to manoeuvre it to oppose the oscillations of the magnetic needle to the extent that the initial heading approaches the lubbers line. Once the compass card has been steadied on the initial heading, the deflector is suddenly removed by picking it up vertically without turning and placing it several metres distant from the compass.

With the dry compass having a 10 inch card, the strength of the deflector of the Thompson type is sufficient for the quarter in which one is working, when one obtains a deviation of 90° ashore with a reading comprised between 12 and 16 on the deflector scale.

Determination of the coefficients of the deviation formula with the

aid of the Deflector. — The general formula of the deflector giving the equilibrium of the deviation needle, is (fig. 33 a):

$$H' \sin \delta = f \cos p$$

This is written :

$H' \sin \delta = f \sin (a + \delta)$ when the position of the deflector is referred to the magnetic meridian.

$$H' = f \frac{\sin (a + \delta)}{\sin \delta} = f (\cos a + \sin a \cot g \, \delta).$$

If we choose for the fixed position of the deflector on all headings the value of $a = 45°$ the formula reduces to :

$$H' = 0.707 \, f \, (1 + \cot g \, \delta) = K \, (1 + \cot g \, \delta).$$

Let us put $W = 1 + \cot \delta$ and compile the table of value of W as a function of δ (*)

On the other hand, we know that

$$H'_N = \lambda H (1 + B' + D') \qquad \lambda H = 1/4 \, \Sigma \, H'$$

$$H'_S = \lambda H (1 - B' + D') \qquad B' = \frac{2}{\Sigma \, H'} (H'_N - H'_S)$$

$$H'_W = \lambda H (1 + C' - D') \qquad C' = \frac{2}{\Sigma \, H'} (H'_W - H'_E)$$

$$H'_E = \lambda H (1 - C' - D') \qquad D' = \frac{1}{\Sigma \, H'} \left[(H'_N + H'_S) - (H'_W + H'_E) \right]$$

(consequently)

The measurements on four headings with the deflector furnish respec-tively :

$$H' = K W_N \qquad \lambda H = 1/4 \, \Sigma \, W$$

$$H'_S = K W_S \qquad \sin B^0 = \frac{2}{\Sigma \, W} (W_N - W_S)$$

and (consequently)

$$H'_W = K W_W \qquad \sin C^0 = \frac{2}{\Sigma \, W} (W_W - W_E)$$

$$H'_E = K W_E \qquad \sin D^0 = \frac{1}{\Sigma \, W} \left[(W_N + W_S) - (W_W + W_E) \right]$$

If, on the other hand H is known, we deduce λ therefrom.

The strength of the deflector is then regulated to bring the deflection closer to the normal $\delta = 90°$. A and E are assumed to be known.

(*) Table IV, page 169, of the Manual of Comdr. Gabriel MALLEVILLE " El Compas Magnetico ", Servicio Hidrografico, Buenos Aires, 1931. This table gives W to 4 decimals for the values of δ comprised between 73° and 107°.

21. *The Practice of Compass Compensation with the aid of a Deflector.* — The first attempt is to equalize the directive force on all headings (for this 5 will suffice) by means of measurements effected with the deflector.

In practice it will suffice to equalise the directive force on the needles on the four cardinal points of the compass, and this will prove adequate when the binnacle is located on the centreline of the vessel, because one can then neglect the oblique quadrantal error arising from the unsymmetrical distribution of the soft iron.

$$\mathcal{A} = 0 \text{ and } \mathcal{C} = 0.$$

The compensation with the aid of the deflector necessitates the use of two compasses, one of which serves as a control while the operations are carried out on the other. The following is one of the methods of procedure.

$1°$ — Put in approximate position the quadrantal correctors, the Flinders and the heel magnet.

$2°$ — *a)* Steer North magnetic with standard compass.

b) Then make a " scrupulous " check on the steering compass.

c) Then deflect the compass under test by $90°$ as explained in paragraph 19 " normal deflection " and note the reading l_1 on the scale of the deflector.

d) Remove the deflector and check to determine if the vessel is still holding the magnetic North.

$3°$ — Carry out the same procedure on East magnetic and measure l_2.

$4°$ — The same procedure on South magnetic and measure l_3. Continue to check the course on South magnetic.

e) The deflector having been removed, place the index of its scale at $\dfrac{l_1 + l_3}{2}$.

f) Introduce or displace the longitudinal magnets in order to obtain the " normal deflection " with the pointer on $80°$ or $280°$. This will nullify the coefficient $\mathcal{B}$.

The red ends of the correcting magnets should be turned towards the point of the vessel which attracts the North of the compass card most strongly, that is, the side exposed to the south on the heading on which the least reading has been obtained; this will be the stern, for example; if the reading on heading South, l_3 is less than the reading on North heading l_1.

$$l_1 > l_3 \text{ — } \text{ red end towards the bow.}$$
$$l_1 < l_3 \text{ — } \text{ red end towards the stern.}$$

$5°$ — Then steer West magnetic, carrying out the same procedure as

above, measure l_4, place the deflector at $\dfrac{l_2 + l_4}{2}$ and introduce or displace the transverse magnets until the " normal deflection " is obtained. This will correct for the coefficient $\mathcal{C}$.

The red pole is always turned towards the point in the vessel corresponding to the strongest directive force, that is, the greatest reading.

$$l_2 > l_4 \quad - \quad \text{red to port.}$$
$$l_2 < l_4 \quad - \quad \text{red to starboard.}$$

Other Rules for Inserting the Magnets :

Heading	Deflector	Place the red end towards
On North	too strong	the stern
	» weak	» bow
East	» strong	starboard
	» weak	port
South	» strong	the bow
	» weak	» stern
West	» strong	port
	» weak	starboard

Note. — If $\mid l_1 - l_3 \mid \geqslant$ 10 divisions, or if $\mid l_2 - l_4 \mid \geqslant$ 10, it is necessary to recommence the swing around the horizon in the same manner after rectifying the position of the longitudinal and transverse magnets.

6°) If $\dfrac{l_1 + l_3}{2} \neq \dfrac{l_2 + l_4}{2}$, there exists a quadrantal deviation $(\mathcal{D})$.

To correct this, continue to check on West magnetic.

g) place the deflector (after the 2nd swing if necessary) at the division $\dfrac{l_1 + l_2 + l_3 + l_4}{4}$.

h) move the quadrantal correctors in or out in order to obtain the " normal deflection ".

When $\dfrac{l_1 + l_3}{2} > \dfrac{l_2 + l_4}{2}$ the spheres should be close together.

When $\dfrac{l_1 + l_3}{2} < \dfrac{l_2 + l_4}{2}$ the spheres should be separated.

7°) If the above mentioned procedure is correctly carried out the deviation should not exceed 1 or 2 degrees.

In order to carry out the procedure with rigorous exactitude, the directive force should be equalised on the exact magnetic headings and not on the compass headings. This is easily accomplished by steering the ship by means of an auxiliary compass, or in the case of external bearings, using the pelorus. In the contrary case, it is well, before undertaking operations with the deflector, to approximate the compensation to about 4 or 5 degrees by means of bearings or comparisons with another compass. This procedure offers the advantage of maintaining the readings of the deflector within the range of the graduated scale. With deviations of the order of 12 to 15°, we should have, in fact, large variations in the intensity of the various directive forces, and all the values could not be measured with the same deflector, in spite of the usual system of control.

If this condition should arise, one should immediately place the proper compensating magnets in place; but the first swing around the horizon will only result in an approximate compensation which must be completed by a second swing.

22. *Graphical Representation.* — The figures given below show what has been successively accomplished in the course of the preceding operations :
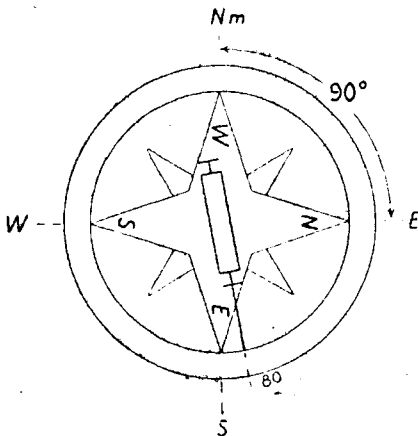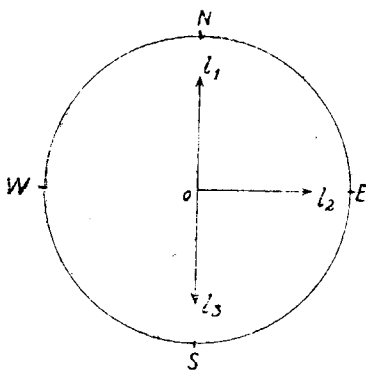

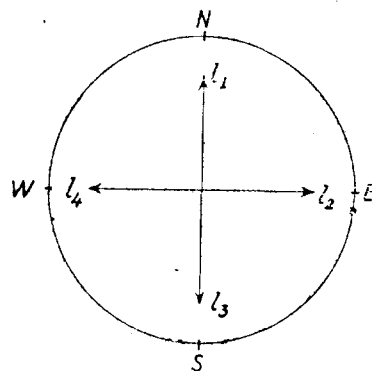
(Fig. 34)
normal deflection of 90°.
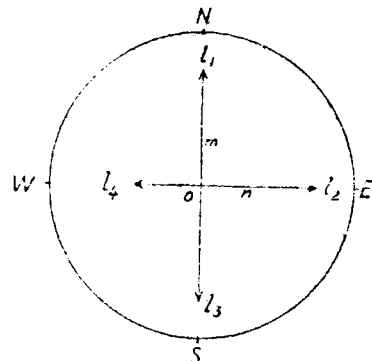
(Fig. 35)
polar diagram of the measured forces

(Fig. 36)

(Fig. 37)

By the introduction of longitudinal magnets, on heading South, one creates the force $B \lambda H$ which, on the polar diagram, refers the middle m of the vector $L_1 L_3$, back to the centre O. This equalises the directive forces on North and South magnetic (Fig. 36).

By the introduction of transverse magnets on heading West, one creates the force $C \lambda H$ which, on the polar diagram, brings back to the centre O the middle n of the vector $L_2 L_4$, which equalises the directive forces on the East and West magnetic headings.

The total forces thus created, as shown in fig. 37, leave the quadrantal deviation still subsisting. The latter is corrected by equalising the lengths $L_1 L_3$ and $L_2 L_4$ of the two axes of the fig. 37, which gives on all headings a common value of the directive force equal to $1/4 K (l_1 + l_2 + l_3 + l_4)$.

We find, furthermore, on page 114 of the 1921 edition of the *Manuel des Instruments Nautiques* (Publication N° 64 of the Service Hydrographique de la Marine, Paris), a verification of the principle of compensation with the aid of the deflector, based on the geometrical representation of the diagram of directive forces on the various headings, or the bicircular dygogram.

23. *Correction to be applied to the readings of the Thompson type deflector.* — In the preceding method, we have assumed that the force of the deflector varies proportionately to the graduations on its scale. This property is not fully verified except within narrow limits and the influence of the lack of proportionality is all the greater the larger the readings, and especially when their differences are large.

The action of the deflector will not be proportional to the separation of the magnets unless the compass card carries a needle which is infinitely small. One cannot assume, further, that it is proportional to the sine of the angle included between the axis of the deflector and the meridian of the compass card; and, besides, one should employ the deflector only in that vicinity and position in which its action is a maximum, so that a small error in the position of the instrument may not be of practical importance.

In order to remedy this source of error inherent in the construction of this instrument, Admiral PERRIN has compiled the following table which gives the correction to be applied to the mean of two readings (on the Thompson type of deflector), in order to obtain the exact reading corresponding to the mean of the directive forces measured. By placing the micrometer on the mean thus obtained, the deflector produces an attraction exactly equal to the mean of the two forces which it is desired to equalize.

TABLE

	CORRECTION TO BE SUBTRACTED FROM THE MEAN OF TWO READINGS					
Difference of readings.	Sum of readings.					
	0 to 20	20 to 25	25 to 30	30 to 35	35 to 40	40 to 45
	turns	turns	turns	turns	turns	turns
2 turns	0,0	0,0	0,0	0,0	0,0	0,0
4 »	0,0	0,1	0,1	0,1	0,1	0,1
6 »	0,1	0,1	0,2	0,2	0,2	0,2
8 »	0,2	0,2	0,3	0,3	0,3	0,4
10 »	0,3	0,4	0,4	0,5	0,6	0,6
11 »	0,4	0,5	0,5	0,6	0,7	0,8
12 »	0,5	0,6	0,6	0,7	0,8	0,9
13 »	0,6	0,7	0,8	0,9	0,9	1,1
14 »	0,7	0,8	0,9	1,0	1,1	1,2
15 »	0,8	0,9	1,0	1,1	1,2	1,4
16 »	0,9	1,0	1,2	1,2	1,4	1,6
17 »	1,0	1,2	1,3	1,4	1,6	1,8
18 »	1,2	1,4	1,5	1,6	1,8	2,0
19 »	1,3	1,5	1,6	1,8	2,0	2,3
20 »	1,5	1,7	1,8	2,0	2,2	2,5

24. *Compilation of the Table.* — A table of this kind might be compiled for any type of deflector and type of compass, by considering the simplified expression for the directive force :

$$F = \frac{2\,m}{R^2} \cos \alpha \quad \text{or} \quad F = f\,(l) \qquad \text{(fig. 38)}.$$
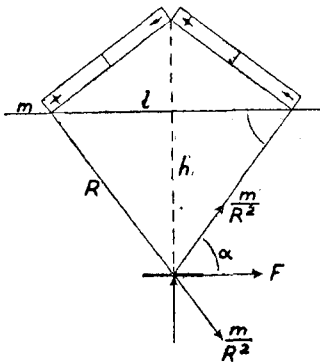
(Fig. 38)
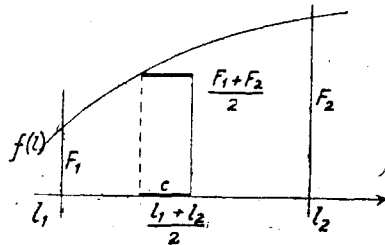
(Fig. 39).

The fig. 39 on which is represented the curve $f(l)$ shows clearly the correction c to be subtracted from the mean of two readings in order to obtain the graduation of the deflector corresponding to the mean of two successive forces F_1 and F_2.

The table of corrections α may also be established experimentally by measuring the deflections at a fixed point ashore.

25. *Numerical Example.* — Conforming to the practical procedure shown above in N° 21, the following measurements were made with the deflector :

Heading North $l_1 =$ 36.5 turns

»   East $l_2 =$ 32.7   »

»   South $l_3 =$ 15.2   »

$|l_1 - l_3| =$ 21.3 turns $>$ being greater than 15 requires another swing around the horizon.

$l_1 + l_3 =$ 51.7 turns

Place the deflector at the division 1/2 i.e. 25.8 turns. Then insert the longitudinal magnets (red forward, because $l_1 > l_3$) in order to obtain the normal deflection at this graduation of the deflector.

Heading West $l_4 =$ 16.9 turns

$|l_2 - l_4| =$ 15.8   » $>$ 10, also requires another swing around the horizon.

$l_2 + l_4 =$ 49.6   »

Let us place the deflector at the half i.e.: 24.8 turns. Then insert the transverse magnets (red to port because $l_2 > l_4$) to obtain the normal deflection at this graduation of the deflector.

A second swing around the horizon is made for a more accurate correction, this time with the mean readings :

Heading North $l_1 =$ 12.6 turns

»   East $l_2 =$ 8.2   »

»   South $l_3 =$ 21.2   »

$|l_1 - l_3| =$ 8.6 turns $<$ 10

$l_1 + l_3 =$ 33.8   »

$1/2 =$ 16.9   »

We place the deflector at the 1/2 correction $=$ 16.9 turns. Then the longitudinal magnets are readjusted (red towards stern because $l_1 > l_3$) in order to obtain the normal deflection.

Heading West $l_4 =$ 13.6 turns

$|l_2 - l_4| =$ 5.4 turns

$l_2 + l_4 =$ 21.8   »

$1/2 =$ 10.9   »

We place the deflector on one half equal 10.9 turns. Readjust then the transverse magnets (red to starboard because $l_2 < l_4$) to obtain the normal deflection at this graduation.

Since $\dfrac{l_1 + l_3}{2} = 16,6 \neq \dfrac{l_2 + l_4}{2} = 10.8,$

there remains a quadrantal deviation. We place the deflector on the corrected graduation 13.5 turns. The spheres were then moved in as far as possible to touch the binnacle without the deflection 90° being obtained. These spheres having thus shown themselves to be too weak, spheres of larger diameter were used which, after several trials, were located at 0.149 m. from the centre of the card. The compensation was then considered completed.

$$
\begin{array}{lcr}
 & & 16,6 \\
 & & 10,8 \\
\hline
\text{diff.} & = & 5,8 \\
\text{som.} & = & 27,4 \\
1/2 & = & 13,7 \\
1/2 \text{ corr.} & = & 13,5
\end{array}
$$

26. *Strength of the Deflector.* — The Thompson type of deflector was developed for the Thompson steering compass in which the slight magnetic moments are of the order of 280 cgs for the 10-inch compass cards (252 mm.) with eight needles. In order to utilise it with the Thompson compass for bearings, it is necessary to unscrew the pivot of the alidade and to replace it by a centering device.

This deflector is not sufficiently strong to be utilised with the modern liquid compasses having large magnetic moments much greater than those of the Thompson dry compass, in the vicinity of 2000 cgs for the Doignon cards of 0.20 m, for instance. We may note, however, that it is not necessary to attain the deflection of 90°. By limiting the deflection to 50°, we see on fig. 40 :
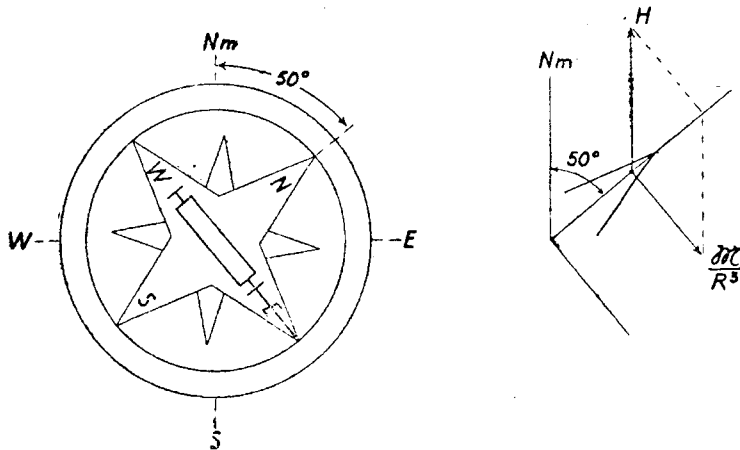
(Fig. 40)

This is :

$$ H \sin 50° = \frac{\mathfrak{M}}{R_3} $$ which permits H to be evaluated by :

$$ H = \left(\frac{2\,\mathfrak{M}}{R^3} \text{ cosec. } 50° \right) \times 1 $$

as a function of *l,* provided the constant deflection of 50° is maintained on all headings while the measurements are being taken.

In the operations for obtaining these measurements, it is essential to adjust the deflector in such a manner that all the forces to be measured are comprised within the length of the graduated scale. For this purpose one may either adjust the worm shaft of the deflector or add the supplementary magnets which are discussed in paragraph 15, note 2, above.

27. *Standardization ashore. - Absolute measurement of the directive forces.* — If we wished to employ the deflector for the absolute measure of the directive forces, it is necessary to graduate it experimentally or to standardise the scale ashore. We measure the deviations φ which it produces for the various separations when it is pointed towards the East or the West of the compass card, in a locality where the horizontal intensity of the field is known, and one then enters in the table the values of $H \sin \varphi$ with respect to the corresponding graduations. If, thereafter, in another locality we observe another deviation φ' under the same conditions, we shall have :

whence : $H \sin \varphi = H' \sin \varphi'$ d'où $H' = H \dfrac{\sin \varphi}{\sin \varphi'}$

28. *More accurate formula.* — To take into account the dimensions of the deflector magnet and the compass needles, we must substitute for the formula $H = F = \dfrac{\mathfrak{M}}{R^3}$, the following more accurate formula which is valid as long as the deflection is in the vicinity of 90° :

$$H = F = \frac{\mathfrak{M}}{R^3} \left(1 + \frac{a}{R^2} \right)$$

in which a is a constant, characteristic of the apparatus. The vertical scales of the deflectors measuring directly the directive force *H,* should be established in accordance with the law $\dfrac{1}{R^3} \left(1 + \dfrac{a}{R^2} \right)$ so that the index of the directive force *H* may be obtained by a simple reading (as in the case of Ogloblinsky's Deflector, type 1922).

29. *Generalisation.* — Both in para. 3 and para. 7 we have considered only the normal positions of the magnets of the deflector with respect to the radius, which from the centre of the card terminates at its centre. Let us consider next a deflector arranged obliquely on the said radius so that it makes the angle φ with the direction red-blue. (fig. 41).

We know that the field of the magnet, if the latter is sufficiently small, acting on the pole (+ 1), may be resolved along the central radius and along a parallel to the direction red-blue of the magnet, each force having the expression respectively :
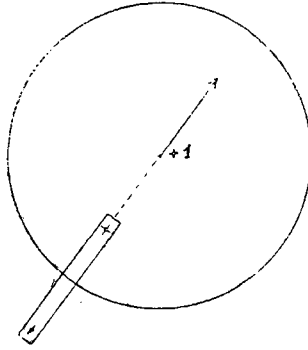
$$G = \frac{\mathfrak{M}}{R^3} \quad \text{and} \, K = 3 \, \frac{\mathfrak{M}}{R^3} \cos \alpha.$$

If $\alpha = 0°$, the repulsive force is $\dfrac{2\ \mathfrak{M}}{R^3}$ (fig. 42).
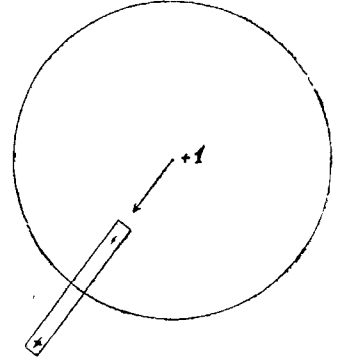
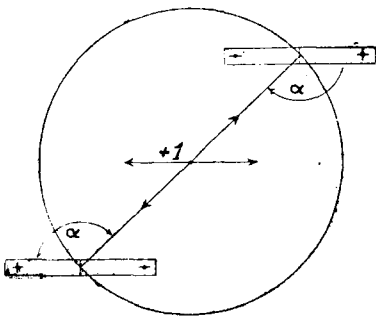If $\alpha = 180°$, the attractive force is also $\dfrac{2\ \mathfrak{M}}{R^3}$ (fig. 43).
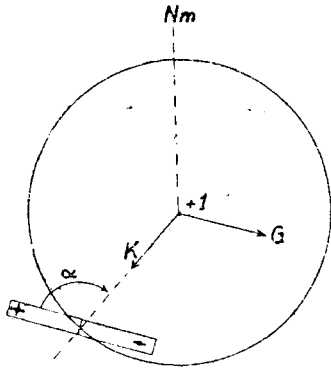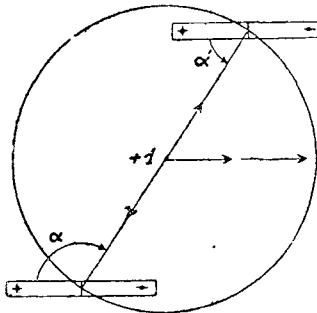

(Fig. 41)   (Fig. 42)   (Fig. 43)
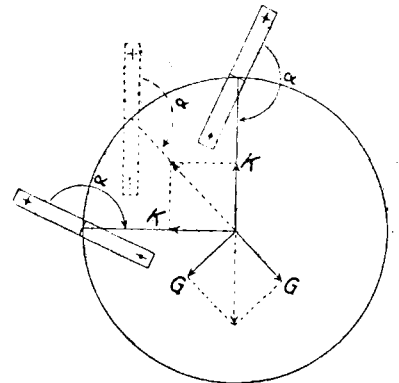

(Fig. 44)   (Fig. 45)   (Fig. 46)

If $\alpha = 90°$, $K = O$. The only active force $G = \dfrac{\mathfrak{M}}{R^3}$ is the force parallel to the magnet which we have considered in all the preceding applications.

30. *Case of two magnets :—*

1°) Two identical magnets, diametrically opposed and in the same positions α, create, at the centre, the component forces G and K which cancel each other (fig. 44).

2°) Two identical magnets diametrically opposed and in supplementary positions α and α', create, in the centre, the force 2 G in their direction red-blue. The two other components K mutually cancel (fig. 45).

3°) Two identical magnets arranged in quadrature with relation to the centre and having the same positions α, are equivalent to a single intermediate magnet in the same position α, placed at $45°$ to each of them and and having the magnetic moment equal to $\mathfrak{M} \sqrt{2}$ (fig. 46).

4°) Two magnets in line, as shown in Fig. 47, act parallel to their common direction red-blue, with an intensity $F = 2 \dfrac{\mathfrak{M}}{R^3} (1 - 3 \cos 2 \alpha)$.
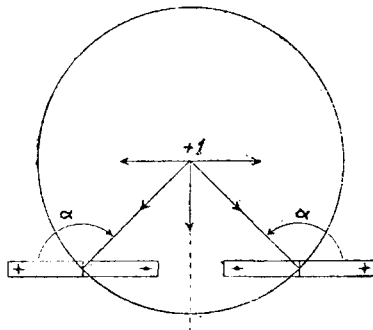
5°) Two magnets in line as shown in Fig. 48, act perpendicular to their common direction when their poles are inverted and with an intensity

$$F = 2 \times \frac{3 \mathfrak{M}}{R^3} \cos \alpha \sin \alpha = 3 \frac{\mathfrak{M}}{R^3} \sin 2 \alpha.$$

The direction of this force is determined by the common colour of the poles closest together.
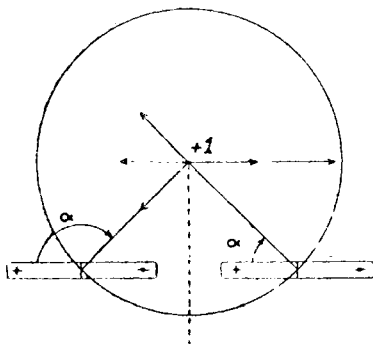

(Fig. 47)   (Fig. 48)

In a more general manner, the deflecting magnet cannot be contained in the horizontal plane of the compass card while having its centre there (fig. 49). Its deflecting action on the positive pole $(+ 1)$ will then have as components the radial component K contained in the horizon and the horizontal component h obtained by projecting G on the horizontal plane of the compass card: the other component v being destroyed by the weight on the pivot.
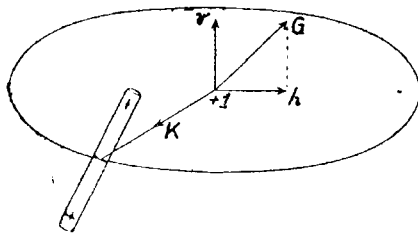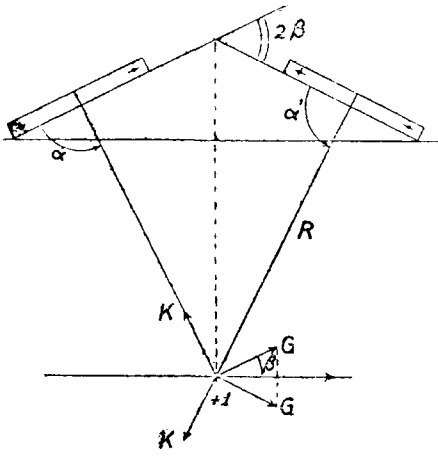

(Fig. 49)

But, in order to obtain a practical arrangement, one must renounce the idea of retaining the magnets in the plane of the compass card; since for the convenience of the operations, it is preferable to place them above the card.

The action of each magnet always remains in the plane defined by the centre of the card; (the latter being supposed to be infinitely small) and by the magnet. Their actions on the positive pole ($+ 1$) are geometrically combined.
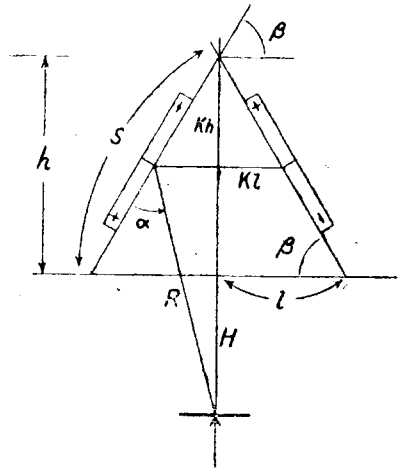
Thus a deflector of the Thompson type placed above the card (**fig. 50**) will have a horizontal action on same :

$$F = 2 \frac{\mathfrak{M}}{R^3} \cos \beta + 2 \frac{3 \, \mathfrak{M}}{R^3} \cos \alpha \; x \cos \left[180° - (\alpha + 2 \, \beta) \right]$$

$$F = 2 \frac{\mathfrak{M}}{R^3} \left[\cos \beta - 3 \cos \alpha \cos (\alpha + 2 \, \beta) \right]$$

(Fig. 50)

(Fig. 51)

R, α and β are further connected by a relation implicit in the construction of the apparatus and its disposition above the bowl containing the compass card (fig. 51).

We have; for instance :

$$l = S \cos \beta$$
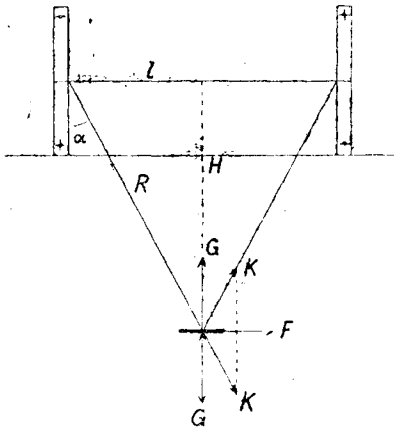
$$h = l \; \mathrm{tg} \; \beta$$

$$R^2 = k^2 \, l^2 = [H + (1 - k) \, h]^2$$

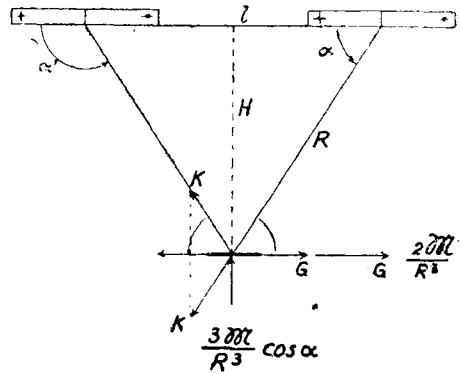$$\alpha = (90 - \beta) + \mathrm{arc} \; \mathrm{tg} \; \frac{k \, l}{H}$$

In the Clausens deflector of Cornelius KNUDSEN (fig. 52) the two vertical magnets give rise to a horizontal deviating force.

$$F = 3 \frac{\mathfrak{M}}{R^3} 2 \cos \alpha \sin \alpha = 3 \frac{\mathfrak{M}}{R^3} \sin 2 \, \alpha$$

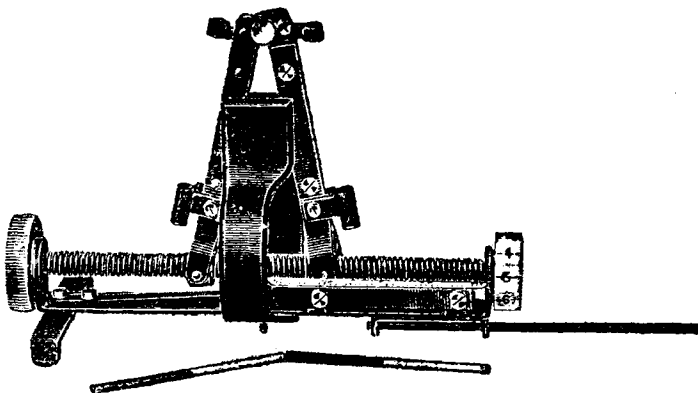We have further $tg\,\alpha = \dfrac{l}{H}$ et $R^2 = H^2 + l^2$.

(Fig. 52)

(Fig. 53)

With the arrangement shown in fig. 53, the deviating force is given by the expression :

$$F = 2\,\frac{\mathfrak{M}}{R^3} - 2\,\frac{3}{R^3}\,\frac{\mathfrak{M}}{R^3}\,\cos\alpha\cos\alpha$$

$$F = 2\,\frac{\mathfrak{M}}{R^3}\,(1 - 3\,\cos^2\alpha)$$

together with : $R^2 = H^2 + l^2$ and $\cos\alpha = \dfrac{l}{R}\cdot$

This latter form of deflector, in which one may make H invariable while retaining the variable l by inserting the small magnets in a slot with millimetric divisions carried in the frame of the wooden alidade, lends itself very conveniently to the makeshift construction which can be carried out with the means available aboard ship, in cases where too great accuracy is not required.

Husun Compass Deflector.