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PREPARATIVE TABLES FOR THE DETERMINATION OF THE LONGITUDE (TIME) BY THE METHOD OF EQUAL ALTITUDES OF VARIOUS STARS.

Calculated and published by the Hydrographic Department of the Imperial Japanese Navy, Tokyo (1921-1940).

A REVIEW

by

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WITH EXTRACTS.

1. *GENERAL.*

The principle involved in the method of equal altitudes for the determination of the time, first described in 1780 by KHOELER, astronomer at Dresden, had been later reported fully in 1808 by GAUSS, Director of the Astronomical Observatory at Göttingen, in the *Monatliche Correspondenz zur Beforderung der Erd-und Himmels Kunde* of the Baron von ZACH, Volume XVIII and Volume XIX (1). It was subsequently generalized, in 1828, by the Russian astronomer KNORRE, Director of the Naval Observatory at Nicolaief and applied by the officers of the Russian Navy, trained under his direction, in their hydrographic surveys of the coasts of the Black Sea and the Sea of Marmara (cf. Astronomischen Nachrichten N° 158, Vol. VII). In 1835, ANGER, Professor at the School of Navigation in Dantzig, developed, in his article published in the *Neueste Schriften der Naturforschenden Gesellschaft,* Vol. III, the methods for the indirect solution of the problem applied to any given number of stars and he strongly recommended it to travelers for use when they had only a simple sextant at their disposal. (2)

In 1888 Lieutenant Edw. PERRIN, of the French Navy, presented a study for the use of mariners and geographers of the direct solution of the problem such as was proposed

(1) DELAMBRE has reported on it in the *Additions à la Connaissance des Temps* for the year 1812.

(2) The original article of KNORRE was published in the Russian language and reproduced for the most part in the second edition of the *Abrégé d'Astronomie pratique* of Sawitsch, also in Russian; there is a German translation of this last work, made in the year 1879 at Leipzig by C.F.W. PETERS. As for the treatise of ANGER, this is found in Vol. III of the *Neueste Schriften der Naturforschende Gesellschaft* in Dantzig under the title : " *Uber die sicherste Bestimmung der geographischen Breite aus Beobachtungen mit einem Spiegelsextanten oder ähnlichen Instrumente.*

by GAUSS himself and the indirect solution by the equation of conditions, according to ANGER and KNORRE, in a work entitled "*Détermination exacte de la Latitude et du Temps du lieu par la méthode des hauteurs égales d'étoiles*", published in the "Annales du Bureau des Longitudes", Vol. IV, Paris, Gauthier-Villars. In particular he gives some simple rules for the preparation of this type of observation with the aid of rapid calculations or of graphic diagrams of the altitudes and azimuths of the stars.

At the beginning of the century the method of equal altitudes received a decidedly fertile impetus as a result of the introduction of the prismatic astrolabe after the year 1903 for operations in the field. Numerous works or manuals have appeared such as that of MM. CLAUDE and DRIENCOURT in France (Paris, Gauthier-Villars, 1910) of Colonel WOODROFE R.E. (London, 1916) of John BALL and H. KNOX-SHAW (Cairo, 1919) of Commander M.J. NOGUEIRA DA GAMA (Rio de Janeiro, 1926) of Captain T.Y. BAKER (London, 1931) of Professor FORNI (Genoa, 1932) of Ingénieur hydrographe A. GOUGENHEIM (Paris, imprimerie nationale ,1936) which develop the principle and the details of its practical application, in particular that of the systematic preparation of the observations by calculations or by graphic methods.

But one does not always have the astrolabe available while many of the field party generally have with them a universal instrument which, in spite of its weak power of magnification, does nevertheless permit of useful expeditious measurements which are easily made without attempting to work on a constant zenithal distance.

It is in this connection that the "Preparative Tables" for the application of the method of equal altitudes just published by the Japanese Hydrographic Department offer such special interest and we propose to report on their use in practice.

The method of equal altitudes, consists, as is well known, of noting the interval of time which elapses between the equal altitudes of stars located on very different azimuths, and of utilising the absolute positions of these stars for deducing the elements sought, *without the intervention of the altitude itself*. In this manner, the systematic errors proper to the instrument disappear entirely from the results, which are not thereafter affected, except by the accidental errors of observation, and these latter are to a large extent compensated. It is entirely independent of the imperfections of the measuring apparatus and further, the observations of stars offer numerous advantages by the fact that observations may be carried out at almost any hour of the night and many observations may be made.

However, the method of equal altitudes should not be confused with the method of corresponding altitudes. This last denomination applies to the equal altitudes of a *one and the same star*, taken on one side and the other of the meridian.

The determination of time by equal altitudes of a single celestial body on both sides of the meridian was practised from quite early times in preference to that by a single altitude. The drawback of the first named method however is that it requires for its performance a considerable interval of time, during which the constants of the instrument and the atmospheric refraction are liable to change.

This defect can be obviated, all its merits being practically retained, if we observe in a short interval of time equal altitudes of two different stars properly selected on both sides of the meridian, instead of those of a single body.

The method of time determination by observing two different stars at equal altitudes, was first brought into practice by the Russian astronomer and geodetist, Prof N. ZINGER. In 1874, he first pointed out that this method, when executed with star-pairs properly selected, is far superior to many others, and it was practically shown that this method enables us to determine the time in only about 10 to 12 minutes, with an error not surpassing a small fraction of a second.

2. *EXISTING LISTS OF STAR PAIRS AND PREPARATIVE TABLES.*

Prof. N. ZINGER, in his work : "Die Zeitbestimmung aus correspondirenden Höhen verschiedener Sterne, 1874", thoroughly studied this method both theoretically and practically, and gave a list of 160 star-pairs, consisting of the brightest stars, and well distributed over the whole sidereal day, by which the time observation can be readily accomplished in any north latitude from 30° to 70°.

KORTAZZI in his work : "Hülfstafeln zur Berechnung örtlicher Ephemiden für die Zeitbestimmungen nach der Zinger'schen Methode, 1891", increased the number of star-pairs to 186. T. WITTRAM, in his work : "Tables auxiliaires pour la détermination de l'heure par des hauteurs correspondantes des différentes étoiles, 1892", increased the number of star-pairs to 200, and published a few auxiliary tables, with which one can rapidly and conveniently prepare the elements for the observation in any north latitude from 30° to 70°. The tables of KORTAZZI and of WITTRAM are computed with the star-coordinates referred to 1900.0.

In order to bring the method of Prof. N. ZINGER into general use in astronomical expeditions and in geodetic surveys, Colonel STCHETKIN of the General Staff, calculated the preparative data for the Wittram's 200 starpairs and for every two degrees of the north latitudes from 40° to 60° for the epoch 1900.0.

Stchetkin's table was published by the Military-Scientific Committee of the Chief Military Staff in St. Petersbourg in 1902, with the title : "Ephemerid zvizd dlia opredilenia vremeni po sposodu Prof. ZINGERA". This extensive volume contains 572 pages

Stchetkin's table was extended to the north latitude 70° by Mr. DOLGOFF, the astronomer of the Emigration Office of the Russian Department of Agriculture.

All the tables hitherto mentioned, however, are only for latitudes higher than 40°. The "Handbuch für Küstenvermessungen" publishêd by the German Department of Navy, in Berlin, 1906, contains a table of auxiliary quantities to facilitate the computations of the zenith distances and the azimuths of 351 star-pairs (Tafel 1a) and is available for all the latitudes from — 90° to+ 90°. The same German work further contains a list of more 528 star-pairs (Tafel 1b), for which however the auxiliary quantities as above mentioned are not given.

These two tables are given for the epoch 1910.0, and the star-pairs are selected in such a manner, that the declinations of the two stars forming a pair do not differ by more than 1° 10', the zenith distances are always within the limits 20° and 70°, and the azimuths do not deviate from the prime vertical more than 40°.

The Preparative Table of observations established by the Hydrographic Department of the Imperial Japanese Navy which we propose to analyse here in the following pages, consisted originally in 1921, in a prolongation of the preceeding tables for North latitude extending from 20° to 40°. Published in the "Bulletin of the Hydrographic Department, Imperial Japanese Navy" Vol. III, Tokyo, 1922, they were calculated by Professor M. KAMENSKY, former Director of the Naval Observatory at Vladivostok, and checked by the Naval Engineer T. NAKANO.

It contains a table of data for 200 star-pairs such that the observer need make only some mental computations which may be carried out at the very moment of setting the instrument for the observation of the star-pair.

The arrangement of material is quite different from that in the foregoing works. This difference permits of a considerable reduction in the size of the Tables, viz— from 572 pages (in Stchetkin's Table) to only 200 pages in the present table. Moreover, the present table being arranged for each complete degree of latitude is much more prăctical and has the advantage of eliminating the errors arising from mental interpolation.

Effort has been made to bring the declinations of the two stars of each pair as near together as possible, to keep their observed azimuths as near as possible to the prime vertical and to limit the zenith distances between 20° and 70°. For purposes of check use has been made of all those star pairs in Wittram's table which satisfy these conditions for the epoch 1930.0 and use of those pairs in the "Handbuch für Küstenvermessungen" which are available for the purpose.

The star pairs are selected in such a manner that the following conditions are satisfied as nearly as possible :—

$-35^{\circ} < \varepsilon < 35^{\circ}$	ε = Half of the difference of the declinations of the pair;
$20^{\circ} < z < 70^{\circ}$	z = Zenith distance;
$230^{\circ} < A_E < 310^{\circ}$	A_E = Azimuth of the east star;
$50^{\circ} < A_W < 130^{\circ}$	A_W = Azimuth of the west star;
$2 < M < 4$	M = Magnitude of star.

These conditions permit a simplification of the formulae for computing the correction of the chronometer from an actual observation of a star-pairs (as it will be shown afterwards).

The last requirement is to beware of magnitude error, which may be produced in the observation of very brilliant or faint stars.

In April 1923, Rear Admiral S. INUZUKA, Director of the Hydrographic Department of the Imperial Japanese Navy ordered the publication of the supplement to the preceding Tables, in the same form, for the North Latitudes extending from 40° to 60°. (Bulletin of the Hydrographic Department, Imperial Japanese Navy, Vol. IV, Tokyo 1923). This publication therefore presents the table of Stchetkin in the reduced and methodical form advocated by Kamensky and brought up to the epoch 1930.0. Finally with the publication in the same form in February 1940 of the third part for the North Latitudes of 0° to 20°, calculated by Naval Engineer Y. TUKAMOTO, under the direction of Vice Admiral S. KOIKE, the Japanese Hydrographic Department has just completed a very important work for facilitating the expeditious determination of longitude in the northern hemisphere which is of considerable interest for the entire zone of equatorial islands in the Pacific. The Bulletin of the Hydrographic Department of the Imperial Japanese Navy, Vol. IX, Tokyo, 1940, furnishes all the necessary data on the subject for the observation of 200 star pairs for each degree of North latitude, with the data brought up to the epoch 1950.0.

3. *DESCRIPTION OF THE JAPANESE PREPARATIVE TABLES.* (*)

For each of the 200 pairs of stars there is given a table of the form reproduced opposite representing one of the pages of the publication.

The pairs are arranged in the order of the sidereal time of the observation, and the following quantities are given for each star pair.

Column 1	φ	=	The latitudes.
Column 2	S	=	The sidereal time at which the two stars attain the equal altitudes simultaneously, viz. the first star is to be observed at S — 2 m. 5 and the second star at S + 2 m. 5.
Column 3	z	=	The common zenith distance when the east star first and the west star next are to be observed.
Column 4, 5	A_E, A_W	=	The azimuths of the east and the west stars, when the east star is to be observed first. The reckoning of Azimuths is from the south in the direction of S-W-N-E-S.
Column 6, 7, 8	dz, dA_E, dA_W	=	The reductions to be added algebraically to z, A_E, and A_W, when the west star first and east star next are to be observed.
Column 9, 10	Δ z, Δ A	=	The correction of z, A_E, and A_W for the 10' variation of the latitude.

The lower marginal table is the annual precessions of S, z, A_E, and A_W.

These corrections are very small and may be neglected till 1960 practically.

(*) *Extracts from Preface to Tables.*

LIST II.

		No.	Star	Mag.	$\alpha_{1930.0}$	$\delta_{1930.0}$
Pair No. 139	E	79	δ Cygni	3.0	19 h 42.8 m	+44° 58′
	W	85	λ Bootis	4.0	14 13.7	+46 25′

φ	S	z	A_E	A_W	dz	dA_E	dA_W	Var. for $\Delta\varphi = +10'$	
								Δz	ΔA E W
+40°	16 h 58.5 m	30° 46′	246.9°	116.0°	−52′	+0.1°	−	−4.2′	+0.25° −
41	16 58.7	30 22	248.4	114.5	−52	+0.2	−	−3.9	+0.26 −
42	16 58.8	29 59	250.1	112.9	−52	+0.3	−	−3.7	+0.27 −
43	16 58.9	29 38	251.7	111.3	−52	+0.3	−	−3.4	+0.28 −
44	16 59.0	29 18	253.4	109.7	−51	+0.4	−	−3.2	+0.28 −
45	16 59.2	29 0	255.1	108.0	−51	+0.4	−	−2.8	+0.29 −
46	16 59.4	28 44	256.9	106.3	−50	+0.5	−	−2.5	+0.30 −
47	16 59.6	28 30	258.7	104.5	−50	+0.6	−	−2.2	+0.30 −
48	16 59.7	28 18	260.6	102.8	−49	+0.7	−	−1.8	+0.31 −
49	16 59.9	28 8	262.4	101.0	−49	+0.7	−	−1.6	+0.31 −
50	17 0.1	27 59	264.3	99.1	−48	+0.8	−	−1.3	+0.32 −
51	17 0.3	27 52	266.2	97.3	−47	+0.9	−	−1.0	+0.32 −
52	17 0.4	27 47	268.2	95.4	−46	+0.9	−	−0.7	+0.32 −
53	17 0.6	27 44	270.1	93.6	−45	+1.0	−	−0.3	+0.32 −
54	17 0.8	27 43	272.0	91.7	−44	+1.0	−	+0.1	+0.32 −
55	17 1.1	27 45	274.0	89.9	−43	+1.1	−	+0.4	+0.32 −
56	17 1.4	27 48	275.9	88.0	−42	+1.2	−	+0.7	+0.32 −
57	17 1.7	27 53	277.9	86.2	−41	+1.2	−	+0.9	+0.31 −
58	17 1.9	27 59	279.8	84.4	−39	+1.3	−	+1.3	+0.31 −
59	17 2.2	28 8	281.7	82.6	−38	+1.3	−	+1.8	+0.31 −
+60	17 2.4	28 20	283.6	80.8	−37	+1.3	−	+2.2	+0.31 −

Annual Precessions of S, z, A_E, and A_W.

φ	δS	δz	δA_E	δA_W
+40°	+0.03 m	0.0′	−0.004°	−0.009°
44	+0.03	0.0	−0.005	−0.010
48	+0.03	0.0	−0.005	−0.011
52	+0.02	0.0	−0.006	−0.012
56	+0.02	0.0	−0.007	−0.012
+60	+0.01	0.0	−0.008	−0.013

EXAMPLE :

Determination of the Time at Monaco (Principality) on 30 June 1941.

Approximate Latitude of the International Hydrographic Bureau :

φ = 43° 44′ 18″.3 N. — Longitude = 0 h. 29 m. 41.9 s. E.

at about 17 hours sidereal time.

An examination of the Preparative Table shows that star pairs N° 139 and 140, for instance, may be chosen for this purpose.

We extract from the Table, page 166 of the Japanese Publication, the following data for star pair N° 139 :—

	S	z	A_E	A_W	Order of Stars
For φ = 43° 00'	16 h. 58 m. 9	29° 38'	251°.7	111°.3	E - W
corr. for + 44'	0.0	— 15'	+ 1°.1	— 1°.1	
For φ = 43° 44'	16 h. 58 m. 9	29° 23'	252°.8	110°.2	E - W
Reduct. for reversal of order	———	— 51'	+ 0°.4	— 0°.4	
For φ = 43° 44'	16 h. 58 m. 9	28° 32'	253°.2	109°.8	W - E

The calculations are made by inspection and one may write the program of observations directly in the following form :—

Place : Monaco $\varphi = + 43° 44'$

Pair N°	Star	T	E/W z A_E A_W	W/E z' A'_E A'_W
		h m		
139	δ Cygni 3.0	16 56.4	29° 23' 252°.8 110°.2	28° 32' 109°.8 253°.2
	λ Bootis 4.0	17 01.4		
140	ζ Cygni 3.0	17 04.2	49° 54' 264°.4 92°.9	49° 00' 92°.1 265°.2
	43 Comae 4.0	17 09.2		
....	....	....	....	....

4. *PREPARATIVE DATA FOR A GIVEN LATITUDE.*

As the two stars forming a pair do not differ much in declination, the local sidereal time at which the two stars attain the same zenith distance must be nearly equal to the mean of their right ascensions. Compute the zenith distances and the azimuths of the two stars for the instants 5 minutes before, and 5 minutes after, this mean, i.e. for $\frac{1}{2}(\alpha_E+\alpha_W)$ — 5 m. and for $\frac{1}{2}(\alpha_E + \alpha_W)$ + 5 m.

(See example of method of calculation and the graphic determination in the "Handbuch der Küstenvermessungen", published by the German Admiralty, Berlin 1906 (second volume).

Plot the four values of zenith distance thus computed on a sheet of millimetre paper in adequate scales, as in Fig. 1, the time being taken as the abscissa and the zenith distance as the ordinate. Connect the two points for each star by a straight line. The two straight lines thus drawn for the two stars would approximately represent the values of their zenith distances for the twenty-minute interval from S_1 to S_2 .

The point of intersection of these two lines would give the zenith distance z_0 and the instant S_0 at which the two stars attain the same zenith distance simultaneously. The observation can not evidently be made at this point, but it must be executed in either of the following positions.

1. A position such as AB where the east star is observed first and the west star next after a certain interval of time corresponding to the length AB. Let us express this order of observation by the notation E/W.

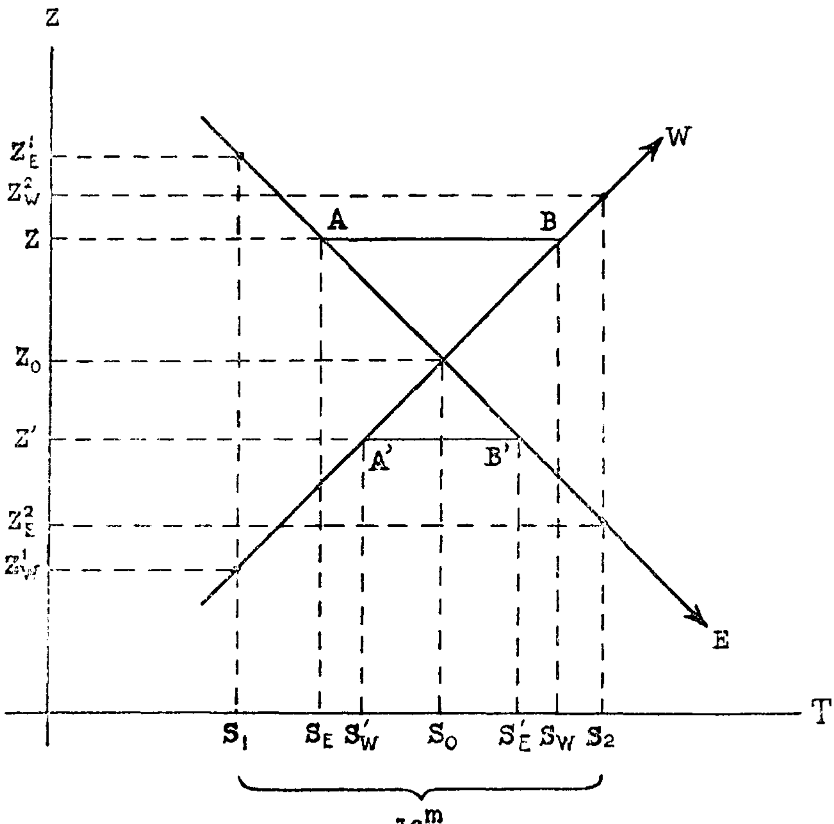

FIG. 1.

2. A position such as A'B' where the west star first and the east star next are likewise observed. Express this order of observation by W/E.

The interval AB or A'B' by which the observations of the two stars are to be separated can be fixed at will according to the circumstances. It is preferable to choose it as short as possible to insure the constancy of the instrumental and the atmospheric conditions during the observation. But on the other hand, we must have a sufficient interval in order to transfer from the observation of the first star to that of the second without haste. Four minutes are quite sufficient for an experienced observer. But five minutes are generally chosen.

5. *FORMULAE FOR CALCULATING THE DATA IN THE TABLES.*

Let α_E and α_W be the right ascensions of the east and the west stars respectively, and δ^E and δ_W, their respective declinations.

Further let us put:

$$S' = \frac{1}{2}(\alpha_E + \alpha_W) \qquad \delta = \frac{1}{2}(\delta_E + \delta_W)$$

$$t = \frac{1}{2}(\alpha_E - \alpha_W) \qquad \varepsilon = \frac{1}{2}(\delta_E - \delta_W)$$

$$K^m = \frac{\varepsilon'}{15 \sin t}$$

$$S_0 = S' + K^m \tan \delta \cos t$$

Then we shall obtain the moment S for any latitude φ, at which the altitudes of the two stars become equal, by the following formula :

$$S = S_0 - K^m \tan \varphi$$

The common zenith distance z at this moment is calculated by :

$$\cos z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos t$$

and the azimuth A of the ficticious star with the declination $\delta = \frac{1}{2} (\delta_E + \delta_W)$ by :

$$\text{cotg } A = \sin \varphi \cos t - \cos \varphi \tan \delta \text{ cosec } t$$

Lastly the azimuth of the east star at the moment S, counted from the south towards the east will be :

$$A_E = A + 15 K^m \sec \varphi$$

and that of the west star, counted from the south towards the west will be :

$$A_W = A - 15 K^m \sec \varphi$$

The values of z, A_E, A_W, as calculated by the processes shown above, are those for the moment S at which the two stars are at exactly the same zenith distance. As it is however impossible to observe them simultaneously at this moment, we arrange our data so that we observe the first star (E. or W. star) 2 1/2 minutes before, and the second star (W. or E. star) 2 1/2 minutes after the moment S, when the two stars simultaneously attain the same altitude. For this purpose we computed the variations of the zenith distance z and the azimuths A_E and A_W in 2 1/2 minutes of time, according to the formulae :

$$dz' = 15 \cos \varphi \sin A \, dt^m$$

$$dA' = 15 (\sin \varphi + f \cos \varphi) \, dt^m$$

$$\text{where } f = \cos A \cot z$$

For the accuracies sought for the final data aimed at, as above mentioned (about 1' for z and 2' for A), it is sufficient to assume for A its mean value calculated by the formula :

$$\cot A = \sin \varphi \cot t - \cos \varphi \tan \delta \text{ cosec } t$$

in calculating dz', but in calculating dA'_E and dA'_W, it is not necessary to take the different values of f for the east and west stars, as this may be computed from the mean values of A and z by :

$$f = \pm \cos A \cot z$$

FORMULAE FOR THE ANNUAL PRECESSIONS OF S, z, A_E and A_W:

Taking, as the basis of deduction, the formulae for precession :

For the E. star :—

$$\Delta A_E = 46''.1 + 20''.0 \sin A_E \tan \delta_E$$

$$\Delta \delta_E = + 20''.0 \cos A_E$$

For the W. star :—

$$\Delta A_W = 46''.1 + 20''.0 \sin A_W \tan \delta_W$$

$$\Delta \delta_W = + 20''.0 \cos A_E$$

$$\Delta \delta = + 0'.33 \cos S' \cos t$$

$$\Delta \varepsilon = - 0'.33 \sin S' \sin t$$

$$\Delta K^m = - 0^m.022 \sin S'$$

$$\Delta S' = + 0^m.051 + 0^m.022 \tan \delta \sin S' \cos t.$$

From these expressions, we can easily deduce the following expressions for the annual variations of S, z, A_E and A_W due to precession :

$$dS = + 0^m.051 + 0^m.022 \tan \varphi \sin S'$$
$$dZ = - 0'.33 \cos S' \cos A$$
$$dA_E = - 0'.33 \cos S' \sin A \cot z + 0'33 \sec \varphi \sin S'$$
$$dA_W = + 0'33 \cos S' \sin A \cot z + 0'.33 \sec \varphi \sin S'$$

6. *PRINCIPLE OF THE METHOD.*

Let T_1 and T_2 be the observed chronometer times of transit of the two stars (1) and (2) at different azimuths over a horizontal wire of a universal instrument supposed to be perfectly leveled. Let next α_1, δ_1 and α_2, δ_2 be the right ascensions and the declinations of the above two stars respectively, and φ the latitude of the place of observation. Lastly let ΔT be the correction of the chronometer to the local sidereal time, and z the common zenith distance at which the stars are observed as above, assuming the atmospheric refractions to be wholly equal for the two cases. We have then the relations :

$$\cos z = \sin \varphi \sin \delta_1 + \cos \varphi \cos \delta_1 \cos (T_1 + \Delta T - \alpha_1)$$
$$= \sin \varphi \sin \delta_2 + \cos \varphi \cos \delta_2 \cos (T_2 + \Delta T - \alpha_2)$$

Hence we have :

$$\sin \varphi \sin \delta_1 + \cos \varphi \cos \delta_1 \cos (T_1 + \Delta T - \alpha_1) =$$
$$= \sin \varphi \sin \delta_2 + \cos \varphi \cos \delta_2 \cos (T_2 + \Delta T - \alpha_2) \qquad (1)$$

If φ be known, the determination of the sole unknown ΔT is possible from the equation (1). Thus, observing the times of equal altitudes of two different stars at a known latitude, we can theoretically determine the chronometer correction.

7. *SELECTION OF STARS.*

Now in actual operations in field, the assumed latitude φ is generally subject to an error of a certain amount. In selecting the pair of stars, the first care must therefore be taken to minimize the effect of an error in the assumed latitude upon the determination of time.

The latitude error does not affect the time determination much, when we approximate the condition :

$$\text{tang } \frac{1}{2} (A_1 + A_2) = 0 ;$$

where A_1 and A_2 are the azimuths of the two stars (1) and (2) respectively or when

$$A_1 + A_2 = 360°$$

The two stars must therefore be observed *in positions symmetrical in azimuths with respect to the meridian.*

Again, in order to diminish the effect of any unavoidable error in the assumed equality of the zenith distances of the two stars, we must observe the stars when they are as *near to the prime vertical* as possible, as can be inferred from the general theory of errors of time determination by altitude. But in this respect the azimuth may deviate in practice from the prime vertical within 40° on its either side, provided the deviations of the two stars take place to the same side, north or south, and by nearly the same amount.

In order to observe two stars nearly on the above conditions and in a short interval of time, we must select two stars of *nearly equal declinations.* The difference of declinations may practically amount to 2° or 3°, without diminishing the accuracy very much.

Lastly the zenith distance at which the two stars are observed should be neither too small nor too large. If it is too small, we have a great inconvenience not only from a too steep elevation of the telescope, but also from a comparatively quick change of the star's azimuth. If it is too large, some inequality of the atmospheric refractions for the two stars, may occur which would affect our result almost directly. The zenith distance is generally limited between 20° and 70°.

These details have lead to the conditions stated above, in paragraph 2.

8. *METHOD OF OBSERVATION.*

The instrument to be used for the purpose may be a universal instrument of any type. The telescope ought to be provided with a certain number of horizontal wires and two or three vertical wires in the middle of its visible field. It is found very expedient from experience to have five horizontal with 2' intervals.

The instrument is set upon a solid pier, and leveled as usual. Observing Polaris, determine roughly the reading of the horizontal circle, when the telescope is turned to the south (or north), i.e. the reading corresponding to the azimuth 0°. If possible, adjust the horizontal circle so that its reading always gives directly the approximate azimuth of the telescope. It is more advantageous to have an observing platform wholly isolated from the tripod or the pier, so as not to disturb the level of the instrument during the observation of each star.

A few minutes before the predicted time of observation of the first star, set the telescope at the zenith distance z or z' at which the pair is to be observed, and set the instrument at the azimuth A_E or A'_W at which the first star is to be observed. Clamp the level firmly to the telescope in such a position that its bubble swings at about the middle of its graduation.

About two minutes before the predicted time of observation (i.e. the time at which the star would pass through the middle horizontal wire), the first star should begin to appear in the field. It should apparently move obliquely from one corner toward the opposite corner. When it approaches the first horizontal wire, carefully turn the instrument in azimuth by means of the fine-motion screw of the horizontal circle, and bring the star in such an estimated position that it will pass the first horizontal wire by the middle vertical wire or better by the middle of a prefixed pair of vertical wires. Then read the level and observe the chronometer time of the passage of the star over the first horizontal wire at a prefixed position relative to the vertical wires.

The star should continue its oblique motion. Turning the instrument in azimuth carefully by means of the fine-motion screw, determine the time of the star's passage over the second horizontal wire by the same vertical position, and proceed likewise for all other successive horizontal wires. The times of the star's transit may be observed by *eye and ear method* or more accurately by the use of a *chronograph.*

Lastly read the level. With this we finish the observation of the first star.

Direct the telescope next to the azimuth A_W or A'_E of the second star, and look at the level. Adjust the level, when necessary, so as to assume nearly the same readings as by the observation of the first star. This adjustment must be made by the foot screws or a tangent screw the touch of which does not affect the connection of the level with the telescope at all. The second star in the meanwhile would begin to appear in the field. It will apparently move in altitude directly opposite to the first star, but in the same azimuth.

Exactly the same procedure is then followed as before, i.e.; read the level, determine the times of passage over the successive horizontal wires by the same vertical position as before, and lastly read the level. The order of the horizontal wires in which the second star is observed is opposite to that for the first star. With this, the observation of the pair is finished.

Utmost care must be taken not to disturb the connection of the level and the telescope during the whole observation of a pair. A complete determination of time consists in the observation of at least two pairs, the first pair being observed in the order E/W and the second pair in W/E, or in the reverse order. A more complete determination would consist of four pairs observed in the following scheme of order :

E/W, W/E, W/E, E/W or W/E, E/W, E/W, W/E

9. *METHOD OF COMPUTATION.*

(I) *Correction of an Observed Time for Inclination.*

We have assumed that the instrument is perfectly leveled. This ideal case cannot be realized generally. But it matters little if we reduce the observed times of one star for inclination of the telescope from the differences of the level readings at the observation of the two stars.

Let i_E, i_W and O_E, O_W be the readings of the two ends of the level bubble, *i* being that nearer to the eye-piece, and O that nearer to the objective of the telescope, and the suffixes E and W denoting that the notations partake to the east and the west stars respectively. When the level readings are made twice before and after the star's observation, we take for each of the above quantities the mean of the two corresponding readings thus noted. Again let μ be the value of one division of the level graduation, expressed *in seconds of arc.*

The reduction in question is evidently proportional to the change of inclination of the telescope as indicated by the difference of the level readings. The factor of this proportionality may be considered to be practically equal to the proportion τ/d in which τ is the difference of the observed times of the star's transits over the two extreme wires whose vertical distance is *d*. From this consideration we have for the reduction the expression :

$$\delta\, Ti = \pm S\,\tau\left[(i_W - i_E) + (O_W - O_E)\right]$$

where

$$S = \frac{\mu}{2d}$$

τ and *d*, the quantities above defined, being here expressed in *seconds of time* and in *seconds of arc* respectively. *S* and τ are always to be taken as positive. S may be tabulated as the constants of the instrument for all combinations of two wires. The *plus* or the *minus* sign of the above expression is to be taken, also in this case, according as the level reading *increases* or *decreases* from *i* towards *o*.

The above reduction may be applied to the observed times of either star. But it is preferable to apply it to those from which the value of τ is derived.

II. *Deduction of the Chronometer Correction.*

Denoting all the quantities referring to the east star with the suffix E and those referring to the west star with W, the equation (I) in paragraph 6 may be written :

$$\sin\varphi\left(\sin\delta_E - \sin\delta_W\right) + \cos\varphi\left[\cos\delta_E\cos\left(T_E + \Delta T - \alpha_E\right) - \cos\delta_W\cos\left(T_W + \Delta T - \alpha_W\right)\right] = 0$$

Here we take for T_E and T_W the means of all the observed times *already corrected for inclination*, of the east and the west stars respectively.

Putting :

$$t = \frac{1}{2}\left(\alpha_E - \alpha_W\right) - \frac{1}{2}\left(T_E - T_W\right)$$

$$r = \frac{1}{2}\left(\alpha_E + \alpha_W\right) - \frac{1}{2}\left(T_E + T_W\right) - \Delta T$$

or

$$t + r = \alpha_E - (T_E + \Delta T)$$
$$t - r = (T_W + \Delta T) - \alpha_W$$

and

$$\delta = \frac{1}{2}\left(\delta_E + \delta_W\right) \qquad \varepsilon = \frac{1}{2}\left(\delta_E - \delta_W\right)$$

the above equation can easily be transformed into :

$$\sin \varphi \sin \varepsilon \cos \delta - \cos \varphi (\cos t \cos r \sin \delta \sin \varepsilon + \sin t \sin r \cos \delta \cos \varepsilon) = 0$$

or

$$\sin r + \frac{\tan \varepsilon \tan \delta}{\tan t} \cos r - \frac{\tan \varepsilon \tan \varphi}{\sin t} = 0$$

Putting :

$$\tan m = \frac{\tan \varepsilon \tan \delta}{\tan t}, \quad \sin n = \frac{\tan \varepsilon \tan \varphi}{\sin t} \cos m,$$

we have :

$$\sin r + \tan m \cos r - \frac{\sin n}{\cos m} = 0$$

or

$$\sin (r + m) = \sin n.$$

Thus we have :

$$r = n - m$$

$$\Delta T = \frac{1}{2} (\alpha_E + \alpha_W) - \frac{1}{2} (T_E + T_W) - r$$

The above expressions are all quite rigorous. The quantity ε is however not large in all the existing lists of star pairs, and m and n therefore are not large as can be seen from their definitions.

Now for a small angle X^s expressed in second of time, we have :

$$\log \sin X^s = \log X^s + \log \sin 1^s - \sigma (X^s)$$
$$\log \tan X^s = \log X^s + \log \sin 1^s + 2 \sigma (X^s)$$
$$\log \cos X^s = - 3\sigma (X^s)$$

where

$$\sigma (X^s) = \log \frac{X^s \sin 1^s}{\sin X^s}$$

But from the equations defining m and n, we have :

$$\log \tan m = \log \tan \varepsilon + \log \left(\frac{\tan \delta}{\tan t}\right),$$

$$\log \sin n = \log \tan \varepsilon + \log \cos m + \log \frac{\tan \varphi}{\sin t}$$

Treating ε, m and n, as small quantities, and substituting for log tan m, log tan ε, log sin n, and log cos m, their equivalent expressions as above given, we have :

$$\log m^s = [\log \varepsilon^s + 2 \sigma (\varepsilon)] + \log (\tan \delta \cot t) - 2 \sigma (m)$$

$$\log n^s = [\log \varepsilon^s + 2 \sigma (\varepsilon)] + \log (\tan \varphi \operatorname{cosec} t) + [\sigma (n) - 3 \sigma (m)]$$

Again the observed times must be corrected for the diurnal aberration by the quantity — 0.021 cos z. Thus we can now summarize the formulae for the computation of the chronometer correction as follows :

$$\delta = \frac{1}{2} (\delta_E + \delta_W) \qquad t = \frac{1}{2} (\alpha_E - \alpha_W) - \frac{1}{2} (T_E - T_W)$$

$$\varepsilon = \frac{1}{2} (\delta_E - \delta_W)$$

$$\log m_0^s = \log \varepsilon^s + 2\,\sigma\,(\varepsilon) + \log \tan \delta + \log \cos t$$

$$\log n_0^s = \log \varepsilon^s + 2\,\sigma\,(\varepsilon) + \log \tan \varphi + \log \operatorname{cosec} t$$

$$\log m^s = \log m_0^s - 2\,\sigma\,(m_0)$$

$$\log n^s = \log n_0^s + \sigma\,(n_0) - 3\,\sigma\,(m_0)$$

$$r^s = n^s - m^s$$

$$\Delta T = \frac{1}{2}\,(\alpha_E + \alpha_W) - \frac{1}{2}\,(T_E + T_W) + 0.^s021 \cos z - r^s$$

The value of σ (X^s) with log X^s as argument is tabulated at Table III of the Japanese Tables. The value of $0^s.021 \cos z$ is given with the zenith distance z as argument in the following table.

CORRECTION FOR DIURNAL ABERRATION.

z	10°	20°	30°	40°	50°	60°	70°
$0.^s021 \cos z$	$0.^s021$	$0.^s019$	$0.^s018$	$0.^s016$	$0.^s013$	$0.^s010$	$0.^s007$

10. *EXAMPLE OF OPERATION.*

The calculation of the chronometer correction for Monaco, on 30th June 1941, about 17 h. 00 Sideral time, of which the preparative example is given above, at the end of paragraphe 3, is shown in the opposite scheme of computation :—

Lieu : Monaco - Bureau hydrographique international.
position estimée : Lat : 43°44′18″.3 N.
le 30 Juin 1941. Long : $0^h 29^m 41^s.9$ Est. vers $17^h 00^m$ T. Sid.

Détermination de l'Heure par des Hauteurs Égales d'Étoiles.

compteur n° 742.

Couple N° 139

E. δ Cygni 3.0 — $19^h 42^m.8$ — +44°58′
W. λ Bootis 4.0 — 14 13.7 — +46 25 — E/W.

Niveau	Est ie	Est Oe	West iw	West Ow
N obs :	11.8	28.7	11.9	28.9
R obs :	11.5	28.5	11.6	28.7

ie = 11.65 Oe = 28.60 $\tau_w = 81^s.5$
iw = 11.75 Ow = 28.80 S = 0.0035
iw - ie = +0.10 Ow - Oe = +0.20 $S\tau_w$ = 0.285
Som = +0.30
$+S\tau_w$.Som = $+0^s.09$

Fil	Est	West	Som
I	$16^h 52^m 55^s.9$	$59^m 27^s.1$	23.0
II	53 16.1	59 06.8	2.9
III	53 36.4	58 46.3	2.7
IV	53 56.8	58 25.9	2.7
V	54 17.0	16^h 58 05.6	2.6
Moy.	16 53 36.44	16 58 46.34	

δ_e = +44°59′18″.45
δ_w = +46 21 42.53
δ = +45 40 30.49
ε = − 41 12.04
ε″ = − 2472.04

log ε″ = 3.39305 n
log 1/15 = 8.82391
log ε = 2.21696 n
σ_ε = 1.1

α_e = $19^h 43^m 10^s.83$
α_w = 14 14 09.83
α = 16 58 40.33
ε_α = 2 44 30.50

Te = 16 53 36.44
Tw = 16 58 46.43
T = 16 56 11.44
ε_T = − 2 35.00

$t = \varepsilon_\alpha - \varepsilon_T$ = $2^h 47^m 05^s.50$

log tg φ = $\bar{1}$.98087
log cosec t = 0.17641
log ε + 2σ_ε = 2.21698 n
log cotg t = 0.04903
log tg δ = 0.01024
log n_o = 2.37426 n
$\sigma_n - 3\sigma_m$ = −2
log m_o = 2.27624 n
$-2\sigma_m$ = −3

σ_ε = 1.1
σ_n = 2.1
σ_m = 1.4
n = $-236^s.72$
m = −188.89
r = $-47^s.83$
−r = $+47^s.83$
α − T = $2^m 28^s.89$
+Aber. = $+0^s.02$
ΔT = $+3^m 16^s.74$

Couple N° 140

E. ζ Cygni 3.0 — $21^h 40^m.0$ — +29°56′
W. β Chevelure 4.0 — 13 08.6 — +28 14 — W/E.

Niveau	Est ie	Est Oe	West iw	West Ow
N obs :	11.7	28.6	11.8	28.7
R obs :	11.3	28.3	11.4	28.6

ie = 11.50 Oe = 28.45 $\tau_w = 72^s.9$
iw = 11.60 Ow = 28.65 S = 0.0035
iw - ie = +0.10 Ow - Oe = +0.20
$\tau_w S$ = 0.255
Som = +0.30
$+S\tau_w$.Som = $+0^s.08$

Fil	Est	West	Som
I	$06^m 44^s.5$	$17^h 00^m 37^s.9$	22.4
II	06 26.6	00 55.9	2.5
III	06 08.8	01 14.6	3.4
IV	05 50.9	01 32.5	3.4
V	17 05 33.3	01 50.7	4.0
Moy.	17 06 08.82	17 01 14.32	

δ_E = +29°59′10″.94
δ_W = +28 10 40.92
δ = +29 04 55.93
ε = + 54 15.01
ε″ = + 3255.01

log ε″ = 3.51255
log 1/15 = 8.82391
log ε = 2.33646
σ_ε = 1.8

α_E = $21^h 10^m 28^s.10$
α_W = 13 09 08.52
α = 17 09 48.31
ε_α = 4 00 39.79

T_E = 17 06 08.82
T_W = 17 01 14.40
T = 17 03 41.61
ε_T = 2 27.21

$t = \varepsilon_\alpha - \varepsilon_T$ = $3^h 58^m 12^s.58$

log tg φ = $\bar{1}$.98087
log cosec t = 0.06445
log ε + 2σ_ε = 2.33650
log cotg t = $\bar{1}$.76924
log tg δ = $\bar{1}$.74522
log n_o = 2.38182
$\sigma_n - 3\sigma_m$ = −2
log m_o = 1.85096
$-2\sigma_m$ = 0

σ_ε = 1.8
σ_n = 2.2
σ_m = 0.2
n = $+240^s.89$
m = +70.95
r = $+169^s.94$
−r = $-2^m 49^s.94$
α − T = $6^m 06^s.70$
+Aber = $0^s.01$
ΔT = $+3^m 16^s.77$

Adop^t ΔT = $+3^m 16^s.76$

Formules :-

r = n - m

$\delta = \frac{1}{2}(\delta_e + \delta_w)$ $\alpha = \frac{1}{2}(\alpha_e + \alpha_w)$ $T = \frac{1}{2}(T_e + T_w)$

$\varepsilon = \frac{1}{2}(\delta_e - \delta_w)$ $\varepsilon_\alpha = \frac{1}{2}(\alpha_e - \alpha_w)$ $\varepsilon_T = \frac{1}{2}(T_e - T_w)$

valeur d'une division du Niveau μ = 5″.82
intervalle des fils I-V d = 832″

$S = \frac{\mu}{2d} = 0.00350$