NOTE ON THE PLOTTING OF BEARINGS ON THE MERCATOR CHART

by

M. GIVRY, INGÉNIEUR HYDROGRAPHE DE LA MARINE, (1829).

(Translated from the original in French).

"In the construction of charts, it had always been considered that the lines of bearing were susceptible of representation by straight lines on the projection of the Mercator Chart. When, in 1819, I commenced the construction from surveys which I had made on the south coast of Brazil, I realized that these lines and in general all geodesics, should be represented on this type of projection by curved lines. In fact, the bearings which, in whatever locality they may be taken, are intersection of the earths surface by the celestial planes, intersect at unequal angles the various meridians which they encounter as the distance from the origin increases, and consequently, in order to preserve the same property on the Mercator charts, where the meridians are represented by parallel straight lines, they must be developped in the form of curved lines.

"EULER was the first to call attention to the fact that the geodesics are projected on the Mercator chart as transcendent curves : he gave the equation for the case where the origin of these lines is located on the equator; but M. LACROIX considering the general aspect of the problem, has found the equations for the different kinds of lines which may be produced by the development of the geodesic on any projection of the globe whatever, and no matter where the origin of the lines is taken. With the aid of the equation of the curve which represents the development of the geodesic on the Mercator chart, the line of bearing may easily be plotted point by point, but one can readily appreciate that much time is consumed in the construction of even a small number of points in this manner. It seemed to me preferable therefore, in order to have only straight lines to plot, to substitute for the curve, regarded as of limited extent, the loxodromic which passes through the end points and which can be plotted on the chart by a straight line. In order to effect this substitution, it will suffice to find the analytical expression of the value of the angle formed at the origin of the curve between the tangent and the cord which subtends the portion of the curve under consideration.

"M. LACROIX, who had the kindness to honour me with his particular good-will has communicated to me the equation of the curve which represents the development of the geodesic on the Mercator chart, and by combining this with the equation of the loxodromic on the same kind of projection of the globe, I have found that the value of the angle sought may be expressed by the following series :—

$$\frac{1}{2} P \sin L - \frac{1}{12} P \sin P \operatorname{cotg} z (1 - 3 \cos^2 L) + \frac{1}{24} P \sin^2 P \sin L \operatorname{cotg}^2 z (1 - 6 \cos^2 L) - \ldots \text{ etc.}$$

in which P is, in seconds of arc, the difference in longitude between the locality of the observer and the point towards which the bearing is directed; z the observed azimuthal angle, or the inclination of the line of bearing to the meridian, and L, the latitude of the locality where the bearing is observed.

When the azimuthal angle z is reckoned from the elevated pole, towards the east or to the west up to 180°, if necessary, the value expressed by the series, or the correction which must be applied to obtain the angle made by the meridian with the substituted loxodromic in the development of the line of bearing, should be added to the observed azimuthal angle: it should be subtracted if the azimuths are reckoned from the depressed pole.

"The above series, which is rapidly convergent, gives with great accuracy the value

of the correction which must be applied to the observed azimuthal angle as long as the difference in longitude between the meridian of the observer and that of the terrestrial object does not exceed two degrees; beyond that limit it may not be sufficiently precise for very exacting operations but unless the operations are being conducted in very high latitudes this condition will not arise, because at 60° latitude and with an azimuth of 90° there may still be twenty marine leagues distance between the observer and the terrestrial object on which the bearing is taken it very rarely happens that bearings are taken on objects at such great distances. and therefore one may make use of the formula as it stands in nearly every case

" I have calculated the tables which give the value of the first two terms of the series; these are in terms of the latitude of the place where the bearing has been observed, and the difference in longitude between the observer and the point observed. This last term cannot be exactly determined except by successive approximations at which one easily arrives in the following manner :— we assume that the terrestrial objects are exactly located on the chart by constructing the azimuthal angles as found by observation; or by developping into straight lines the lines of bearing; the differences of meridian which result from this assumption will serve as arguments for taking from the tables the first value for the corrections, which on being applied to the bearings necessarily change the position of the objects which we have assumed to be accurately plotted, and they then already approach much closer the position which they should occupy. With the aid of this first change in position of these points we obtain, by operations similar to the above, a second approximation of the arguments sought, with which we take from the tables new and more accurate corrections than those which we took at first. These are applied to the observed azimuths and produce a slight displacement in the position of the terrestrial objects on the chart. If we wish to obtain even greater accuracy we seek, from these last positions the arguments for the table, exactly as was done in locating the preceeding positions. The final positions thus obtained are still more accurate than those which were obtained by the preceeding corrections. Finally, by proceeding thus in successive steps to positions corrected more and more we arrive at consecutive corrections where the difference is no longer appreciable.

" In practice one may usually be content with two successive trials to obtain a value for the corrections sufficiently accurate because the limits of accuracy of the azimuthal angles is never less than three or four minutes of arc, and it will therefore suffice to make the correction to the nearest minute. This consideration shows that one can dispense with the use of the terms beyond the first. This alone will suffice for correcting the azimuths taken under sail. But if one neglects to apply to these angles the corrections given by this term, one runs the risk of incurring considerable errors, especially where the surveys are being carried out in high latitudes ".