PRECISE DETERMINATION OF THE GIVRY CORRECTION

by

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1) The Givry correction, which bears the name of the hydrographic engineer who first demonstrated its necessity and calculated its expression, is the difference between the azimuths of the loxodrome and the orthodrome joining two points A and B on the surface of the earth. As this quantity is principally used in graphic constructions, the flattening of the earth is neglected.

If the two points under consideration are not very far apart, it is permissible to assume that the difference in azimuth between the two curves is the same at the two extremities of the arc AB and that consequently it is equal to one half the convergence of the meridians between the two points; this is given very exactly by the Napier analogy :—

$$(1) \qquad \operatorname{tg} \frac{\gamma}{2} = \operatorname{tg} \frac{G}{2} \frac{\sin \frac{\varphi_2 + \varphi_1}{2}}{\cos \frac{\varphi_2 - \varphi_1}{2}}$$

where φ_1 and φ_2 designate the latitudes of the points under consideration and G their difference in longitude.

Frequently the approximate formula will suffice :—

$$(1\text{ a}) \qquad \gamma = G \sin \varphi_0$$

by calling φ_0 the mean of the latitudes φ_1 and φ_2 ; but the double entry tables which give the value of k to be used for the Givry correction are generally established with the aid of the equation

$$(1\text{ b}) \qquad \operatorname{tg} k = \operatorname{tg} \frac{G}{2} \sin \varphi_0$$

having an accuracy comparable with that of formula (1 a).

2) The use of radiogoniometry in maritime and aerial navigation has led to the application of the Givry correction to pairs of points separated by several hundred miles.

The Givry hypothesis, which is tantamount to the merging of the image of the orthodrome on the Mercator projection with its osculatory circle between the points A and B, still remains valid; the error which is involved is practically negligible for distances up to a thousand miles, with respect to the accuracy of radiogoniometric bearings which, at present, is about one degree.

If however one is obliged to use greater distances or if, as a result of improvements in the science of radiogoniometry, one is forced to seek even greater accuracy, then it will be necessary to add a corrective factor to the ordinary expression for the Givry correction, or to have resort to a more rigorous formula.

3) In his *Traité d'Hydrographie* (1882), the hydrographic engineer GERMAIN shows the corrective factor equal to $-\frac{G^2 \cot Z_1}{12}$ or $-\frac{m^2 \sin 2 Z_1}{\cos^2 \varphi_1}$ representing by Z_1 the azimuth of the orthodrome at the point A of latitude φ_1 and by m the distance AB (expressed in terrestrial radians);

These terms may also be written $-\frac{G\, d\varphi}{12 \cos \varphi_0}$ or $\frac{G}{12}(S_1 - S_2)$ by putting $\varphi_2 - \varphi_1 = d\varphi$ and $S(\varphi) = \text{Log} \tan\left(45^\circ + \frac{\varphi}{2}\right)$ the meridional part on the sphere. It is in this latter form $\frac{G}{12}(S_1 - S_2)$ that it is given in the German and Italian works treating of the practical manner of obtaining the Givry correction. These works contain two double entry tables, one of which gives the principal part of the Givry correction by the application of formula (1 b) mentioned above; $\tan k = \tan \frac{G}{2} \sin \varphi_0$, the other giving the quantity $W = GS/12$ as a function equally of G and of φ.

The method adopted in these works is the following :—

There is first determined an approximate value k of the correction from the first table. Then one looks for the exact semi-convergence of the meridians; this is given by $\tan \frac{\gamma}{2} = \frac{\tan k}{\cos \frac{d\varphi}{2}}$ which may be written $\tan k = \tan \frac{\gamma}{2} \sin\left(90^\circ - \frac{d\varphi}{2}\right)$. In this form we see that $\frac{\gamma}{2}$ may be obtained as a function of k and $d\varphi$ equally with the aid of the first table by entering it in a different manner and provided also that the value of the latitude, which is usually stopped at 60 or 70°, is extended to the vicinity of the pole, $90^\circ - \frac{d\varphi}{2}$ reaching nearly to 90°.

Then one enters the second table for the pairs of values G, φ_1 and of G, φ_2, the corrective factor is equal to the difference of the values of W furnished by the table.

The Givry correction to be used is finally $\frac{\gamma}{2} + W'_1 - W_2$, which is exact to about the 4th order by considering G and $d\varphi$ as infinitely small of the first order of magnitude.

But this method is not very advantageous because it necessitates the extension of the usual tables as far as the poles, the introduction of a special table to furnish the quantity W, and finally four entries into the tables to obtain the correction.

4). We shall see that it is possible to obtain the Givry correction with about the same degree of accuracy by a more simple method, based on entering the usual table, not with the mean latitude φ_0, but with an auxiliary latitude given in a special table as a function of φ_0 and of $d\varphi$.

Let us designate by Z_1 the azimuth of the orthodrome at A' and by x the difference sought between the azimuth of the orthodrome and that of the loxodrome, which will consequently be $Z_1 + x$.

By calling, as we have stated, S the meridional part, we have immediately $\tan (Z_1 + x) = \frac{G}{S_2 - S_1}$

On the other hand the Napier analogy gives : $\tan\left(Z_1 + \frac{\gamma}{2}\right) = \frac{\cos \varphi_0}{\sin \frac{d\varphi}{2}} \tan \frac{G}{2}$.

We see therefore, that it is easy to obtain x as a function of the given quantities by the elimination of Z_1 between the two equations.

The calculation is simplified a little if we resort to the use of hyperbolic functions. In fact the relation $S = \text{Log} \tan \left(45^\circ + \frac{\varphi}{2}\right)$ may be written $th\ S = \sin \varphi$; $sh\ S = \tan \varphi$, etc.

We deduce from this $\text{th} \frac{S_2 - S_1}{2} = \frac{\sin \frac{d\varphi}{2}}{\cos \varphi_0}$ and consequently $\tan \left(Z_1 + \frac{\gamma}{2}\right) = \frac{\tan \frac{G}{2}}{\text{th} \frac{S_2 - S_1}{2}}$.

By combining this equation with $\tan (Z_1 + x) = \frac{G}{S_2 - S_1}$ we have

$$\text{tg}\left(\frac{\gamma}{2} - x\right) = \frac{\frac{S_2 - S_1}{2} \text{tg} \frac{G}{2} - \frac{G}{2} \text{th} \frac{S_2 - S_1}{2}}{\frac{S_2 - S_1}{2} \text{th} \frac{S_2 - S_1}{2} + \frac{G}{2} \text{tg} \frac{G}{2}} = \text{tg} \frac{G}{2} \text{th} \frac{S_2 - S_1}{2} \frac{\frac{\frac{S_2 - S_1}{2}}{\text{th} \frac{S_2 - S_1}{2}} - \frac{\frac{G}{2}}{\text{tg} \frac{G}{2}}}{\frac{S_2 - S_1}{2} \text{th} \frac{S_2 - S_1}{2} + \frac{G}{2} \text{tg} \frac{G}{2}}$$

However $\text{tg} \frac{G}{2} = \text{tg} \frac{\gamma}{2} \frac{\cos \frac{d\varphi}{2}}{\sin \varphi_0}$, consequently $\text{tg} \frac{G}{2} \text{th} \frac{S_2 - S_1}{2} = \text{tg} \frac{\gamma}{2} \frac{\sin d\varphi}{2 \sin 2\varphi_0}$.

Putting finally, to simplify the terms; $\tan \frac{G}{2} = u \quad \text{th}\frac{S_2 - S_1}{2} = v$ and using the development in series $\frac{G}{2} = u - \frac{u^3}{3} + \frac{u^5}{5} - \frac{u^7}{7} + \ldots$

$$\frac{S_2 - S_1}{2} = v + \frac{v^3}{3} + \frac{v^5}{5} + \frac{v^7}{7} + \ldots$$

By substituting in the expression for $\tan\left(\frac{\gamma}{2} - x\right)$ we have after simplification :—

$$\text{tg}\left(\frac{\gamma}{2} - x\right) = \text{tg}\frac{\gamma}{2}\frac{\sin d\varphi}{\sin 2\varphi_0}\frac{\frac{1}{3} + \frac{v^2 - u^2}{5} + \frac{v^4 - u^2v^2 + u^4}{7} + \ldots}{1 + \frac{v^2 - u^2}{3} + \frac{v^4 - u^2v^2 + u^4}{5} + \ldots}$$

$$= \text{tg}\ \frac{\gamma}{2}\frac{\sin d\varphi}{3\sin 2\varphi_0}\left[1 + \frac{4}{15}(v^2 - u^2) + \frac{8}{35}(v^4 - u^2v^2 + u^4) - \frac{4}{45}(v^2 - u^2) + \ldots\right]$$

$$= \text{tg}\ \frac{\gamma}{2}\frac{\sin d\varphi}{3\sin 2\varphi_0}\left[1 + \frac{4}{15}(v^2 - u^2) + \frac{4}{315}(11\,v^4 - 4\,u^2v^2 + 11\,u^4) + \ldots\right]$$

We then deduce, by neglecting the 5th order :

$$\text{tg}\,x = \text{tg}\frac{\gamma}{2}\left[1 - \frac{\sin d\varphi}{3\sin 2\varphi_0}\left[1 + \text{tg}^2\frac{\gamma}{2} + \frac{4}{15}(v^2 - u^2)\right]\right]$$

And by neglecting the 4th order

$$(2) \qquad \text{tg}\,x = \text{tg}\frac{\gamma}{2}\left(1 - \frac{\sin d\varphi}{3\sin 2\varphi_0}\right) = \text{tg}\frac{G}{2}\ \frac{\sin \varphi_0}{\cos\frac{d\varphi}{2}}\left(1 - \frac{\sin d\varphi}{3\sin 2\varphi_0}\right).$$

If we call the *auxiliary latitude* the angle ψ defined by :

$$(3) \qquad \sin \psi = \frac{\sin \varphi_0}{\cos\frac{d\varphi}{2}}\left(1 - \frac{\sin d\varphi}{3\sin 2\varphi_0}\right)$$

We have (4) $\tan x = \tan\frac{G}{2}\sin\psi$, equation in the form analogous to equation (1 b) giving the usual correction *k* as a function of *G* and of φ_0. The tables applying the formula (1 b) may therefore be utilized directly for obtaining the complete Givry correction *x*, provided there is attached thereto a table giving the auxiliary latitude ψ as a function of φ_0 and of $d\varphi$.

Since 1937 the work "*Radiosignaux à l'usage des Navigateurs*" published by the Hydrographic Office of the French Navy, contains a table of the auxiliary latitude (*), as well as a diagram giving the auxiliary latitude (formula 3) and the Givry correction (formula 1 b).

(*) *Note by the I.H.B.* — The table for the auxiliary latitude is reproduced at the end of this article by permission of the *Service Hydrographique*.

5) Starting from the results mentioned above, we readily find other expressions for x. In fact, since $\tan \frac{\gamma}{2} \frac{\sin d\varphi}{\sin 2\varphi_0} = \tan \frac{G}{2} \operatorname{th} \frac{S_2 - S_1}{2}$ we may write :

$$\operatorname{tg} x = \operatorname{tg} \frac{\gamma}{2} - \frac{1}{3} \operatorname{tg} \frac{G}{2} \operatorname{th} \frac{S_2 - S_1}{2}$$

and, neglecting always the 4th order :

$$x = \frac{\gamma}{2} - \frac{G(S_2 - S_1)}{12}$$

the formula in the German works.

In this form, we see that the corrective term $\frac{G(S_2 - S_1)}{12}$ is proportional to the surface of the rectangle triangle formed on the Mercator projection by the loxodrome AB, the meridian at A and the parallel at B: and we thus have a simple means of evaluating and determining the magnitude of this term.

For the rest $S_2 - S_1 = G \cot (Z_1 + x)$ and consequently,

$$x = \frac{\gamma}{2} - \frac{G^2 \cot (Z_1 + x)}{12}.$$

By neglecting the terms of the third order we arrive at the formula given in the Germain treatise : $x = \frac{\gamma}{2} - \frac{G^2 \cot Z_1}{12}$.

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In this treatise (pp. 183 to 187) we find an article devoted to the "correction which must be applied to the azimuth before being laid down on the mercator chart". The expression for the correction is given to the first order, but the author refers to a more profound treatment of the question appearing in the Essay on astronomy given before the Faculty of Sciences in 1837 at Lyon by Ensign Bravais.

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(*) *Here after reproduced by the care of I.H.B.*

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TABLE PROVIDING THE AUXILIARY LATITUDE Ψ

(Extracted from Ouvrage N° 11 - 2 - Radiosignaux à l'usage des navigateurs, 1er volume - published by Service Hydrographique de la Marine, Paris, 1938).

LATITUDE MOYENNE φ_0	ÉCART EN LATITUDE $d\varphi$												
	LATITUDE MOYENNE NORD.												
	− 30°.	− 25°.	− 20°.	− 15°.	− 10°.	− 5°.	0.	+ 5°.	+ 10°.	+ 15°.	+ 20°.	+ 25°.	+ 30°.
	LATITUDE MOYENNE SUD.												
	+ 30°.	+ 25°	+ 20°	+ 15°.	+ 10°.	+ 5°.	0	− 5°.	− 10°.	− 15°.	− 20°.	− 25°.	− 30°.
°	°	°	°	°	°	°	°	°	°	°	°	°	°
0..........	5,2	4,3	3,4	2,5	1,7	0,8	0,0	− 0,8	− 1,7	− 2,5	− 3,4	− 4,3	− 5,2
2..........	7.3	6,3	5,4	4,5	3,7	2,8	2,0	+ 1,2	+ 0,3	− 0,5	− 1.4	− 2,2	− 3,1
4..........	9,4	8,4	7,5	6,6	5,7	4,8	4,0	3,2	2,3	+ 1,5	+ 0,7	− 0,2	− 1,1
6..........	11,5	10,5	9,5	8,6	7,7	6,8	6,0	5,2	4,3	3,5	2,7	+ 1,8	+ 1,0
8..........	13.6	12,6	11,6	10.6	9,7	8.9	8,0	7,2	6,3	5,5	4,7	3,9	3,0
10..........	16,8	14,7	13,7	12,7	11,8	10,9	10,0	9.2	8,3	7,5	6,7	5,9	5,1
12..........	17,9	16,8	15,8	14,8	13,8	12,9	12,0	11,1	10.3	9,5	8,7	7,9	7,1
14..........	20,1	18,9	17,9	16,8	15,8	14.9	14,0	13·1	12,3	11,5	10.6	9,8	9,1
16..........	22,3	21,1	20,0	18,9	17,9	16.9	16,0	15,1	14,3	13,4	12,6	11,8	11,0
18..........	24,5	23,3	22,1	21,0	19,9	18,9	18,0	17,1	16,2	15,4	14,6	13,8	13,0
20..........	26,8	25,4	24,2	23,1	22,0	21,0	20,0	19,1	18,2	17,3	16,5	15,7	14,9
22..........	29,0	27,6	26,4	25,2	24,0	23,0	22,0	21,1	20,1	19,3	18,5	17,7	16,9
24..........	31,3	29,9	28,5	27,3	26,1	25,0	24,0	23,0	22,1	21,2	20,4	19,6	18,8
26..........	33,7	32,1	30,7	29,4	28,2	27,1	26,0	25,0	24,1	23,2	22,3	21,5	20,7
28..........	36,0	34,4	32,9	31,6	30,3	29,1	28,0	27,0	26,0	25,1	24,2	23,4	22,6
30..........	38,5	36,7	35,2	33,7	32,4	31,2	30,0	28,9	27,9	27,0	26,1	25,2	24,4
32..........	40,9	39,1	37,4	35.9	34,5	33,2	32,0	30.9	29,8	28,9	27,9	27,1	26,2
34..........	43,5	41,5	39,7	38,1	36,6	35,3	34,0	32,8	31,7	30,7	29,8	28,9	28,0
36..........	46,1	44,0	42,1	40,3	38,8	37,3	36,0	34,8	33,6	32,6	31,6	30,7	29,8
38..........	48,6	46,5	44,4	42,6	40,9	39,4	38,0	36,7	35,5	34,4	33,4	32,4	31,5
40..........	51,6	49,1	46,9	44,9	43,1	41,5	40,0	38,6	37,4	36,2	35,1	34,1	33,2
42..........	54,5	51,8	49,4	47,2	45,3	43,6	42,0	40,6	39,2	38,0	36,9	35,8	34,8
44..........	57,7	54,6	52,0	49,6	47,6	45,7	44,0	42,5	41,0	39,8	38,6	37,4	36,4
46..........	61,0	57,6	54,6	52,1	49,8	47,8	46,0	44,4	42,8	41,5	40,2	39,0	37,9
48..........	64,7	60,8	57,4	54,6	52,1	50,0	48,0	46,2	44,6	43,1	41,8	40,5	39,4
50..........	69,0	64,2	60,4	57,3	54,5	52,1	50,0	48,1	46,3	44,8	43,3	42,0	40,7
52..........	74,3	68,1	63,7	60,0	57,0	54,3	52,0	49,9	48,0	46,3	44,8	43,3	42,0
54..........	82,4	72,8	67,2	63,0	59,5	56,6	54,0	51,7	49,7	47,8	46,1	44,6	43,1
56..........	•	79,2	71,3	66,2	62,2	58,9	56,0	53,5	51,3	49,2	47,4	45,7	44,2
58..........	•	•	76,6	69,8	65,0	61,2	58,0	55,2	52,8	50,6	48.6	46,7	45,0
60..........	•	•	86,0	74,1	68,1	63,6	60.0	56,9	54,2	51,8	49,6	47,6	45,7
62..........	•	•	•	79,8	71,5	66,2	62,0	58,5	55,5	52,8	50,4	48.2	46,2
64..........	•	•	•	•	75,7	68,9	64,4	60,0	56,7	53,7	51,1	48,6	46,4
66..........	•	•	•	•	81,4	71,8	66,0	61,5	57,7	54,4	51,5	48,8	•
68..........	•	•	•	•	•	75,2	68,0	62,8	58,5	54,9	51,6	•	•
70..........	•	•	•	•	•	79,5	70,0	63,9	59,1	•	•	•	•
72..........	•	•	•	•	•	87,5	72,0	64,8	59,3	•	•	•	•