A STUDY OF PHOTOTHEODOLITE CAMERAS

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INTRODUCTION.

The construction of phototheodolites involves the determination of each camera focal length in accordance with its adjustment. For this purpose, the photographs of surveyed points can be used. The focal length may be inferred from the measurements of the opposite intervals of the sighting point pictures and from the presumably known angles of field of vision of the sighting points. We will in the first place deduce the restitution formulae and at the same time give a few practical examples. A similar study has been published by WERKMEISTER (1). Thereby the operation is restricted to the deduction of the camera focal length and the plate principal point position from a few aiming situated close to the plate margin, so that the distorsion errors would not be assessed.

DEDUCTION OF THE FORMULA FOR THE DETERMINATION OF THE FOCAL LENGTH AND POSITION OF THE PLATE PRINCIPAL POINT.

The knowledge of the focal length and position of the plate principal point is needed for the restitution of photographs with phototheodolites. The plate principal point should be understood to mean the foot of the perpendicular from the centre of projection to the image plane. As the image originates from a central projection, the picture side projection centre of which is the central point of the exit pupil, so the perpendicular also falls from the central point of the exit pupil to the plate plane. This point is often wrongly described as the point of intersection of the optical axis with the image plane. The plate principal point is generally recorded by a series of marks on the photographic plates. For instance, two pairs of points, printed at the same time as the image and whose connecting lines are perpendicular, are used for the purpose.

The point of intersection of these connecting lines is described as the registering frame center R. If the adjustment of the phototheodolite camera be correct, it must at least coincide with the plate principal point to the extent that, when plotting the plate, it can be considered with sufficient accuracy as the plate principal point. The focal length f, in connection with this investigation, is not to be taken in the sense of geometrical optics as paraxial focal length, but as a camera constant, which gives the distances from the image points to the plate principal point, the *sizes of the photograph,* 1', in the most accurate possible support of the simple formula

$$(1) \quad l' = f \text{ tang } \omega$$

as a function of the angle of the field of vision ω.

The photogrammetric calculation is based on the fact that the angle of inclination of the object side of the bundle principal beam ω is equal to that of the picture side ω'_i. The projection centres must also possess the characteristics of the nodal points, that is, the pupils must coincide with the nodal planes. This requirement places the diaphragm in one of the two principal planes.

(1) *Determination of the inner orientation of a phototheodolite camera. Zeitschrift für Instrumentenkunde 50, 1930, pages 246-254.*

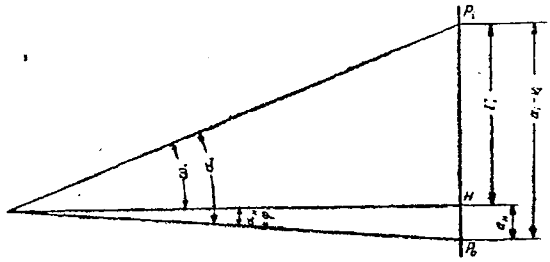

FIG. 1.

We would suggest the following :

Let us take for any point whatever P_i, l'_i for the free from lens distortion picture size, $l'_i + v_i$ for the measured and distorted picture size, as well as ω_i; and ω'_i for the objet side and picture side angles of inclination respectively. Here, the angles of the direction of the perpendicular from the projection center to the picture plane and the picture sizes from the foot of this perpendicular to the picture plane are to be calculated. The signs l'_i ω_i and ω'_i respectively, departing from the standard use of geometric optics — must always be the same. All picture sizes on the one side of the axis and the angles of field of vision belonging thereto must be reckoned as positive, and negative on the other side. The distortion v_i is positive, if the actual picture point, deduced from the free from lens distortion picture point is shifted in the direction of the growing picture sizes: Of course, v_i does not represent only the error of lens distortion but includes the error of measurement as well. However, these two errors can be separated only on the assumption that the progress of distortion has some functional relation to the picture size or the angle of field of vision.

When examining a newly constructed phototheodolite camera, the exact position of the plate principal point is at first supposed to be unknown. Let us take, in Fig. 1; H for the plate principal point and P_0 for a near picture sighted as the same time. The actually measured picture size of a picture point P_i, must be deduced from the point P_0 and designated as a_i. We have therefore $a_i = P_0 P_i$. This picture size a_i corresponds to the angle of field of vision α_i. The signs a_i and α_i being always the same, and indeed in agreement with the selected signs l_i and ω_i. On Fig. 1, P_i might be the free from lens distortion picture point of any sighted point. We therefore obtain the following illustration :

$$l'_i = a_i - v_i - a_H \quad \text{and} \quad l'_0 = - a_H$$

$$\omega_i = \alpha_i - \varphi \qquad \omega_0 = - \alpha_H = - \varphi$$

and :

$$l'_i = f \operatorname{tg} \omega_i$$

(2) $$v_i = a_i - a_H - f \operatorname{tg} (\alpha_i - \varphi).$$

Under the condition to be complied with by these photographs that φ is only small, it is possible, according to Taylor's development, to obtain the formula :

$$f(x + h) = f(x) + h f'(x)$$

and by taking away what follows the second term :

(3) $$v_i = a_i - a_H - f \left\{ \operatorname{tg} \alpha_i - \frac{\varphi}{\rho} \frac{1}{\cos^2 \alpha_i} \right\}$$

φ must be expressed in seconds or minutes of arc, and correspondingly ρ stands for the quantity of seconds or minutes of arc, so that $\rho = 206\,265''$ or $3438'$. As the abscissa of the plate principal point a_H is ranging about one or more tenths of a millimeter, the freedom from lens distortion may be taken for granted and we have :

(4) $$a_H = f \operatorname{tg} \varphi \doteq f \frac{\varphi}{\rho}$$

The error of distortion of point P_i will then be :

$$(5) \quad v_i = a_i - f \left\{ \operatorname{tg} \alpha_i - \frac{\varphi}{\rho \cos^2 \alpha_i} + \frac{\varphi}{\rho} \right\}.$$

Having regard for point P_0 to an error of distortion or what is perhaps more manifest to an error of measurement, the value a_H whould be replaced by $a_H - v_H$. On principle, the error v_H is determined by means of adjustment. In this case, we start from the formula:

$$(6) \quad v_i - v_H = a_i - a_H - f \operatorname{tg} (\alpha_i - \varphi).$$

In equation (2) v_i should be replaced by the difference $v_i - v_H$ only. On the other hand, in the representation adopted by us, each error v_i stands for the difference between the error of P_i and that of P_0.

THE ADJUSTMENT OF MEASUREMENTS.

In principle, the focal length and the plate principal point position can be calculated strictly on the basis of only three sighting points, by means of equation (5), if the determination of the distortion is left out. The main advantage of the calculation by compensation is that many more collimating points can be utilised, and apart from the quantities sought for, it affords evidence of its accuracy and finally makes the application of the error of distortion as no longer evanescent.

Before proceeding with the compensation proper, let us take an approximative value for the focal length f and lay down $f = f_0 + \Delta f$ so as to have only small quantities to determine for the compensation ($\Delta f, \varphi, v_i$) and be satisfied with a relatively rough calculation. After leaving out the quadrate terms in the quantities φ and Δf, the formula becomes

$$(7) \quad v_i = a_i - f_0 \operatorname{tg} \alpha_i + \frac{f_0 \varphi}{\rho} \operatorname{tg}^2 \alpha_i - \Delta f \operatorname{tg} \alpha_i.$$

for the sake of abbreviation

$$(8) \quad A_i = - \operatorname{tg} \alpha_i, \quad B_i = \frac{f_0}{\rho} \operatorname{tg}^2 \alpha_i. \quad \omega_i = a_i - f_0 \operatorname{tg} \alpha_i,$$

we then obtain the equation of error :

$$(9) \quad A_i \Delta f + B_i \varphi + \omega_i = v_i \qquad \text{for } i = 1, 2, \ldots\ldots k.$$

The normal equations are then deduced thereupon in the usual symbolic form of compensation calculation according to the least square method :

$$(10) \quad \begin{aligned} [AA] \Delta f + [AB] \varphi + [A\omega] &= 0 \\ [AB] \Delta f + [BB] \varphi + [B\omega] &= 0 \end{aligned}$$

in order to check the formation of totals, we also calculate the auxiliary quantities

$$(11) \quad S = A + B + \omega$$

and the product totals

$$[SA], \quad [SB], \quad [S\omega], \quad \text{et} \quad [SS].$$

should the values of quantities A, B and ω be very different, it would be advisable to select new units for quantities φ, Δf and v, before proceeding with the calculation of the totals, the error of distortion v in μ or the angle error φ being expressed, for instance, in 100" as unit. For this adjustment, it is usual to omit to attribute various weights to the respective measuring points, as, in the case of lenses somewhat free from distortion the result is practically undependent of the weights and calculation with weights requires greater labour.

In practice, the solution of normal equation and determination of errors are obtained through a special schema, so that restitution can also be carried out by auxiliary means which naturally must be worked in to that effect.

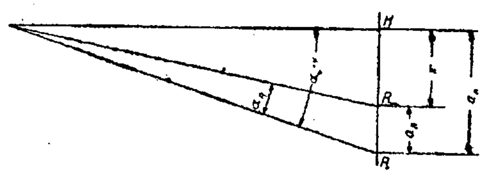

FIG. 2.

The relations between the registering frame centre and the plate principal point are shown in Fig. 2. If we describe by R the registering frame center and by a_R its co-ordinate, which is also determined by the admeasurement of the plate, we obtain

$$(12) \quad R\,H = x = a_H - a_R$$

which gives the desired value for the position of the plate principal point, in relation to the registering frame center.

METHOD OF CALCULATION.

The plate is measured with a measuring machine and the picture sizes $a_i = P_0\ P$ are determined for a series of points. Two approximate values are calculated from two points placed as symetrically as possible, for the focal length, by means of formula :

$$f = \frac{a_i}{tg\ \alpha_i}$$

The mean value of both numbers is selected as the approximate value f_0. This method of determination of the approximate value f_0 has the advantage that the distance $P_0 H$, which is still unknown, gives rise only to an insignificant error in the approximate value f_0.

The quantities A_i, B_i, ω_i are then calculated by means of (8) and S_i by means of (11) and finally the product totals [AA], [AB] etc..., which appear in (10) are made up. Then comes the solution of the ordinary equations (10) by the usual process of adjustment calculation. We obtain thereby Δ f and φ as well as their mean errors. Finally the errors of distortion v_i are calculated, quite simply indeed, from formula (9) as only small numbers are to be determined, for which a slide rule is quite adequate.

RESULTS OBTAINED FROM A FEW PHOTOTHEODOLITE CAMERA MEASUREMENTS.

Table 1 gives the focal length f and the distances x from the plate principal point to the registering frame center R, for two phototheodolite cameras, as well as their mean errors (m.F.) resulting from adjustment. The cameras are fitted with Askania phototheodolite objectives type R, with a focal length of 37 cm. and 1 : 5,5 ratio of lens aperture. The designing of this objective entails a maximum freedom from distortion.

It is to be noted on Table 1, that the focal length is obtained with sufficient accuracy, but that the mean error for the plate principal point represents in both cases a quantity similar to the sought for value itself. The plate principal point error is even frequently considerably smaller than its mean error. In the cases given in Table I, the plate principal point error is always still smaller than three times the mean error, which means that the value of the plate principal point error is not guaranteed.

TABLE I. — CAMERA CONSTANTS.

N° of the apparatus.	f	m.F.	x	m.F.
304 735	369,83	± 0,015	+ 0,24	± 0.009
304 736	369,86	± 0,019	— 0,18	± 0.013

In Fig. 3 for both the above mentioned cameras, the residual error v_i in relation to the picture size l'_i and the angle of vision ω, are represented as abscissa. The sighting points are designated by letters C, D, ... Z. in Fig. 3. As shown in Fig. 3, the residual errors (2) are throughout under 0,01 mm. No regular line that might be construed as representing distortion is to be observed. The errors v_i occur rather quite irregularly and are therefore to be considered mainly as errors of measurement. It can be inferred from these curves that the actual errors of distortion are well under 0.01 mm. As, from these curves, the degree of accuracy of these measurements, is assumed to be about 0.01 mm., the objectives of these phototheodolite cameras may be described as "*completely free from lens distortion*".

SUMMARY.

The formulae for the determination of the position of the plate principal point and the focal length of the photo theodolite cameras have been computed from photographs of known sighting points. The results of two such calculations are given as examples and discussed.

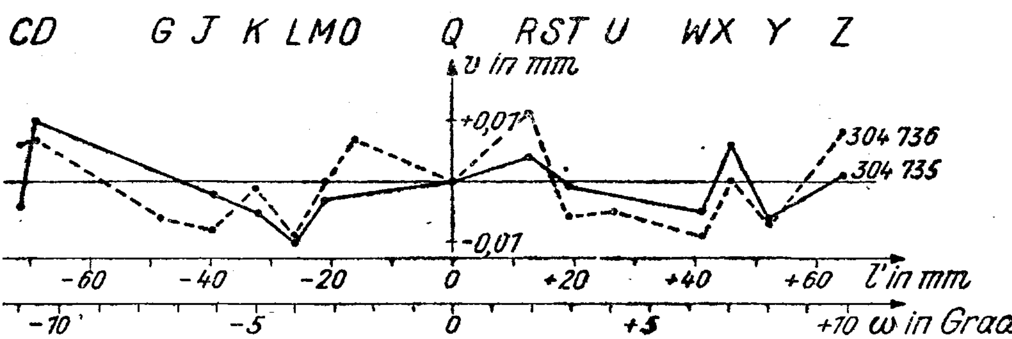

FIG. 3.

(2) *Under more favorable weather conditions than those prevailing at the time when the photographs were taken, on account of the refraction anomalies over the roofs of a great City the errors of measurement, should be sensibly smaller.*