

BEARINGS CARD AND POSITION CHARTS BY RADIOGONIOMETRY

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In order to utilise radiogoniometric bearings, a compass card must be available at the spot from which they are taken. If the employed projection were conformal, the card divisions would be equidistant, but the bearing beams would be curved lines. In order to make them into straight lines, use is made of the gnomonic projection which is not conformal, except in its centre.

Let P be the earth's pole, M, the centre of the projection, O the spot from which bearings are taken; P' M' and O', the representations of these points on the gnomonic projection with centre M', d, the distance OM, a, the bearing of any point, z, that of the centre M taken from point O, a' and z', the distorted angles on the chart.

The author calculates the difference $a - a'$, and denoting by δ the quantity $\tan^2 \frac{d}{2}$ reaches the expression :

$$\tan (a - a') = \delta \frac{\sin 2(a - z) + \sin 2z + \delta \sin 2a}{1 + \delta [\cos 2(a - z) + \cos 2z] + \delta^2 \cos 2a},$$

which, for low values of δ , gives :

$$\tan (a - a') = \delta [\sin 2(a - z) + \sin 2z],$$

and, for low values of $a - a'$:

$$a - a' = \frac{d^2}{4} \tan I' [\sin 2(a - z) + \sin 2z],$$

which is generally sufficient in most practical cases.

The maximum distortion of the card occurs when a is equal to 90° or 270° ; it is then : $\pm \frac{d^2}{2} \tan I'$ for low values of d.

A more accurate result is obtained by adopting the variable x such as: $\sin 2x = \tan^2 \frac{d}{2}$.

The maximum distortion is then : $a - a' = \pm 4x$.

These extreme values occur on bearings equal to double the chart centre bearing.

The author gives a representation on a polar stereographic projection of the curves, which are places of the bearing taking spots where the maximum distortions occur for different values of the latitude of the employed projection centre, symmetric curves in respect of the central meridian should be added.

The author also calculated a development of $a - a'$, according to the even powers of d and gives a table of the maximum values of the first three terms of this development for different values of d.

By adopting the simplified formula given for $a - a'$, the maximum values of the differences will be given by :

$$\tan (a - a') = \tan^2 \frac{d}{2} (\sin 2z \pm 1).$$

The author compiled a simple diagram giving for each spot from which bearings were taken, the extreme values of $a - a'$, from the values of z and d and where the lines of equal extreme values of $\tan (a - a')$ are straight lines parallel to the directions $z = 45^\circ$ and $z = 135^\circ$.

Lastly, Professor Immler settles the question of the extent to which, a card computed for a place P (d and z) may also be employed for a neighbouring point P' (d' and z').

He puts

$$\tan \frac{d'}{2} = \sqrt{\sin \left(2z \pm \frac{\Delta}{2} \right)},$$

and fixes point P' by its polar coordinates p and q , in relation to centre O.

He then finds :

$$p = \frac{\pm 2 \Delta}{d \tan r' [1 + \sin (2z \pm q)]}$$

The boundary lines thus defined are arcs of parabolas, which limit a lens-shaped field whose thickness is half its diameter.

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