ALTITUDE POSITION LINES ON CHARTS ACCORDING TO GARCIA

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§ 1) Altitude tables.

In practice, altitude position lines are generally obtained by means of tables.

1° These tables may be mere logarithm tables. The calculation of observed star altitudes and azimuths is made through trigonometric formulae with trigonometric functions, whose logarithms are taken from an ordinary logarithm table, unless logarithms especially arranged for the purpose are available, such as for instance the well-known N° 7 of Fulst's nautical tables.

2° Use is made of tables permetting to do away with part of the logarithmic calculation, certain intermediate quantities of the solution are tabulated, by means of which calculation can be made shorter when following a definite schedule. The calculation of azimuth by means of table A, B, C, of Fulst's nautical tables is a wellknown example. As regards altitudes lines calculation, these tables are mostly based on the splitting up of the basic spherical triangle into two right-angled triangles. Table F is an example of this sort of table.

3° A third possibility is the tabulation of altitude and azimuth without the observer having any need of logarithmic calculations. These tables deserve the name " altitude tables " in the true acception of the word. It may be remarked, incidentally, that such tables may constitute hour angle and azimuth tables for the longitude method (1). As regards the altitude method, there are only Wedemeyer's and Ball's tables. Wedemeyer's tables are in some degree close-meshed ; they furnish the quantity Δ h in such a way that position lines must be drawn on stereographic instead of sea charts. Ball's tables are wider-meshed and require considerable interpolation work. Despite the fact that Wedemeyer's tables are excellent and possess great advantages for untrained personnel, they are not in general use. As regards Ball's tables an effort has been made (E. Krause, *Seewart* 1933, p. 214) to facilitate interpolation, but even that, did not make the tables any more popular.

J. Garcia, a Spanish Naval Officer, starts from the idea that actually, only altitude tables such as Wedemeyer's or Ball's really do away with calculation. The large size of these tables together with the interpolation work which they necessitate, will always stand against their propagation. J. Garcia has therefore asked himself the question of how this difficulty is to be met. He has consequently evolved a completely new method of work which will be described in what follows (2).

§ 2) Group of altitude position lines and selection of particular central curves.

J. Garcia's method merits special consideration, because it proceeds from quite general assumptions which are applicable not only to altitude position lines but also to other curves such as those of equal azimuth and great circles. The altitude curves of a heavenly body, for instance, on the earth's surface, are circles round the representation point of the star, as a centre. These circles form a group of curves with a common centre. On Mercator's chart, we have also in the altitude curves of a star a group of curves following a special mathematical law. Should we wish to draw one of these curves to correspond to an observed

(1) The Sumner Line of Position. Washington, 1924. Hydrographic Office, n° 203 and 204.
(2) Metodo de trazado de las curvas de altura. (*Revista General de Marina*, Nov. 1940.)

altitude, we could do it without calculation, if we possessed an altitude table containing the whole of these curves. Such a table should present for every curve an adequate series of pairs of values of geometrical co-ordinates φ and λ. The same considerations hold good for curves of equal azimuth or great circles, except that their concentration into one group is governed by other stand points than with altitude curves. Therefore what follows should only mention altitude curves and merely indicate occasionally the possibility of general application.

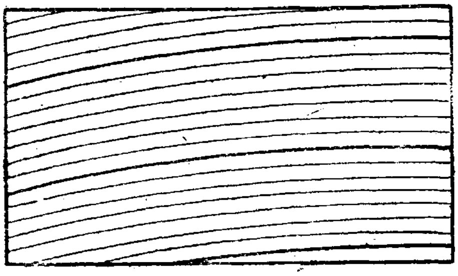

Fig. 1

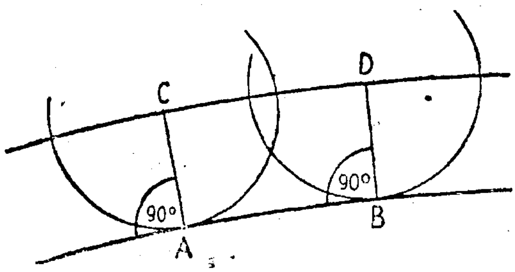

Fig. 2

It stands to reason that a table which contains sufficient data for all altitude curves round a representation point, would already prove very extensive. Still, for practical purposes, such a table would not be sufficient, more tables should be added for other representation points. Their great volume would prohibit the compilation of tables for drawing altitude curves so as to include from the outset all curves of every group, even if the theoretical infinite group were in practice limited to different minute-to-minute altitude curves. Garcia then set himself this problem : Is it possible to draw the other altitude curves, when only a limited number of them are tabulated ? Figure 1 shows a section of such a group of altitude curves on Mercator's chart. Garcia calls the thick drawn curves central curves. His question may then be put as follows : how is it possible with available tabulated elements for these central curves to obtain the other curves of the group ? Let now the altitude curve (*Fig.* 2) running through A and B, be any such afore-mentioned curve and the altitude curve through C and D, a central curve. It is first of all assumed that the altitude curve through A and B is already known. Perpendiculars are then raised in A and B and intersect the central curve in C and D. Let us now imagine in C and D, a circle with radius CA or DA, described round C and D as centres, the altitude curve through A and B is then a line tangent to both circles. We can imagine the same construction being made for all points between A and B. The altitude curve passing through A and B is tangent to all these circles and is characterised by Garcia as the envelope of all these circles. At each point of tangency the envelope curve may be replaced by an element of the tangent to the circle. The whole of the tangents put together give the envelope curve. Assuming that we know the radii of these circles, we can actually construct the altitude curve passing through A and B, by starting from the central curve passing through C and D, and draw a series of circles of a known radius, with points on the central curve as their centres, together with the tangent common to every two of them. The element of this tangent practically coincides with the envelope curve and consequently with the altitude curve when these circle centres are close enough to the central curve.

The law giving these circle radii for groups of curves will generally be somewhat involved and depend on the points of the central curve as well as on the gradients between the particular central curves. In the case of altitude position lines, this law is however very simple, as from curve to curve, with an interval space $\Delta h = 1$ minute, the distance is exactly one minute. On the sphere, distances between particular altitude curves are constant, on Mercator's chart, they follow the law of that chart's scale. When the central curves are drawn sufficiently close, it is at once possible to deduce intermediate altitude curves from them as the altitude difference between the central curve and the curve sought gives the radius of all particular circles to be drawn.

§ 3) Using a table of central curves for altitude curves.

The altitude table, which is to be used, according to Garcia, as a basis for drawing altitude position lines, must therefore permit the drawing of a series of central curves for every representation point. The construction of other intermediate altitude curves is then directly possible. How will this altitude table look ?

It is sufficient to calculate in advance a number of central curves for all representation points on one meridian, because although the representation of a heavenly body on the terrestrial sphere depends on its hour angle in relation to Greenwich, the outline of the altitude curves round the representation point does not change therewith. On the contrary, the position of individual altitude curves in relation to the representation point is given by the latitude and local hour angle of the heavenly body. Value pairs of latitude and hour angle therefore determine individual central curves. They must be indicated sufficiently close together, to make sure that the small circles drawn therefrom furnish envelope curves that actually coincide with the altitude curves sought for.

The representation point of the star on the meridian is determined by the star declination. The declination in this case is therefore the group value. It characterises a whole group of equal altitudes. But it is not advisable to chose it as a value for entering tables, as it is the only quantity for which no interpolation is necessary. It is more expedient to chose for entering tables the altitude which characterises the central curve. The tables will therefore look like the following illustration (see table 1).

For entering the table, take an even degree value of the altitude, as horizontal argument, the declination as vertical argument, the latitude and as hour angle, the tabulated value.

TABLE I

Part of a numerical altitude table, according to Garcia

φ \ δ	h = 20°												
	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°
40°	6329 *09*	6425 *09*	6520 *09*	6614 *09*	6707 *09*	6800 *09*	6851 *09*	6943 *08*	7033 *08*	7123 *08*	7215 *08*	7302 *08*	7351
41	6304 *09*	6401 *10*	6458 *09*	6554 *09*	6650 *09*	6744 *09*	6838 *09*	6931 *09*	7024 *09*	7116 *09*	7208 *09*	7259 *08*	7349
42	6236 *10*	6326 *10*	6436 *10*	6534 *10*	6631 *09*	6728 *09*	6824 *09*	6919 *09*	7014 *09*	7108 *09*	7202 *09*	7255 *09*	7347
43	6207 *10*	6310 *10*	6412 *10*	6512 *10*	6612 *10*	6711 *10*	6809 *10*	6906 *09*	7003 *09*	7059 *09*	7155 *09*	7250 *09*	7345
44	6137 *11*	6242 *11*	6346 *11*	6449 *10*	6551 *10*	6652 *10*	6753 *10*	6852 *10*	6951 *10*	7050 *10*	7147 *09*	7245 *09*	7342
45	6104 *11*	6212 *11*	6319 *11*	6425 *11*	6529 *11*	6633 *10*	6735 *10*	6837 *10*	6939 *10*	7039 *10*	7139 *10*	7239 *10*	7338
46	6031 *12*	6141 *12*	6250 *11*	6359 *11*	6506 *11*	6612 *11*	6717 *11*	6821 *11*	6925 *10*	7028 *10*	7130 *10*	7232 *10*	7333
47	5954 *12*	6108 *12*	6220 *12*	6331 *12*	6441 *11*	6549 *11*	6657 *11*	6804 *11*	6910 *11*	7015 *11*	7119 *11*	7224 *11*	7327
48	5916 *13*	6032 *13*	6148 *12*	6301 *12*	6414 *12*	6525 *12*	6636 *12*	6745 *11*	6854 *11*	7002 *11*	7109 *11*	7215 *11*	7321
49	5835 *13*	5955 *13*	6113 *13*	6230 *13*	6346 *12*	6500 *12*	6613 *12*	6725 *12*	6837 *12*	6947 *12*	7057 *11*	7206 *11*	7314

In order not to needlessly increase the volume of the table, a permutation is made, from a certain position of the altitude curves, of the vertical argument and tabulated value; because for a course in the region where the star azimuth is near 90°, the altitude curve cuts quite a number of latitude degrees in quick succession, it is therefore desirable to take latitudes for entering the table. When the star azimuth exceeds 45°, the points of intersection with the hour circles become more frequent, and consequently the hour angle is more expedient for entering the table. By printing the table in a special way, for instance, the top in black and the bottom in red, one can make sure that no confusion will occur.

The table may be arranged in a numerical form, like Table I or in a graphic one. As already pointed out, there should be little interpolation work to do, if declination be taken as horizontal entry, because the star declination is fixed by nature once for all. Garcia recommends also different types of print for tabulated values and interpolation values. The tabulated values are groups of 4 or 5 figures, the last two of which are minutes of arc, the preceding ones are degrees. Signs are purposely not incorporated in the table. Neither is it indicated, in the body of the table, but only in the directions for use, that interpolation values are tenths of minutes of arc. A special small auxiliary table is also used for interpolating, it is a sort of multiplication table, whose form does not differ from well-known similar ones.

If a graphic table form is selected, interpolation work can be saved and tabulated values can be furnished by scales, to a precision of one minutes (*Fig.* 3). This form of graphic representation corresponds to the scales of a sliding-rule. Reading off also results only from vertical entry, which is facilitated by the fact that the horizontal entry line is repeated from time to time. Garcia recommands to use a transparent rule with a line set on two of such horizontal scales, so as to obtain tabulated values with adequate accuracy.

The procedure for passing on to the plotting of altitude curves on the sea chart, according to Garcia, is therefore as follows; the nearest even degree is placed on the observed altitude.

Two points on the central curve should be determined for this altitude. The horizontal is entered with declination δ, which determines a vertical column that furnishes two pairs of values of latitude and hour angle. Entering is always done with latitude degrees nearest to the D. R. latitude, or else the hour angle would necessitate some calculation. Either entering is done with the even latitude degree which is found in the vertical entry, or, if it so happens that the nearest latitude degree of the D. R. latitude is no longer in the vertical entry, two tabulated values are sought in the already-found column, whose rounded off values no longer furnish two pertinent rounded-off values of the hour angle. From the hour angles thus obtained and by means of the hour angle of Greenwich meridian, the longitude of both points on the central curves is calculated. The Greenwich hour angle must always be computed by means of the Nautical Almanac. The two points, whose φ and λ have been determined are then plotted on the chart. Two circles are described with these two points as centres and the neglected minutes of the observed altitude as radii.

	0	1	2	3	4	5	6	7	8	9	10	11	12	
40	64	65	66	67	68	69	70	71	72	73				
41	64	65	66	67	68	69	70	71	72	73				
42	63	64	65	66	67	68	69	70	71	72	73			
43	63	64	65	66	67	68	69	70	71	72	73			
44	62	63	64	65	66	67	68	69	70	71	72	73		
45	62	63	64	65	66	67	68	69	70	71	72	73		
46	61	62	63	64	65	66	67	68	69	70	71	72	73	
47	61	62	63	64	65	66	67	68	69	70	71	72	73	
48	60	61	62	63	64	65	66	67	68	69	70	71	72	73
49	60	61	62	63	64	65	66	67	68	69	70	71	72	73
	0	1	2	3	4	5	6	7	8	9	10	11	12	

FIG. 3

The tangent to the two circles is the altitude curve sought. The arrangement of the tables must be in such a form that the rectilinear plotting of this tangent is always sufficiently accurate as to coincide with the altitude curve. On this account, it will be sufficient, for instance, to give the table entries from degree to degree between 10° and 80°, over and under these quantities, half-degrees should be shown as entries. For higher latitudes, it may even be necessary to allow vertical entries to proceed by half degrees or even closer intervals. In higher latitudes, it may also be desirable to draw a curved altitude line. In this case, not only two value pairs of φ and λ but a larger number are taken and therewith a broken tangential line is drawn.

The obvious advantage of this method is that the calculation of the azimuth is not necessary. Moreover, this method is certainly the only one which permits, in a simple manner, the approximate drawing of curved altitude lines, because the taking of more points on the central curve is practically no more trouble than taking only two. A volume of Garcia's altitude tables contains about 500 pages of normal book size.

§ 4) Example.

On September 7th 1940, from a position whose D. R. co-ordinates were $\varphi = 46° 30'$ N., and $\lambda = 9° 15'$ W., was measured a sun altitude of 20° 2.5', the observer's elevation being 5 m. The calculation of the hour angle was made somewhat differently from the usual practice aboard German vessels. At the same time, the Greenwich hour angle was calculated :—

Chronometer	17 h. 7 m. 37 s. 5	$\odot$ 20° 2',5	δ = 5° 56' 0 N.
Error	— 3 m. 11 s. 5	+ 9',5	
G. M. T.	17 h. 4 m. 26. s. 0	20° 12',0	
E.	— 2 m. 6 s.		
GHA	17 h. 2 m. 20 s.		

t (W Greenwich) 5 h. 2 m. 20 s. in degrees = 73° 35'.

Enter the table with the altitude nearest to the one observed, therefore 20° 12' in the table for h = 20° and with the same sign as δ. For the central curve h = 20°, three pairs of values of φ, λ, are found, in order to prove that this requires hardly any more work than finding the two indispensable pairs of values. In the numerical table, we would enter column δ = 6° and by using a small interpolation table which can be contained in a single sheet and detached from the table, we would obtain for latitudes 45°, 46° and 47° the corresponding values of the local hour angle. With the table in graphic form these three values of *t* are determined directly under δ = 5° 56'.

The following pairs of values are thus obtained :—

φ = 45°	t = 67° 31'	λ = 8° 42' W.
φ = 46°	t = 67° 13'	λ = 8° 22' W.
φ = 47°	t = 66° 53'	λ = 8° 42' W.

The value of λ, given in the last column, are derived directly from the local hour angle and Greenwich hour angle in degrees = 75° 35'. The three points, determined by the aforementioned pairs of values of φ and λ, are points on the central curve for the altitude line 20° at the time of the measurement. They are plotted on the chart (*Fig.* 4) and round them as centres are described three circles with a radius of 12 nautical miles, this distance of 12 miles being taken off the side of the chart. The tangents to be drawn are described in the west as the heavenly body is in the west. The same consideration applies as for the plotting of the altitude position line through the intercept ht-hc. The position line is the curved line furnished by the tangents, whose curve certainly remains imperceptible in the present case.

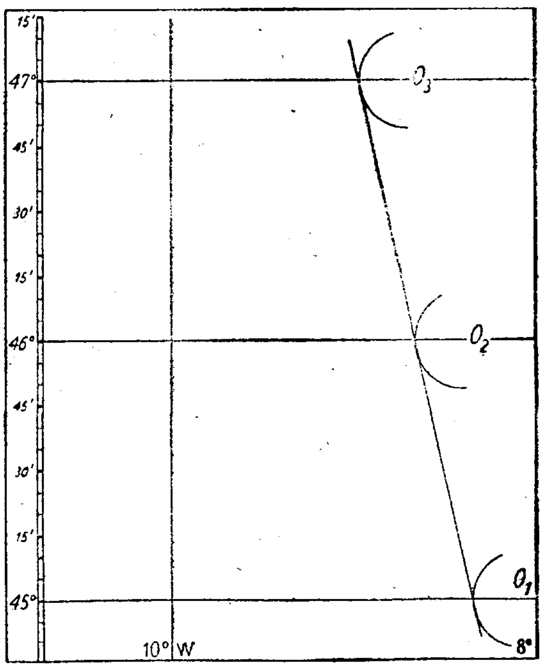

FIG. 4

§5) A few critical remarks.

The method is a new departure, especially if one considers that it is quite extensively based on the notion of curve groups. There exists in it a certain reminiscence of the position line chord method as well as of the Longitude method (*Navigation Hand-book,* by Meldau-Steppes, Edition 1935-1945, § 107), likewise of the latitude method (*ditto,* § 116), and equally of the already mentioned tables for the longitude method of the Hydrographic Office of Washington. Still, Garcia's method is superior to the last-mentioned system in so far as it is applicable to every position of the altitude line, because Garcia contrives to select another arrangement for his tables when azimuths are between 45° and 135° or 225° and 315°. The use of Garcia's method assumes that plotting is made on a chart or a Summer chart (or on cross-section paper). Navigation according to a position line is possible only if this line has been plotted, not only calculated. Now, it is often contended, with justification, that in order to plott a position line by means of D. R. position, two position lines can only conveniently be combined after being observed at separate times. In commercial navigation there are also many objections to drawing position lines only on sea charts or Summer charts, as it is the custom, at sea, to effect position line drawings in the Observation book.

The method of calculation of the hour angle must depart from that which is employed in Germany and work from the notion of the Greenwich Hour Angle of a star. Only this is calculated and transformed into degrees. The method would therefore prove particularly simple, if it were used in connection with a Nautical ephemeride which already gives Greenwich hour angle or time angle (Zeitwinkel) in degrees (see *Neuzeitliche Navigation*, Seewart 1937, pp. 272-279). The use of a chronometer with divisions in degrees might also be considered. In connection with his work, Garcia also suggested the utilisation of a chronometer with a degree dial.

The method reminds one very much of what has been accomplished within the frame work of the "Nautical researches" carried out by the Navy. Tests made aboard ship with Garcia's tables, which it is not very difficult to arrange for definite latitudes, should be submitted for expert judgement. But apart from this, one can already appreciate the quite new and remarkable idea on which the theoretical method is strongly based, as well as the minute achievement attained, especially the arrangement of the graphic table, which is also very original and reveals the great practical experience of its framer.

Summary

The above described Tables by Juan Garcia are entitled *Tablas de Lineas de Posición de Altura* (Astronomical Position Lines Tables) of a convenient size (17 × 25 cm. = 6,5 × 10 inches) are divided in 3 small volumes of 125 pages each. Volumes I and II are conveniently

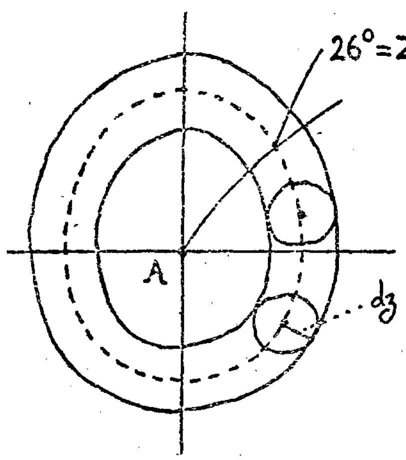

FIG. 5

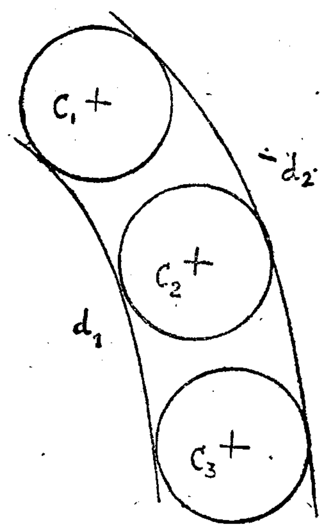

FIG. 6

arranged for the sun, moon and planets (declination from 25° N. to 25° S.) and covering latitudes from 30° N. to 30° S. in volume I and from 30° to 60° N. and S. for volume II. Volume III is specially computed for the navigational stars with declinations of more than 26° and for latitudes from 60° N. to 60° S. They have been published in 1944 by "Editorial Naval", Montalban, Madrid—Explanations concerning the use of the tables are given in Spanish and English.

Commander Juan Garcia has developed the mathematical theory used in his tables in the article " Sobre el metodo de las curvas de altura envolventes " published in the *Revista General de Marina* of January 1943. Fig. 5 illustrates the system thereof.

On the celestial sphere all the altitudes circles centered at A have been grouped within a number of zones according to their angular distance to the heavenly body A, each zone has been given a width corresponding to a round number of degrees : for instance, zone 26° relates to all altitudes circles of 25° 30' to 26° 30' ; the 26°00' altitude circle being the central one of the zone.

Garcia's Tables are construted so as to give directly for every central position line of each zone the values of the hour angle and latitude of a *series of successive points from each central position circle* with the accuracy of 1/4 of minute of arc.

The altitude (h) rounded to every whole degree of altitude determines the table to be entered : the horizontal argument in this table is the declination in whole degree, the latter determines the column to be used.

The tabulated hour angles corresponding to several round values of the latitude expressed in whole degrees as given on the left hand side of each line of the tabulation are taken out. (Of course those values should be taken nearest to the D.R. latitude.) The hour angles are given in degrees and minutes, for tabulation purposes the table values are printed, as for example, 2615 instead of 26° 15' and 11527 instead of 115° 27'. The red printed numbers by the side of hour angles values give the change in tabulated values due to a change of 10' in declination, this facilitates the interpolation for the actual value of declination in minutes. For this purpose a multiplication table printed on a movable cardboard sheet is provided with the table : this is the only one interpolation to be made.

It suffice to add GHA (Greenwich Hour Angles) to each local hour angle taken out from the table to obtain the longitudes of the corresponding points of the *central position circle*. These points will be plotted on the chart by their latitude and longitude C1, C2, C3.

Difference dz between hc the latitude of the central position circle and observed altitude gives the radius of the small circles (*fig.* 6) the " envelope " of which (enveloppe arc d) represents on the chart the geometrical locus of the observed fix, from this is derived the name " envelopes methods " (metodo de las involutas) selected by the Author.

If the true altitude (hc + dz) is larger than the central altitude (hc) arc d1 towards the heavenly body should be selected, otherwise the remote arc d2 should be selected.

H. B.

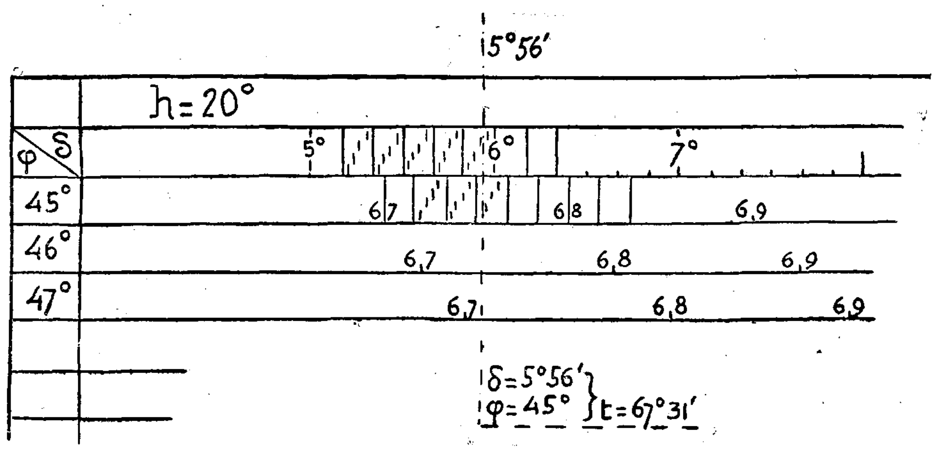

FIG. 3bis

Direct graphic interpolation on Garcia's Tables
(Accuracy 1 minute)