REFRACTION

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An investigation of refraction prevailing in the Victoria area was undertaken by the Geodetic Survey of Canada over a seven-week period in 1940, comprising part of May, all of June, and part of July.

Every effort was made to insure reliability of results through the use of electric signal lights as day-time targets set over the triangulation stations, the reading of zenith distances being obtained with Wild 5 1/4" theodolites reading to single seconds. Staff gauges were set and connected to the level system to establish the state of the tide for the reduction of the readings on the sea horizon. Barometer and temperature readings were made at each station, and in addition, the temperature of the sea water and of the air at one-metre height was also observed in time as near as possible to the observations for zenith distances.

In brief, we may say that the area is one adjacent to mountainous country and the observations over the period indicate that inversions of temperature are not infrequent and exist during the day as a general condition. A return to normal cooling with height occurs only when winds are strong, say, over 25 miles per hour. Winds of less strength than 15 miles per hour are not sufficient to keep the air thoroughly mixed and lenses of various densities then occur over the area. For winds of less than 15 miles no approach to the accepted mean value " m " = 0,07 is possible. For higher winds sea horizon values of " m " are well represented by 0,115 and other lines by 0,084.

The grand mean for the sea horizon is " m " = 0,127, and for other lines " m " = 0,112.

A study of the extremes of zenith distances indicates that a range of 6 minutes of arc is not an unusual occurrence at Macaulay (elevation 72',8).

The extreme coefficients of refraction observed during the period May 16-July 6 are :— a low of " m " = 0,020 ; a high of " m " = 0,592 (rejected). Note the high values on the sheet GONZALES-RACE ROCKS L.H. for June 18th and 28th.

As examples of abnormal refraction, the following summary, for the line GONZALES to RACE ROCKS L.H. are given for June 28th and July 1st :—

GONZALES (June 28). — " Hot and very smoky today. Fog banks lying off south shore obscuring the sea horizon. Evidence of unusual refraction again today. Race Rocks L.H. between 7,5 and 8,5 hours was acting like an accordian ".

Hour	Zenith distance	« m »	
7 h. 32	90° 10'47",5	.379	
7 h. 37	50",3	.374	
7 h. 40	90° 12'21",0	.236	Faling barometer.
8 h. 11	90° 12'37",0	.211	Light clouds.
9 h. 07	90° 12'23",5	.232	Light S. W. winds.
10 h. 51	90° 12'55",5	.181	Sta. Temp. 19°,5 C.
11 h. 11	90° 13'02",3	.171	No Temp. Gradient.
13 h. 00	fog		
15 h. 10	90° 11'07",0	.350	
18 h. 39	90° 12"10',0	.252	

GONZALES (July 1). — " Smoke and haze very thick again. Could scarcely see Mary Hill at 7 hours. Triangle and Sea Horizon obscured at 7 hours. Triangle M^{tn} visible at 11 hours but no sign of signal light and air boiling. Very bad at 15 hours and at 16,5 hours everything blotted out by dense smoke and haze. Abnormal refraction again today with Race Rocks L.H. going up and down very rapidly. "

Hour	Zenith distance	« m »	
7 h. 37	90° 13'34",0	.122	
9 h. 07	90° 11'26",5	.320	No Temp. Gradient.
11 h. 00	90° 08'30",0	.592	
13 h. 08	90° 10'25",0	.416	
15 h. 01	90° 12'43",5	.199	

The mixed condition prevailing over the area in the lower strata is indicated by the coefficients of refraction not conforming to the daily curve, in which the means by hours should approach a minimum at the time of maximum temperature.

An examination of the large sheet — GONZALES to RACE ROCKS L.H. — shows coefficients both in the early morning and late evening which would produce abnormal seeing. They occur too frequently to be a curiosity and must be regarded as of common occurrence. They are not the result of erroneous pointings, as by reference to other lines in the lower strata, values for the same hour indicate that they are in agreement and excessive refraction is present.

Evidence is also available which shows that the value of " m " increases suddenly for the 15 hours period. This is a feature contrary to the normal daily curve, which has a minimum at about 14 hours. This occurrence is apparent for the higher lines at the same hour.

To present a clear picture of the varying effect in refraction, copies of the results for three lines from stations Macaulay, Albert Head, and Gonzales, to Race Rocks L.H. are attached. A conventionnal system is used to indicate the strength of wind and its direction with relation to the side of an octagon, thus oblique strokes are used for the Northeast, Southeast, Southwest and Northwest sectors. In other respects it is believed that the tabulation is self-explanatory.

The immediately following tabulations establish that for the calculation of geographic ranges, the formula nautical miles $= 1{,}23\sqrt{H'}$ should be used.

Average values of the coefficient of refraction (= m), where K = 2m, arranged according to wind strength in miles per hour.

a) *Station :* Macaulay ; *Elevation :* 72,8 feet ; *Object :* Sea horizon.

Wind strength :	0 - 5	5 - 10	10 - 15	15 - 20	20 - 30	Over 30
Coefficient value	.200/9	.202/3	.163/1	———	.100/1	.102/6
and weight :	.241/4	.204/3	.194/2	———	.086/1	.101/6

Average value for 36 *observations* = .151.

b) *Station :* Albert Head ; *Elevation :* 143,6 feet ; *Object :* Sea horizon.

Coefficient value	.141/7	.176/8	.165/3	.151/8	.135/23	———
and weight :	.118/8	.150/9	.151/3	.134/8	.121/21	———

Average value for 98 *observations* = .138.

c) *Station :* Gonzales ; *Elevation :* 214,1 feet ; *Object :* Sea horizon.

Coefficient value	.131/16	.124/21	.140/6	.123/27	.116/34	.097/9
and weight :	.128/27	.134/21	.133/8	.112/23	.101/42	.088/14

Average value for 248 *observations* = .117.

d) *Station :* Mary Hill ; *Elevation :* 379,2 feet ; *Object :* Sea horizon.

Coefficient value	———	———	———	.117/6	.118/3	.098/1
and weight :	———	———	———	.106/3	.107/2	.099/1

Average value for 16 *observations* = .111.

In the above tabular statement the coefficient value and weight is understood to be as follows :— for .200/9, 9 values of the coefficient for the given wind strength average .200.

The application of the usual formula for the horizon distance, i. e. : " nautical miles = 1,14 $\sqrt{\text{elevation (feet)}}$ ", may thus be seen to result in distances too short for this area, because of the definitely indicated increase in the coefficient of refraction when lover elevations are used.

Station	Elevation	Average Coefficient	N° of observations	Deduced Formula
Macaulay	72',8	m = 0,151	36	1,27 $\sqrt{H}$
Albert Head	143',6	m = 0,138	98	1,26 $\sqrt{H}$
Gonzales	214',1	m = 0,117	248	1,22 $\sqrt{H}$
Mary Hill	379',2	m = 0,111	16	1,21 $\sqrt{H}$
		m = 0,127	398	1,23 $\sqrt{H}$

For 398 observations on the sea horizon the weighted value of " m " = 0,127 and the deduced formula then equals 1,23 $\sqrt{H}$.

Objection may be raised that the sea horizon is often indefinite and thus does not permit accurate readings of zenith distances.

As a comparative study is available, for lines in a geodetic sense lying relatively close to the sea surface, from observations made at three stations upon a definite mark on Race Rocks L.H., a summary (on large sheets) of these observations is appended for three lines. The information is further collected below for a comparison with the above table.

Plate 1 Planche 1

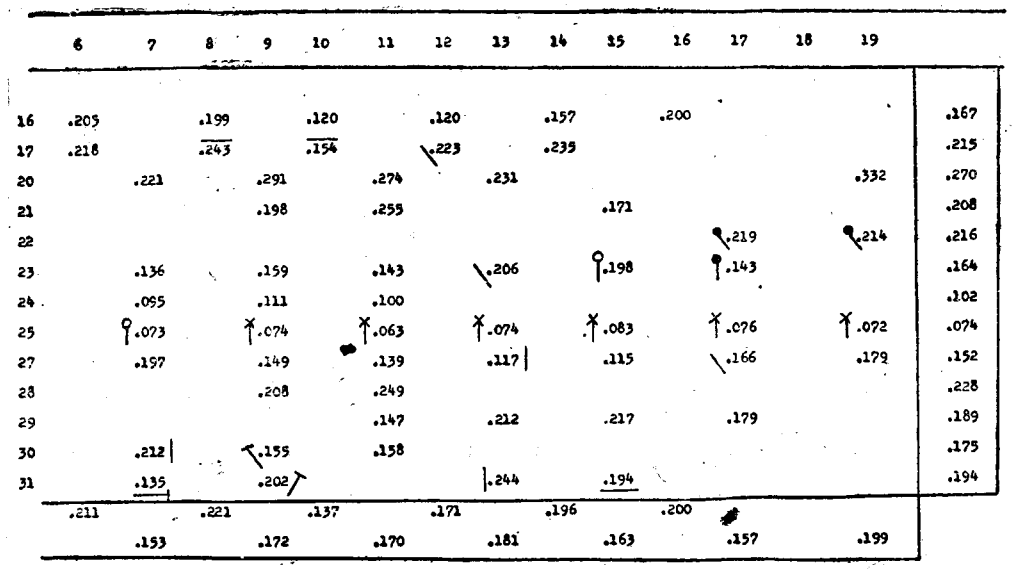
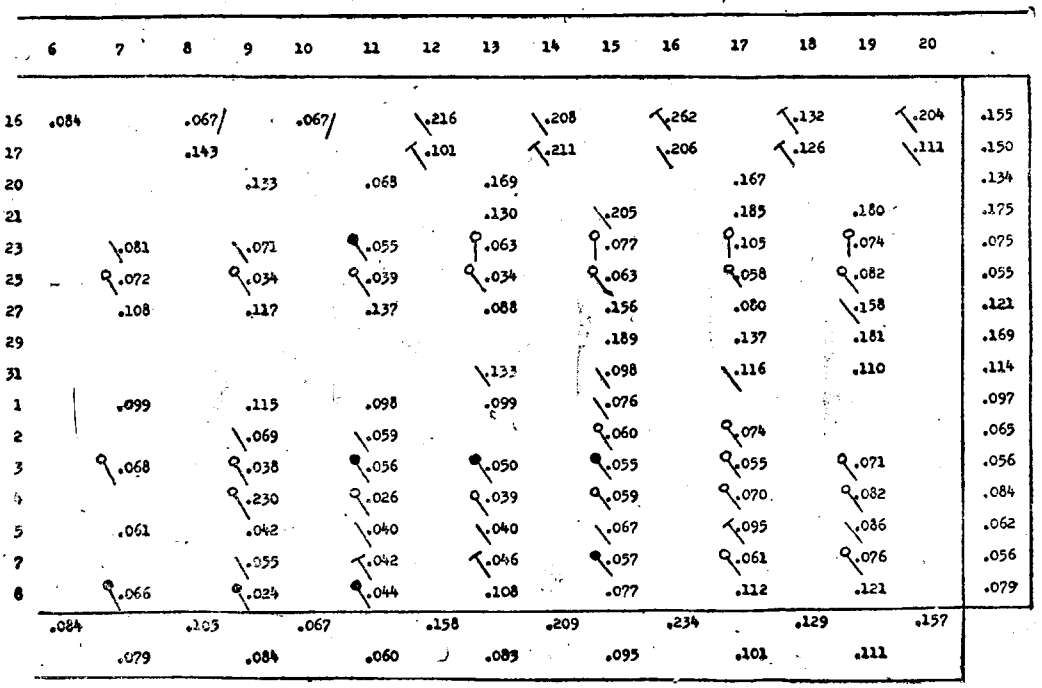


Plate 2 Planche 2



Average value of " m " according the Wind Strength in Miles per hour :
Valeur moyenne de « m » d'après la force du vent en miles à l'heure :

PL. 1. — MACAULAY-RACE ROCKS L.H. (*Phare*)

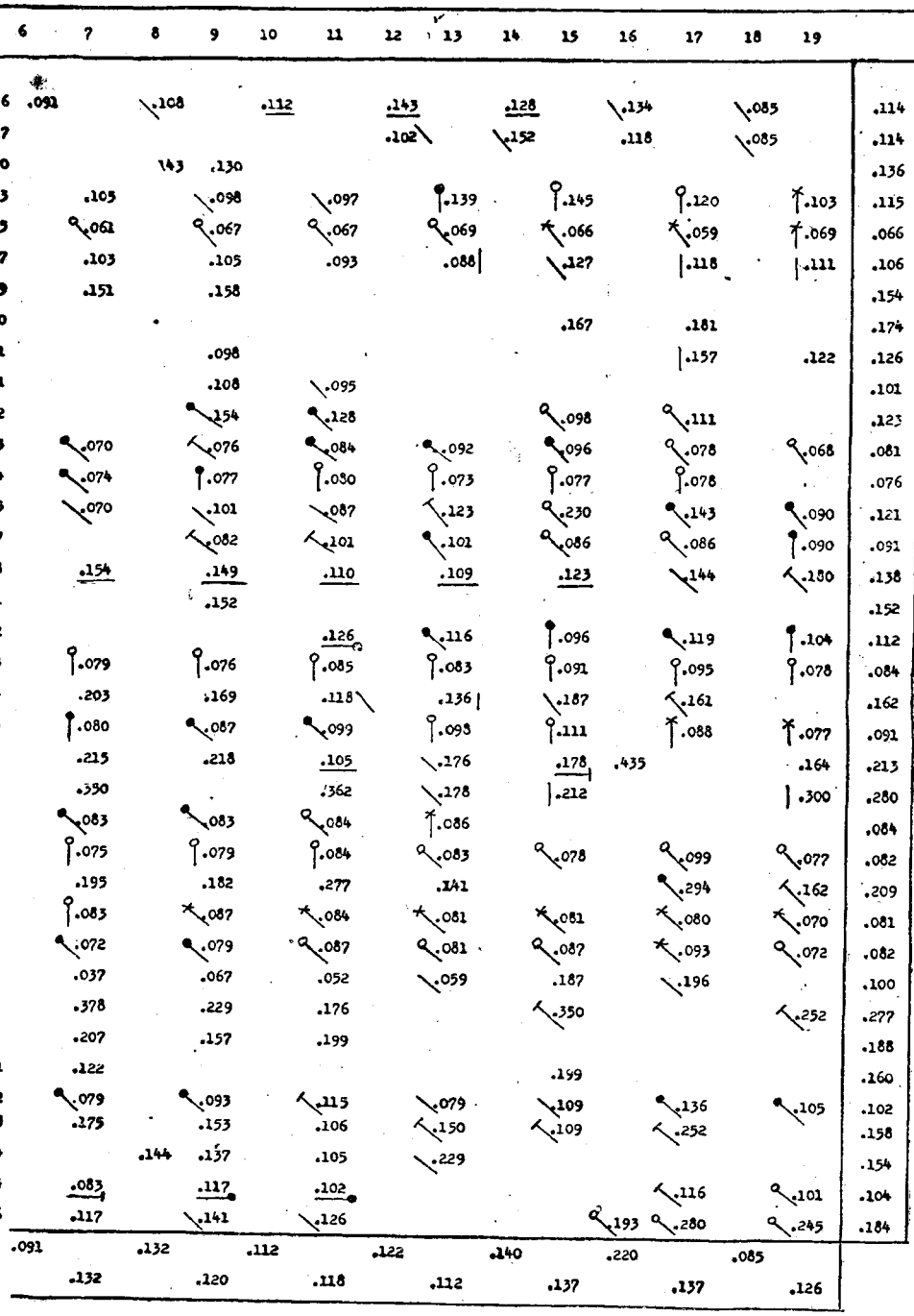
0 - 5	5 - 10	10 - 15	15 - 20	20 - 30	over 30
.186/34	.195/9	.164/3	.192/3	.135/2	.074/6

Elevation : Macaulay, 72.83 feet ; Race Rocks L.H., 40.9;
log length (meters) = 4.2087166 ; 8.726 nautical miles.

PL. 2. — ALBERT HEAD-RACE ROCKS L.H. (*Phare*)

0 - 5	5 - 10	10 - 15	15 - 20	20 - 30	over 30
.119/30	.108/20	.135/9	.045/9	.068/25	——/0

Elevation : Albert Head, 143.63 feet ; Race Rocks L.H., 40.9 feet;
log length (meters) = 4.0310200; 5.796 nautical miles.

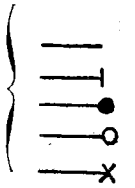


PL. 3. — GONZALES-RACE ROCKS L.H. (*Phare*)

0 - 5	5 - 10	10 - 15	15 - 20	20 - 30	over 30
.167/45	.135/42	.156/16	.107/32	.099/43	.080/14

Elevation : Gonzales, 214.08 feet; Race Rocks L.H., 40.9 feet;
log length (meters) = 4.3001059 ; 10.769 nautical miles.

Wind Diagram (wind strength in miles per hour)
Force du vent en milles à l'heure



Line	Nautical Miles	Mean Height	Average coefficient	N° of Observ^ons	Deduced Formula
Macaulay (72',8) to Race Rocks L. H. (40',9)	8,726	56',8	m = 0,173	57	$1{,}32\sqrt{H}$
Albert Head (143',6) to Race Rocks L. H. (40',9).....	5,796	92',3	m = 0,097	93	$1{,}18\sqrt{H}$
Gonzales (214',1) to Race Rocks L. H. (40',9)	10,769	127',5	m = 0,128	192	$1{,}23\sqrt{H}$
				342	$1{,}23\sqrt{H}$

Thus for 342 observations on lines lying moderately close to the sea surface, the deduced formula is again $1{,}23\sqrt{H}$.