REFRACTION OCCURRING IMMEDIATELY ABOVE THE WATER SURFACE

by

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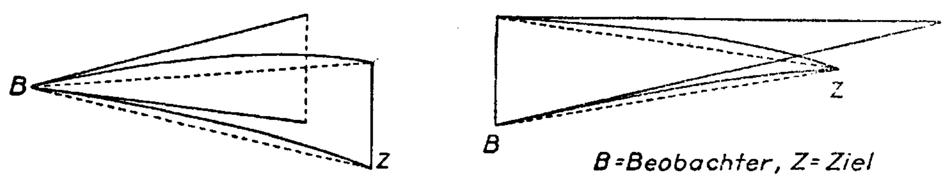
In collaboration with G. Prüfer, the author has examined and analyzed observations on the dip of the horizon ; so far, however, it has been possible to publish only a few extracts from this work (1). As a result of this analysis, it has been demonstrated by means of the following theoretically derived formula :

$$Kt' = 5.04'\sqrt{0.1123\ h + T_0 - T_h}$$

that the thermic conditions in the lower atmospheric layers can be elucidated. When other refraction observations in the lower atmospheric layers were dealt with, therefore, the application of a similar method seemed to be indicated. Observations of this kind made by the author in 1937 were already available. By means of these observations, an attempt was made to determine the distance of an accurately-known target by measuring a vertical base of low altitude. By such measurements, a comparison at low altitude is made between the path of an upper light ray and that of a lower ray. A false determination of the distance is caused by the differences in the curves of the light rays. This influence produces different effects according to the method applied, i. e., depending upon whether the base vertical to the target is shifted so that the distance is determined by measuring the angle through which the vertical base lines are seen ; or whether the distance from the target is determined by means of vertical base lines located near the observer (Fig. 1) (as in the case of all telemeters).



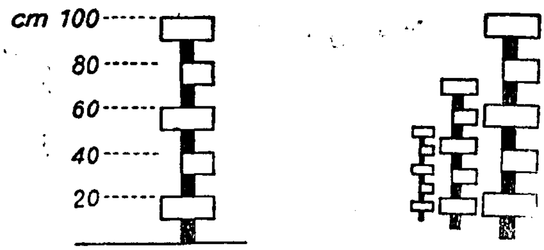
Under the influence of a fairly pronounced curve of the light ray, for instance, the angle through which the base at the target is seen, appears to be greater than it is according to measurement of the lines from eye to extremity of base. Such curves therefore simulate a distance less than the true distance while, under identical refraction conditions, the distance measured by a vertical located near the observer appears greater, since the tangents to the light rays intersect at an appreciable distance from the apertures of the base arrangement.

As a matter of fact, practical observations have revealed the existence of such phenomena. However, these measurements, made in the Baltic and

(1) Dtsch. Hydrogr. Z. 1,26 (1948) et 2,78 (1949).

North Sea in the vicinity of Heligoland, were difficult of accomplishment, since accurate determination of the distance of a raft moored in the sea is subject to various errors even if the distance be measured by means of the angle formed by the horizontal base lines located near the target, and by means of a horizontal placed near the observer. The factor that chiefly interfered with operations was the unsteadiness of the object to be measured.

The author moreover decided to follow certain often reiterated proposals (1) and to observe the path of the light rays along sighting-rods placed in line one after the other. The rods were shaped as shown in Figure 2, which made approximate observations easier. After the three rods have been aligned with the telescope, the image shown below (Figure 2) is obtained. In this way the path of the light through 4 positions is immediately observed, or only 3 positions when the farthest rod is no longer visible.



With the application of these two methods, an attempt was made to relate light ray measurements to temperature measurements made at the same time. The formula used by geodesists to measure height, which takes refraction into account, was no longer applicable to the realization of this relation. This formula

$$h = e \operatorname{tg} a + \frac{1 - k}{2r} e^2$$

in which e is the distance, h the relative height of the point sighted under the angle a, r the earth's diameter and k the coefficient of refraction (= 1 : radius of curvature of light path) is not applicable because the constant values of the refraction coefficient are inadequate. It is rather the difference between the curvature of the upper light ray and that of the lower light ray which intervenes in the differential measurements.

If we assume that the layers are concentric, we can deduce the effect of refraction on the angular difference measured, by means of the vertical base lines and of heights marked on the sighting-rods by a light ray. With regard to measurements by means of a vertical base located near the observer, these should be treated as angular values. The difference between an angle modified by refraction and the theoretical angle gives in this case the following formula :

$$+ \int_0^e \frac{1}{n} \frac{dn}{dh} \, de \ \text{(upper light ray)} - \int_0^e \frac{1}{n} \frac{dn}{dh} \, de \ \text{(Lower light ray)}$$

while, with reference to verticals at the target, the formula takes the opposite sign.

(1) Kohlschütter, Ann. d. Hydrogr. u. marit. Meteorologie, 31, 533 (1903).

Leaving aside humidity, always almost saturated and fairly constant, we have :

$$\frac{1}{n}\frac{dn}{dh} = \frac{c\,b}{(1 + b\,T)^2}\left(0.034 + \frac{dT}{dh}\right) \quad \begin{matrix} c = 0.000293 \\ b = 0.003665; \end{matrix}$$

giving for the angular difference the approximate value of :

$$\pm\ 0.0037' \int_0^e \left(\frac{dT}{dh}\ \text{up} \ - \frac{dT}{dh}\ \text{down}\right) de.$$

This means that we can elucidate the behaviour of the temperature gradient if we possess measurements made at different heights of eye. Measurements at 2 metre and 0.5 metre heights of eye were made (the centres of the telemeters were set at these altitudes) and it was possible to combine them so as to admit, for instance, the values dT/dh as being constant for certain areas. The results were only partially satisfactory because the resulting conditional equations are not independent, i. e. only gradient differences result which, however, correspond to the temperature measurements ; moreover, temperature variations of short duration were not included and consequently some details were not evident, with the exception of extreme temperature gradients.

The second method using marked rods is less subject to difficulties of this kind but is not applicable to observations at sea. However, the variations observed here as to the path of the light rays are noteworthy and instructive. Their analysis was made so that the height might be calculated at which a light ray, starting from a point of the farthest rod and the path of which would be linear, would pass to one of the other sighting-rods.

$$h_r = h_o + \frac{e_i^2}{2r} + \frac{e_i}{e_3}\left(h_3 - h_o - \frac{e_3^2}{2r}\right)$$

Let h_3 be the true altitude of the point of the third rod observed at a distance e_3 ; h_o the height of eye of the observation, e_i the distance from the 2nd or ist rod, for which the height h_r is calculated. Because of refraction, h_r is not observed; but

$$h_b = h_o + e_i \tan(90^\circ - z_o) - e_i \int_0^{e_i} \frac{k}{r}\left(1 - \frac{e}{e_i}\right) de + \frac{e_i}{2r}$$

It becomes a question of examining the difference $h_o - h_r$.

The angle z_o of the light ray starting from the observer is unknown, but account being taken of the fact that the formula is correct for the whole length of the light ray and consequently for the starting-point of the light ray on the farthest rod ($e_i = e_3$) so that at that point $h_b - h_r = 0$, the following formula is valid at any point for the difference to be studied :

$$h_b - h_r = \frac{e_i}{r}\left(\int_0^{e_3} k\left(1 - \frac{e}{e_3}\right) de - \int_0^{e_i} k\left(1 - \frac{e_i}{e}\right) de\right).$$

As $k = -\frac{r}{n}\frac{dn}{dh}$ and introducing the numerical values, the formula takes the following form :

$$h_b - h_r = 0.000\,001\,074\, e_i \left(0.034 \left(\frac{e_3}{2} - \frac{e_i}{2} \right) + \int_0^{e_3} \frac{dT}{dh} \left(1 - \frac{e}{e_3} \right) de - \int_0^{e_i} \frac{dT}{dh} \left(1 - \frac{e}{e_i} \right) de. \right)$$

(in metres)

If a constant temperature gradient for certain layers of limited thickness is assumed, integration calculations can be made for a few sections of the whole distance. As the sighting-rods indicate several altitude values at the same time, a large number of equations is obtained in which the different temperature gradients repeat themselves, accompanied by successively new coefficients. There results from this a large number of conditional equations from which the most plausible values of the temperature gradient can be deduced by the method of least squares.

As an example, the following table shows the variations to which the indications are subject during a relatively short period, involving only the beginning and the end of a development which could be followed at all stages during the observations.

Observations made on 29th June, 1949,
at 32 cm. height of eye

$h_b - h_r$

	III	II	I	III on II	III on I	II on I	TEMPERATURE		
11 h. 10 m.	cm.	cm.	cm.	cm.	cm.	cm.			Dip of the horizon — 3,7'
							10 cm.	17,6°	
	100	58	44	— 5	— 3	0	20 »	17,4°	angle
	80	44	36	— 9	— 5	— 1	50 »	16,6°	100. III + 0,1'
	70	30	20	— 17	— 18	— 9	200 »	16,4°	100. II + 2,2'
		100	69			+ 2	water	18,0°	
14 h. 20 m.									DIP OF THE HORIZON
	100	66	54	+ 3	+ 7	+ 5	10 cm.	20,7°	
	50	45	38	+ 8	+ 5	+ 1	20 »	20,6°	— 1,0'
	10	22	30	+ 6	+ 8	+ 5	50 »	20,4°	angle
		100	71			+ 4	200 »	20,6°	100. III + 0,7'
Distance en meters	1970	1000	550				water	19,1°	100. II + 2,2'

At 11 h. 10 m. the last rod was invisible at a height of 70 cm. and part of the traverses of the three rods were reflected in the water and part appeared twisted. This phenomenon was due to a shortening of the horizon; the horizon was still in the vicinity of the surface of a lake 2 kilometres in length. At 14 h. 20 m. all the rods were visible without distortion, but the dip of the horizon was still somewhat too pronounced. The image of the horizon had shifted several times ; it had moved upward. Occasionally, uniform streaks could be observed rising above the horizon, their upper edge forming a straight line. These atmospheric streaks enabled a second line of horizon to be fixed. It was there that, in the course of a few minutes, the new horizon line formed.

The path of the light rays was examined on the sketches which had been drawn up. It was assumed, for instance, that there were three layers of constant optical density, one of them over 32 cm. high ; another between 32 and 60 cm.; and a third over 60 cm. high. It was therefore possible to calculate their temperature gradients from the various $h_b - h_r$, deducing the extent of each area from the point where the light ray in question penetrated the different layers. The following shows the result for the three gradients from the lowest upward :

1110 H.M. — 0.52° — 0.18° and — 0.02° ; but
1420 H.M. + 0.14° + 0.12° and + 0.04°.

From the first of the preceding formulae (page 1), the result for the dip of the horizon is : $T_0 - T_b$ at 1110 H., 0.54°, and at 1420 H. — 0.04°. Moreover, on the assumption — not used so far — of the rays being circular, there may be deduced from the observed angles which were determined for the upper ends of the rods, a constant temperature gradient for the whole length of the path of the light ray. The result is :

			H.M.		
For Sighting-rod	111.100	at	1110	the constant temperature gradient of	—0.22°
» »	111.100	at	1420	» »	—0.04°
» »	11.100	at	1110	» »	+0.04°
» »	11.100	at	1420	» »	+0.04°

Finally the reflections may be used to deduce the temperature gradients. At 1110 H., for instance, the reflection of rod 11.30 was thrown on rod 1.8; thence a temperature gradient of —0.86° can be deduced. The resulting larger gradient can reasonably be admitted, as the light ray producing the reflection was not exclusively limited to following a path lower than 32 cm.; it reached layers much lower than those reached by the ray from the non-reflected image passing rod 1.20.

Another day, at the of a morning characterized by sultry weather, during a storm followed by a short but violent shower, refraction conditions changed in the opposite direction for a still shorter period. This observation as well as the others all tally closely with the calculated temperature gradients. However, it is difficult to measure the temperature in as detailed a way as can be done by measurement of the light rays. Besides, the determinations deduced from optical observations are arbitrarily subject to the assumption that here are certain layers for which constant conditions are admissible. It is nevertheless surprising to note to what extent the path of the light ray can shift upwards or downwards within such small distances. It is remarkable that there should be, by this method or by similar methods, a means of ascertaining temperature conditions which manifest themselves immediately above the surface of the water.