WORK OF THE TRIANGULATIONS SECTION OF THE INTERNATIONAL ASSOCIATION OF GEODESY AT THE IXth GENERAL ASSEMBLY HELD IN BRUSSELS

by Captain A. VIGLIERI,
Director of the Hydrographic Institute of the Italian Navy

The work of the First Section (Triangulations) of the International Association of Geodesy took place on the 22nd, 23rd, 24th, 27th, 28th and 29th August of this year, under the presidency of Professor Jacquinet. The duties of Secretary were undertaken by Professor Marussi in the absence, due to reasons of health, of Professor Tienstra.

The questions dealt with by this Division were particularly interesting, as the titles of those brought under discussion and of the papers presented before the Congress, given below, fully demonstrate.

The subjects submitted for discussion during the meetings are the following:

— A Factual Account of the European Adjustment, by Col. Floyd W. HOUGH (U.S.A.).

— Adjustment of European Triangulation, (together with the presentation of a general map of European geodetic Nets), by W. C. A. WHITTEN (U.S.A.).

— Co-ordination of the World Networks (Paper presented by Captain A. VIGLIERI, Director of the Istituto Idrografico della Marina Italiana), in view of the use of radio methods of navigation, and for the collaboration between hydrographers and geodesists.

— Proposal of the Spanish National Committee concerning the geodetic unification of the Mediterranean.

— Tableau d'ensemble des Déviations de la Verticale en Europe et utilisation des résultats pour la Détermination du Profil du Géoïde en Europe (*General information concerning Plumb Line Deflections in Europe and the use of the results for the determination of the Profile of the geoid in Europe*), Report by Secretary-General P. TARDI (France).

— Measurement of distances with the Geodimeter, by Prof. BERGSTRAND (Sweden).

— General Report No. 3 concerning geodetic Applications of wireless telegraphy (with special reference to Radar-type methods), by Dr. C. A. HART (India).

— Shoran Triangulation in Northern Canada, by J. E. R. ROSS (Canada).

— Nouvelles formules destinées à faciliter le calcul des très longues lignes géodésiques avec Table à cet effet.
(*New formulae to facilitate calculation of extended geodetic lines, together with appropriate Table*), by M. DUPUY, of the National Geographical Institute (France).

— Die sphärische Berechnung von Streckennetzen (*Paper concerning the calculation of trilaterals by means of the conformal projection of the ellipsoid on the Sphere*), by Prof. C. F. BAESCHLIN (Switzerland).

— Report of Projections, by C. A. WHITTEN (U.S.A.).

— Provision General Report No. 1 — Triangulation; Base Lines; Laplace points, by Prof. TIENSTRA (Netherlands).

— Remarks on the resulting Calculations of the Denmark-Norway connection by flare triangulation 1945, by O. SIMONSEN (Denmark).

— Measurement of a comparative Base of international character, by Prof. HEISKANEN & Dr. PESONEN (Finland).

— Proposal of Argentine Republic concerning new rules for primitive Triangulations of 1st order (See: « Bulletin Géodésique », N° 27, pages 257-289).

— Discussion of subject mentioned in the « Bulletin Géodésique » No. 15 of 1950 and No. 19 of 1951 concerning the method of representation of Earth area on the reference ellipsoid, in cooperation by Prof. BAESCHLIN (Switzerland), Prof. VEINIG MEINESZ (Holland) and Prof. MARUSSI (Italy).

The following papers have been distributed:

— Report on the activities of the Italian Military Geographical Institute in the domain of Triangulation and Levelling from 1948 to 1950, by General L. MOROSINI (Italy).

— Geodetic Operations in Canada, by J. E. R. ROSS (Canada).

— Report of geodetic Works in Turkey, by the Turkish Geodetic Survey (Turkey).

— Brief Summary of the geodetic Work of the Geographical Survey Institute for the Period 1947-1950, by M. KATSUHIKO MUTO (Japan).

— Transfer of Triangulation results of Poland on the international ellipsoid and their introduction in a standard system based on common points, by S. MILBERT (Poland).

— Geodetic Works carried out in « Tauernkraftwerk », by F. LÖSCHNER (Austria).

— Revision of the First Order Triangulation in the region disturbed by the Mankaido Earthquake of 1946, by KATSUHIKO MUTO (Japan).

— The horizontal Displacement of the Earth Crust accompanying the great Kwanto Earthquake 1923, by E. INOUE & H. ONO (Japan).

— Die absolute Lotabweichung in Potsdam und die geodätischen Ausgangswerte des gesamteuropäischen Netzes auf dem Hayfordschen Ellipsoid (*Absolute plumb line deflection at Potsdam together with the geodetic results of the European net system on Hayford ellipsoid*), by K. LEDERSTEGER (Austria).

— Vorschläge für die Vereinheitlichung der europäischen Kartenwerke (*Proposal for standardization of European Cartography*), by K. LEDERSTEGER (Austria).

— On a new Method of Adjustment of extensive triangulation Network, by Dr. B. GOUSSINSKY (Israël).

— Generalization of least squares method using matricial algebra processes, by R. MARCHANT (Belgium).

— Application of the calculus of matrices to method of least squares with special reference to geodetical Calculations, by Dr. BJIERHAMMAR (Sweden).

— Rapport additif sur les Projections géodésiques employées en France et en Afrique du Nord (*Additional Report on the Geodetic Projections used in France and in North Africa*), by the National Geographical Institute (France).

— Procédé actuellement employé au Service Géodésique français pour le Calcul de la Réduction à la corde en projection Lambert, avec des remarques générales sur la présentation de nouvelles Tables pour la Projection Lambert française (*Process presently used by the French Geodetic Service for the calculation of cord reduction in Lambert projection, together with general*

remarks concerning the presentation of new tables for French Lambert projection), by M. DUPUY (France).
- Short Methods in the Transverse Mercator Projection, by Y. C. B. REDFEARN (Great Britain).
- Carta estudiata para la mapa de Espagna (*Grid developed for the map of Spain*), by Santos Anadon LAPLAZA (Spain).
- The length of a geodetic Line as a Function of the plane Gauss-Co-ordinates of the end Points, by G. A. RUNE (Sweden).
- Projection on the Earth Ellipsoid of Distances measured above the Sea Level, by G. A. RUNE (Sweden).
- Tabeller till Gauss' Hannoverska Projektion (*Table for Gauss' Hanoverian Projection*), by G. A. RUNE (Sweden).
- Calibration of a 25 m. Jäderin wire in terms of luminous waves, etc., by N. WATANABE & M. IMAIZUMI (Japan).
- Proprietà geometriche delle linee di posizione della navigazione iperbolica e loro tracciamento sull'ellissoide di riferimento (*Geometric Properties of Position Lines in hyperbolic Navigation and their Layout on the Reference Ellipsoid*), by Prof. S. BALLARIN (Italy).
- Progrès réalisés dans l'étude de la réfraction géodésique (horizontale et verticale), (*New Progress in the Study of geodetic Refraction, horizontal and vertical*), by M. LEVALLOIS (France).

In what follows, a detailed study of those subjects which more directly affect hydrography is given.

Overall adjustment of European geodetic nets

The first question on the agenda was the Report concerning the overall adjustment of the European geodetic nets carried out by the United States' geodesists under the auspices of the International Association of Geodesy.

Colonel Floyd H. Hough (U.S.A.) of the Army Map Service gave an historical summary of the subject from the year 1945 onwards. He told that in April, 1945, the Americans occupied the town of Friedrichsroda in Turingia, where the Trigonometrical Division of the *Reichsamt für Landesaufnahme* had been evacuated; at that time this Division, under the direction of Erwin F. Gigas, was working on the adjustment of the « Greater Germany » network ordered by the German Headquarters. This calculation, starting from Eastern Prussia, was almost finished; but the adjustment method used, the choice of which had been determined by reasons of military character, made it unsuitable for scientific purposes.

The work of overall adjustment of the European network was handed over to this group of German technicians directed by Gigas, and it was recommended to use the Bowie method, which had already been successfully employed in the United States of America; it was, however, modified in the assignment of weights as indicated in Whitten's works.

The German geodesists, who had meanwhile been transferred to Bamberg, were provided with modern electric computing machines; after the return to America of the Intelligence Team which had contacted Gigas, charge of the work's technical direction was given to the Geodetical Division of the Army Map Service.

Exactly on schedule, in June, 1947, the adjustment of the Central European Area was completed; the area covered by this adjustment is 1,000,000 square kilometres.

The observations on which this work is based are those made by German geodesists during the last 140 years; from these observations, by means of an analysis that took six months' work, 714 triangulation stations, 106 Laplace stations, 50 base lines and 77 astronomical stations were selected, avoiding the stations for which an abnormal deflection of the vertical could be anticipated because of morphological discontinuity of the terrain.

Calculation of the adjustement includes 1,332 condition equations; the 905 triangles observed close with the average approximation of 0''.515 and for only 25 of them the closure error exceeds 2''. The mean angular error, calculated by the Ferrero international formula, is ± 0''.348.

In this way a geodetic datum was obtained which cannot be considered as depending upon an initial point but upon the whole area. All the bases were reduced to sea level and to the international metre. Geographical co-ordinates are computed on the international ellipsoid. The datum so obtained was called « Central European datum », later renamed the « European datum »; and it may be considered as largely independent from deflections of the vertical owing to the considerable number of well-distributed astronomical stations included in it and the care with which they were selected. The above-mentioned definite results testify that the selection made has led to the work being based on incontestably trustworthy observations.

Immediately after this adjustment of the Central European net, Gigas was given charge of a similar operation for the South-east European triangulations. For this area too the method previously used was employed, with the difference that in this adjustment everything dealing with the already adjusted network of Central Europe (geographical positions, azimuths, lengths of sides, etc...) was assumed without further modification, so that the whole South-east area has been included in the « European datum ».

This new work, which covers an area of about 900,000 square kilometres was completed in January, 1950. The observational data used include 700 triangulation stations, 47 base lines and 71 Laplace stations. 1,358 condition equations were established for the computation; the 592 observed triangles close with an average approximation of 1''.157. The mean angle error according to the Ferrero formula is ± 0''.608.

In 1946, while work of the adjustment of the Central European area was going on, the Chief of the Geodetic Division of the Army Map Service established contacts with the Offices of several European States, which were followed up during the International Conferences of Paris (1946-1947) and Oslo (1948), where it was definitely agreed that each adherent country would supply to the Army Map Service, through the intermediary of the I.A.G., a dossier of all the most recent first order observations. For its part, the A.M.S. was to take charge of the adjustment calculations, and to communicate the results only to each interested country, and to the I.A.G. only the deflections of the vertical.

The data supplied by the various nations were first examined by Monsieur Tardi, Director of the Central Bureau of the I.A.G. and by his assistant Colonel Laclavère; the work of adjustment computations was then assigned to the Coast and Geodetic Survey, U.S.A. which, under the technical direction of Whitten, completed in 1950 the work of South-west

Europe and, in 1951, the one referring to Northern Europe. Thus was accomplished the wish of so many eminent scientists: to-day the whole of the European networks west of 30° E. and those of North-west Africa form one homogeneous geodetic framework which, based on the International metre, covers an area of about 5,000,000 square kilometres with a population of more than 360,000,000.

The work accomplished represents an enormous contribution to scientific research and to collective security, while being at the same time a proof of the remarkable co-operation and common purpose which has animated the European and American geodesists.

W. C. A. Whitten, representative of the U.S.A. Coast and Geodetic Survey who had undertaken the technical direction of the South-west and Northern Europe adjustment, has described the procedure adopted in the calculation of the adjustment and the results obtained in the two abovementioned areas.

Contrarily to the initial decision of the Oslo Conference (1948), the modified Bowie method was not adopted as it had been done for the Central European area; a new procedure was used which, making it possible to use punched cards, has created the possibility of a simultaneous adjustment of several sections of the network.

This new method consists in first adjusting several base expansions situated near the junction points of triangle chains and then to retain as fixed the results obtained, for lengths and angles, from this preliminary adjustment. In this way two aims are attained: first, an accurate determination of the linear elements of the principal lines of the triangulations scheme is obtained and, second, computation is reduced by a great amount as the particular nets obtained by the first adjustment can be shifted in azimuth, latitude and longitude, but serve nevertheless to interrupt accumulation of the coefficients of the equations which must be carried through the triangulation chains.

The condition equations for azimuths, lengths, latitudes and longitudes relating to these elements which have already undergone preliminary adjustment, were written in such a way as to keep the angles invariable. 86 points were included in this first adjustment; they depend on 10 azimuth stations and 7 base lines.

This first part of the work having been accomplished, the block of South-west Europe was divided in four parts to allow four groups of calculators to work simultaneously.

Thanks to experience gained during the previous operations, it was possible to predetermine the order of elimination of the equations before forming the normal equations which reached the number of 2,348 and were solved by means of the Gauss-Doolittle elimination method. It is interesting to note that, once all the calculations over, observations relative to a North African base in proximity of Navarin were received. A local adjustment was then made to introduce this new base, as well as the stations connected with it, into the previously adjusted network. Ascertained differences were of the order of 1/50,000, which demonstrates the quality of accuracy obtained also in marginal areas.

The definite results of this block, which includes North Africa, Italy, France, Belgium, Spain, Portugal, Switzerland, Austria and Gibraltar, can be summarized as follows: 2,391 triangles with average closure of 1''.21.

While work on this block (ended in June, 1950) was going on, preliminary studies were begun concerning the method to be employed in the adjustment of the Northern block covering Estonia, Lithuania, Finland, Norway, Sweden and Denmark. On the basis of the experience gained in the adjustment of the South-western block, it was decided to use, instead of the standard method of condition equations, that of the variation of co-ordinates which, although leading to more laborious calculation, offers the possibility of a higher degree of mechanization.

For lengths and azimuths, condition equations were formed and then added to those resulting from the method of co-ordinates variation, as for the South-east block. The calculation was organized in such a way that different groups of operators might carry it on simultaneously from different points of departure. The 1,042 triangles observed have an average closure of 0''.70.

Whitten ends his description by the salient comment that the experience gained during these two colossal operations and especially the practice acquired in calculations and use of computing machines, enable us to-day to consider as possible the realization of adjustment projects still vaster than the present ones, which did not include the entire network of each country but only chains of triangles selected as convenience dictated.

After Mr. Whitten's report, the President announced that an International Committee would be appointed for further study of the adjustment of the European nets and that such Committee would include one member for each nation represented.

Proposal concerning the adjustment of world geodetic nets

During the second meeting, Captain Viglieri, Director of the Hydrographic Institute of the Italian Navy, spoke on the question of the adjustment of geodetic nets of the world.

He recalled how already, at the Oslo Conference (1948), the President of the Directing Committee of the I.H.B., Admiral Nares, had announced his intention to establish contacts between the International Union of Geodesy and Geophysics and hydrographers in order to solve the question of obtaining geographical co-ordinates of fixes at sea with the greatest possible degree of accuracy.

Experiments made using the new radio-electric methods (Decca, Loran, etc.), by which a fix may be obtained at great distances from the coasts and with great accuracy, made it evident that when one is obliged to make simultaneous use of points belonging to the nets of several countries, differences may be found which are no longer admissible. Also Captain Jensen, Hydrographer of the Royal Danish Hydrographic Office, reporting on experiments over the Danish chain made from the Surveying Ship *Freia,* stated that it had been possible to determine the position of the ship with an approximation of 50 metres and that, on account of the discrepancies existing between the Danish, Norwegian and Swedish nets, appreciable differences had been ascertained between the positions determined by radio-electric methods and those obtained by means of land points.

The Triangulations Section had entered the above communications in its records.

Captain Viglieri then said that the overall adjustment of the European networks, recently achieved, represents an initial and noteworthy step towards meeting the wishes of hydrographers. As far as it is known the International Hydrographic Bureau has not yet obtained from the I.A.G. the results of the work done in this field and, on the other hand, the requests put forward were of a *qualitative* character and require to be completed *quantitatively* by specifying the order of approximation necessary for the present-day needs of navigation.

The chief point is now to establish which is the order of approximation required in the values of the geographical co-ordinates of the stations forming part of a group of radio aids for navigation.

With stations about 100 kilometres apart, the relative distances must be known with an approximation of a few metres and, for this purpose, the accuracy that can be obtained by means of a first order national net is more than sufficient; still, it may be necessary to include in a same group several radio-electric stations connected with nets referring to different orientation ellipsoids or it may happen that stations may be used from a zone not linked up with them and which consequently does not possess co-ordinates homogeneous to those of the stations.

These considerations put in evidence the importance, for the needs of modern navigation, of the co-ordination of world networks.

In view of the fact that the degree of accuracy required for the geographical co-ordinates is not excessive, the values of them obtained from the adjustment (first order approximation) of the networks by the improved Bowie method, or any similar one, as it has been done for the European networks, are already sufficient ; this is admissible, even if it is only by examination of vertical's deviations, obtainable after co-ordination of the world network, that a criterion will be available for establishing the origin and orientation of one single, or of several reference ellipsoids, to be selected as most suitable.

Some islands or island groups can be linked up with the continents by using, if necessary, parachuted or radio-controlled flares and theodolites with photographic registration (e.g. the Gigas type), or Radar-based methods.

Other island groups, however, which on account of their distance cannot be connected with the adjusted continental network by adopting the above systems, can be covered by a continuous *independent* network based on accurate astronomical determination. It will thus be possible to obtain also in these cases the homogeneity in the values of geographical co-ordinates that is indispensable for modern navigation in those zones.

It is true that with systems based on total or annular solar eclipses or on star occultation or gravimetric methods, it will be possible to link up the continents — or islands with continents — geodetically with one another ; but hydrographers do not feel that this is necessary at present, because after leaving an area in which it is possible to fix the ship's position by radio-electric systems, bound for a similar area beyond the ocean, navigation between the two must be controlled astronomically, and consequently, in the case of any island that may be met on the way, the accurate astronomic determination of a point of it supported by local geodetic triangulation is sufficient.

From the foregoing it is easy to realize the immediate practical importance of the planned coordination of the networks, and bearing in mind the order of approximation required, it is obviously desirable that the I.A.G. communicate forthwith to the I.H.B. the results obtained in the adjustment of the European networks as well as the programmes that it intends to carry out in future for the adjustment of world networks.

In its turn the I.H.B., taking an active part in this project, ought to bring its influence to bear upon the various countries, through the hydrographic offices, requesting such countries to co-operate fully towards the attainment of the planned coordination.

The cooperation of all concerned will unfailingly achieve an early and complete solution of this complex problem, undoubtedly one of fundamental importance for the science of hydrography.

A lively discussion followed Captain Viglieri's speech ; agreement concerning the question discussed was general. The meeting was also unanimous in recognizing the advantages that would accrue from adopting for cartography and for radio-electric navigation a single international network similar to the one existing in Europe since the recent achievement of the adjustment work.

Admiral Nares, President of the Directing Committee of the International Hydrographic Bureau, requested that results of the European adjustments be forwarded officially to the I.H.B., the Bureau undertaking to forward them to the Hydrographic Offices of its States-Members.

Report on the use of the geodimeter

M. Erik Bergstrand brought forward the subject of the use of the « geodimeter » for distance measurements. So far as the principle of its working is concerned, this instrument closely resembles the standard arrangement worked out by Fizeau for measurement of the velocity of light.

In diagram form, it is composed of a source whose light is made to pass through a *Kerr* cell inserted between two crossed *Nicols* prisms, so that the intensity of transmitted light is a function of the voltage difference which exists between the condenser's plates.

A circuit, controlled by a quartz oscillator, causes the sign of this voltage difference to reverse, so that a succession of luminous flashes, having a frequency of the order of 100 per second, is obtained.

These flashes, after having been reflected by a plane mirror placed at the other extremity of the base to be measured, converge onto a photoelectric cell fed by the above-said oscillator and whose sensitivity varies therefore with the same frequency with which the light flashes are transmitted. In this way the reflected flashes are more or less synchronized with the instants of high sensitivity of the cell from which a current is produced which is therefore a function of the phasing between these instants and those of the arrival of the reflected flashes, i.e., a function both of the distance to the mirror and of the frequency at which the flashes are transmitted. Specifically, the current of the photo-electric cell has a zero intensity for all positions of the mirror located at multiple distances (according to a round number of times N) of the wave-length corresponding to the frequency of the flashes.

The points constituting this succession of positions which are at equal intervals the one from the other, are called « zero points ». The number N can be determined when an approximate value of the distance.to be measured is known, or by effecting the measurements with two different wave-lengths, i.e. on different flash frequencies.

It must be borne in mind that the velocity of light, which constitutes the physical element upon which this method of determination is based, must be known with very great accuracy. Its value in vacuum is henceforth accurately known thanks to the works of numerous researchers who have studied the subject, and the agreement existing between different values determined by different methods is significant, as may be seen from the following table, which summarizes the most recent determinations:

OBSERVER	METHOD	RESULT
Bergstrand	Geodimeter	299 793 1 ± 0.26
Essen	Cavity resonator	299 792 ± 3
Aslakson	Shoran	299 792 ± 2.4

On the contrary it is necessary to determine the value of the actual velocity through air at the time of each experiment and for this it is necessary to know with great accuracy the temperature, barometric pressure and humidity of the air along the track followed by the ray; it is therefore necessary to extrapolate when the ray path runs at an altitude above the ground where it is not practicable to set up measurement instruments. For instance, with regard to temperature, recourse has been had to a curve giving the vertical gradient in the atmosphere at different hours of the day; of course it must be remembered that this vertical gradient is also subject to change due to the influence of wind, etc.

The technique of observation is as follows: the mirror is placed at the extremity of the side and the frequency made to vary slightly so as to bring to zero the value of the current in the cell; the distance from the mirror to nearest zero point is then determined from the change imposed upon the frequency. The zero point nearest the geodimeter is similarly determined and the two extremities of the distance to be measured are thus obtained. The distance to the nearest zero point is a fundamental constant of the instrument which must be determined on a measured base.

The results so obtained with the geodimeter are summarized in the following table:

GEODETIC DISTANCE	DISTANCE ACCORDING TO GEODIMETER	NUMBER OF OBSERVATIONS
20 203.60 ± 0.10 m.	20 203.60 ± 0.04 m.	7
30 920.76 (1)	30 921.53 ± 0.07 m.	17
23 506.25 ± 0.07 m.	23 506.33 ± 0.04 m.	17
	28 071.76 ± 0.03 m. May	28
28 071.88 ± 0.10 m.	.79 ± 0.05 m. October	32
	.74 November	1
11 032.06 ± 0.08 m.	11 032.01 ± 0.03 m.	2 (2)

The measurement of each side has required, on an average, four nights; in the sides measured, the correction taking into account earth curvature is already incorporated.

The limits of approximation of values deduced from the geodimeter are reckoned for an error of 0°.5 (C.) temperature.

Report on geodetic application of radioelectric procedures

Dr. C. A. Hart, University of Roorke (India) submitted a report on geodetic applications of radar-type procedures. Having run over the first experiments made in this field during the war when radar was used for military purposes (bombing, photographic reconnaissance, etc...) and successively in Italy, Great Britain, Canada and Australia, the speaker passed in review the more recent experiments carried out after the date of the communication he had presented to the General Assembly of the I.A.G. at Oslo in 1948.

Recent improvements in this field affect chiefly the following points:

a) Crossing the base line by plane following a « figure 8 » track which causes a change of distance such as to eliminate observation errors (dependent upon operator's hysteresis) which took place when using the first method adopted which consisted in flying across the side on a perpendicular course;

b) Determination of the distance obtained by the least squares method;

c) Improvement in determination of altitude of the plane when it must fly above a non-surveyed area;

d) Corrections to velocity of electro-magnetic wave according to effective air conditions, checked by a second plane during the measurement;

e) Improvement of methods for reduction to the geoid of distances measured by plane;

f) Instrumental improvement in Shoran apparatus, particularly with regard to the possibility of correcting change of signal intensity and the introduction of a cinematographic recording camera in order to eliminate the personal errors of the observer.

---

(1) Uncertain.

(2) With the new geodimeter.

In Australia the quadrilateral Sydney - Canberra - Tamworth - Condobolin was determined by measuring the four sides and the two diagonals, having lengths comprised between 158 and 311 miles, with an average of 20 measurements for each side; the mean approximation obtained in the distances is about 24 metres, which is equal to a ratio of 1/14,285 of the distance.

In the U.S.A., after many experiments the purpose of which was to improve the working conditions of the instruments, a project of radar-trilateration has been worked out in Florida in order to check the present degree of accuracy which this sort of triangulation is capable of affording; for this purpose 6 geodetic vertices were selected. The measured distances vary from 40 to 320 miles with linear errors which, after adjustment, reach a maximum value of about 4.5 metres with a mean error — obtained by comparison with the known geodetic distances — of about 1.8 metre, as shown by the following table in which the definite shoran distances are entered.

GEODETIC DISTANCES	SHORAN DISTANCES	DIFFERENCE
Statute Miles		(in feet)
40 6131	40 6122	— 4
96 7171	96 7174	— 13
100 3098	100 3105	+ 4
118 9953	118 9951	— 1
133 0113	133 0122	+ 5
134 9698	134 9684	— 7
139 1225	139 1251	+ 14
145 8487	145 8421	— 3
145 8884	145 8913	+ 15
190 5047	190 5060	+ 7
199 1914	199 1912	— 1
226 9903	226 9893	— 5
235 5264	235 5241	— 12
277 0569	277 0572	+ 2
320 1519	320 1521	+ 1

At present, in the U.S.A., a new project is being prepared which will take into account 32 stations and 130 measured distances.

In Canada, the sides of a triangle (Senneterre - Orleans - Sudbury) covering between 217 and 273 miles have been measured and an approximation of between 1/83,000 and 1/26,000 has been obtained; following this experiment, an extensive work of shoran survey was carried out which is undoubtedly the most important of all those so far achieved. This work will be described in detail in the following report by J. E. R. Ross, the conclusions of which can be anticipated, namely, that the measurements of distances made are of a very high degree of accuracy: that, with shoran, it cannot be guaranteed that the exact orientation in azimuth will be maintained and that it is, therefore, necessary to measure supplementary sides (diagonal lines) for each vertex; that, in view of the length of the lines, it is not generally possible to carry out simultaneously the check in azimuth, and that consequently Laplace stations must be provided for.

As a conclusion to all these experiments, Dr. Hart affirms that in the present state of development of radar technique, it is beyond all doubt that the geodetic measurement of distances by radar-type systems permits determination of positions with an accuracy comparable to that obtained with standard first order triangulation. In particular, for the connection of points for which the optical method cannot be applied, the results obtained are certainly superior to those which can be reached by astronomical determinations.

Up till now no exact estimate of the cost of this new method of geodetic surveying has been established; but it is obvious that, for instance, having to extend the triangulation network in the non-surveyed areas of Northern Canada, South America and Africa, as for linking up between North and South America, this method will require a very much shorter time for completion of the work.

Dr. C. A. Hart ended by saying that since the method had arrived at a satisfactory point so far as the technical side is concerned, he hoped that geodesists would lend all possible co-operation toward its improvement, as had been accomplished in the case of standard triangulation.

Report on shoran trilateration in Canada

J. E. R. Ross then read a report concerning shoran trilateration operations in Canada where a net has been selected starting from the geodetic side (Sprague - Camp Hugues, 296 km.), and closing on the other geodetic side (Beatty - Vermillon, 406 km.) with an axial length of roughly 1,760 km. The mean length of the sides is 336 km., with a maximum length of 496 km. and minimum of 128 km.

The preliminary work was completed in 1948; this consisted of the reconnaissance of 17 selected stations, for two of which accurate determinations of latitude, longitude and azimuth were made with a view to their use as Laplace stations.

In fact, a shoran trilateration requires a check in azimuth, but it is still doubtful whether the correction can suitably be made use of because of the smallness of the triangles which contain the side on which the azimuth check is made. One thing is certain: that two stations, reciprocally visible and forming part of a shoran network, cannot reasonably be used except for this purpose and that the azimuth check will at least serve for orientation of the side.

The author then goes on to describe the method used and the observations regarding the important problem of « calibration », that is, the exact determination of the time delay elapsing between reception of the signal by the ground station and re-transmission of the same signal.

In measuring each side, the crossings made by the plane were always at least 16 in number, and the method used the one of figure-eight line crossing; the route flown was chosen in order to form, with respect to the normal, such an angle as to have, during the measuring operation, a variation in distance, relative to each of the two stations on the ground, of the order of 1 mile. Two symmetrical routes (with reference to the baseline) were followed, first in one direction then in the other. As the time necessary for each flight is that required for the camera to take at least 31 photographs of the indicator, the distance flown by the plane during the period of measurement is thus determined, and consequently is established the route, which, as previously said, should enable a variation in distance of 1 mile to be obtained. In general, the route flown in line crossing forms with the baseline's normal an angle between 14 and 8 degrees.

Measurements were begun in 1949 and completed in 1950; during the latter year, thanks to the experience acquired, the work could be carried out much more efficiently.

When the two partial distances measured by the plane are added together, the distance relative to each line-crossing is obtained. These values are plotted on a graph and a parabola is drawn across them ; the minimum value deduced from it represents the distance measured along the two lines plane-stations.

When the lines are very short, as those used on the sides for azimuth control, better results are obtained by having the plane cross the baseline on the outer side of the stations and about 100 km. away from them. The distance between stations is obtained by the same procedure, but in this case it is represented by the maximum value of the difference between the two distances measured by the plane.

Once the plane-stations distances have been obtained, corrections must be applied, the first of which is the reduction to the horizon of the plane-stations distances. For this purpose it is necessary to know the elevations both of the stations and the plane, and as this method of surveying is particularly destined for use in areas that are little explored and where altitudes of the ground are unknown, it may be interesting to describe the way in which a practical solution to this problem was obtained in Canada.

While operations were being prepared a set of aneroid barometers was placed in the vicinity of the stations, and readings were taken every 3 hours simultaneously with international synoptic time. The data thus obtained were handed over to the Canadian Meteorological Service, where through comparison with the weather charts, the pressure reduced to sea-level was obtained by interpolation and careful analysis for all points considered and for each observation made every 3 hours.

Using all these values, a mean value was obtained for each station (for a period of 14 days) of the difference between the atmospheric pressure measured and the pressure reduced to sea-level deduced from the weather charts.

A graph was then constructed, having as ordinates, for a certain number of meteorological stations in the area, the differences between the pressure measured at these stations and the pressure reduced to sea-level for each of them, and the altitude values of the stations themselves as abscissae. A straight line was then made to pass through the points thus plotted, and this line was drawn in such a way as to give greater weight to the points representing meteorological stations of approximately the same elevation as the shoran stations and with similar morphological characteristics.

By means of this graph the elevation of the shoran stations was then found by linear interpolation. This method supplied a degree of accuracy to within approximately 2 metres in areas where meteorological stations were numerous, and it is believed that where distances between shoran and meteorological stations amount to 100 or 200 miles, a 10-metre degree of accuracy can be obtained; under least favourable conditions encountered during the surveys, the error may be as much as 30 metres.

Determination of the elevation of the plane is made in a similar way, by means of ground-stations making observations at the time of the line-crossings and by using the most current data supplied by the Meteorological Service in order to determine the altitude of a specific isobaric surface. Pressure is measured on board at the time of line-crossing, and in order to determine the height of the plane, the difference in altitude between the two isobaric surfaces remains to be determined. To do this the mean density of the air must be known, and this is obtained by having measurements of the temperature and humidity of the air taken by the plane before, during and after crossings.

It should be remarked that an approximation of 45 to 50 metres can be obtained by this method, although larger errors may be expected if conditions are unfavourable.

The distances obtained should then be corrected for variation of the velocity of propagation and curvature of the path actually followed by the waves. This correction can be figured from tables prepared for the purpose.

A geodetic survey party composed of 8 persons followed operations and made corrections immediately after the flights. Stations could not be moved until the results of adjustment were known, and measurements were repeated for lines requiring too great an adjustment. In 1950 measurements were repeated for 7 out of 56 measured lines, but in 1949 the number was greater owing to lack of experience and faulty instruments.

A far larger amount of work was accomplished during 1950, owing partly to more favourable meteorological conditions, but also because while in 1949 it was decided to take no measurements when warm or cold fronts or inversions existed in the areas, for fear of serious inaccuracy, in 1950 measures were made whenever weather conditions were safe enough for flying. The results obtained were quite satisfactory although their precise evaluation is not possible until a qualitative meteorological analysis has been made. Improvements were moreover made in air-borne equipment and in the technique of its operation, with the result that a record of 40 line-crossing flights were made in one day.

With regard to the adjustment of the shoran sides, J. E. R. Ross points out that the lengths determined are subject to three types of errors: constant, systematic and accidental. « Constant » error is considered the one due to inaccurate knowledge of delay, following inadequate calibration, causing an error equal in amount and sign in all measured sides, irrespective of their length; and by « systematic » error is understood that which may be due to an inadequate knowledge of physical constants occurring in computations, such as the velocity of wave propagation.

The method used in adjustment is Helmert's, consisting of a system of differential formulae supplying the relation between the variations in length and azimuth of a geodetic line and the corresponding variation of the geographic co-ordinates of its extremities. The scheme of adjustment is as follows:

I. A first adjustment is made using the shoran data which may still contain the error in delay calibration;

II. To this first adjustment are added the data referring to the two Laplace stations;

III. In a second adjustment the necessary data for bringing into coincidence the two ends of the shoran net with the Beatty and Vermillon terminal geodetic stations are taken into consideration. Possible instrumental errors and those due to calibration are therefore greatly reduced. As calibration was performed differently in 1949 and 1950, it will be necessary to have two distinct values.

IV. The data with reference to par. II are readjusted with due regard to the calibration constant determined during the second adjustment (par. III).

The positions obtained for the terminal ends of the net for each of the four steps of the adjustment have been listed in the following table, where the difference between the values IV and the geodetic co-ordinates represent the final closure. As said before, the values III coincide with the geodetic values.

BEATTY

		latitude	longitude
Shoran values	I	52°54'07'',741	104°48'28'',011
	II	08'',490	28'',212
	III	08'',182	27'',046
	IV	08'',181	27'',783
Geodetic values		52°54'08'',182	104°48'27'',046

VERMILLON

		latitude	longitude
Shoran values	I	53°17'27'',891	110°52'57'',270
	II	29'',349	57'',474
	III	29'',632	55'',052
	IV	28'',968	56'',230
Geodetic values		53°17'29'',632	110°52'55'',052

Final closure shows that Beatty's shoran position requires a 12.64 m. correction eastward and 0.03 m. northward, and Vermillon's 21.79 m. eastward and 20.54 m. northward.

If even the largest of these errors in position is referred to the axial length of the net, the resulting closure error is 1/59,000.

The length of the Beatty-Vermillon side obtained by adjustment IV shows a difference of 6.86 m. with the geodetic length, and as this difference applies to a side 406 km. long, after a development of 1,760 km., it is an excellent proof of the quality of the method, and also shows that the technique of observation has attained a satisfactory degree.

The examination of the corrections to be applied to the 74 sides (due to adjustment III) has shown that maximum corrections were relative to sides originating from two stations in both of which operational difficulties as well as variations in the calibration setting were encountered.

Final adjustment does not appear to show any systematic error, as the number of positive corrections is practically the same as that of negative corrections (39 versus 35) and the sum of the two groups is almost equal. The ratio between the correction and the length of the sides is less than 1/30,000 for 59 lines and 1/50,000 for 49 out of the 74 lines. The lines for which the ratio is highest are 13 in number and fall between limits of 1/15,000 and 1/30,000. The most glaring errors occur only in connection with the two lines used for azimuth control (1/5,000 and 1/4,000) and this was due to the impossibility of measuring short lines by shoran (in the present case 32 and 26 kilometres) with the same accuracy as the long ones.

Reference has previously been made to the advantage of Laplace stations: their use is particularly worthwhile, but whenever possible the length of the azimuth line should be at least one-fifth of the lines of the net : in the present work, however, owing to difficulties encountered in the field, the ratio was 1 to 10.

Comparison has also been made between the final shoran and astronomical positions for a certain number of vertices, but accurate methods had been used in the determination of the astronomical co-ordinates of only three among them, the others hiving been obtained from exploratory fixations.

As regards these three positions, a maximum discrepancy (in the Churchill vertex) of 123 metres along the meridian and 60 metres along the parallel was found; the difference is certainly due to deflection from the vertical and may serve to give an idea of the approximation that an astronomical station can give.

From this first important operation of trilateration, J. E. R. Ross draws the conclusion that measurement of distances by the shoran method supplies a high degree of accuracy and that it can be used to extend a network of stations spaced widely apart with an accuracy of position determination far exceeding that obtainable by astronomical methods.

Laplace stations must be introduced at frequent intervals for azimuth control, however, as the shoran method does not enable orientation of a network to be maintained with the same degree of accuracy as by ordinary triangulation methods. Observed azimuths should have reference to lines having the order of magnitude of the net lines, but as this is in general impossible, special care must be taken to introduce Laplace stations where maximum distance may be attained.

Results obtained may be summarized as follows:

(*a*) Lines having an average length of 340 km. may be measured with an accidental error of the order of 5.50m.;

(*b*) When care has been taken to measure a number of sides for each vertex amounting to approximately twice the number that would be needed in determining the station geometrically, probable latitude and longitude errors are of the order of 8 metres;

(*c*) The error in the length of the closure side determined at the end of a 1,760 km. network is + 6.8 metres, which is equivalent to 1/59,000.

It appears, therefore, that accidental error may be evaluated at 1/60,000.

The author ends by pointing out improvements that might be made with regard to instrumental equipment; he likewise recommends that in future operations the elevation of the surface of a certain number of lakes in the area to be surveyed should be determined, so that reference may be had to them in order to eliminate the majority of errors due to inaccurate determination of station and plane altitudes; he also suggests the use of the radiosonde with a view towards improving corrections to be made for wave paths.

Miscellaneous Reports

Following the report on trilateration, M. Dupuy of the French National Geographic Institute informed the commission that new formulae have been studied by the N.G.I. for the computation of very long geodetic lines with a high degree of accuracy. Tables in connection with this method have been prepared.

Professor Baeschlin then described a method enabling computation of very long geodetic lines with an accuracy of 1:200,000, and the computation of trilateration network adjustment by means of the conditioned observations method. This method is based on conformal representation of the ellipsoid on the sphere and enables Laplace's equation to be easily accounted for. Tables have been computed that greatly simplify the method.

Dr. O. Simonsen presented a report on the calculations of the connection made by means of parachuted flares between Denmark and Norway in 1945.

The connection calculations, already worked out by 1948, were based on preliminary adjustment of the Danish net. New values were then determined for the nets following adjustment of the Baltic ring and the inclusion of astronomical observations.

Recomputation of the connection has therefore become necessary and the paper presented describes the methods followed and the results obtained. A complete report will be published by the Danish Geodetic Institute.

Resolutions and recommendations of I.A.G.

At the close of proceedings, the Closing General Session of the International Geodetic Association, after examining the work of F. W. Hough and W. C. A. Whitten on the overall adjustment of European nets, expressed its congratulations and thanks to the Army Map Service and the U.S. Coast and Geodetic Survey for their magnificent job in fulfilment of a long-expressed desire of the Association.

The following four proposals relating to the Triangulations Division were then approved:

1. CONSIDERING that an adjustment of the triangulation around the Eastern Mediterranean will supplement the European network as already adjusted.

CONSIDERING that a geodetic connection between Crete and North Africa was recommended in different instances to the interested nations on the occasion of previous General Assemblies and that modern technique now gives the possibility of a very satisfactory connection.

CONSIDERING that geodetic operations now under way make it possible to contemplate this adjustment in the very near future, provided that a limited number of triangulation junctions still unobserved, be observed.

RESOLVES THAT under agreement between the interested governments, geodetical connection between Crete and Cyrenaica be made as soon as possible, *that* the triangulation junction between Turkey and Syria, necessary for the adjustment, be also observed as soon as possible.

2. CONSIDERING that the International Association of Geodesy on the occasion of several previous General Assemblies expressed its very deep interest in the completion of the 30th Arc of Meridian which crosses the whole African Continent, and

CONSIDERING further that the gap existing in this Arc of Meridian covers only a thousand kilometres approximately.

RESOLVES that under agreement of the interested governments the observations should be carried out as soon as possible and that this chain should be used in the future as a basis for a general adjustment of the geodetic network of the African Continent.

AGREES to facilitate as far as possible the execution of these observations.

3. CONSIDERING that there is an obvious advantage to use in the different countries the same projection system.

CONSIDERING that the transverse Mercator projection (Gauss projection) seems to be the most adequate system, as already proposed by the International Association of Geodesy, for general use up to latitude 80°.

CONSIDERING that detailed Tables for this system already exist.

EXPRESSES the wish that the Transverse Mercator Projection in 6° belts in the sheet line of the 1,000,000 International World Map with a scale factor of 0,9996 should be used in preference to any other system for the computation of geodetic co-ordinates and for mapping and charting, whenever the interested country finds it adequate and practicable.

There is no question of making an obligation to any country where geodetic and topographic surveys are well developed to adopt a new system of projection.

But it is recommended that the proposed projection should be used in preference to any other:

I) in countries recently opened to Geodesy and Topography, for recent surveys and for surveys to be undertaken in the future ;

II) in the countries already covered by geodetic surveys and topographical maps, whenever a replanning of the topographic and cartographic program is undertaken.

4. CONSIDERING the high accuracy obtained in the measurement of a standard base-line in Finland with a light interference apparatus,

RECOMMENDS that such basis be measured by a similar method in different countries by the interested organizations, and

ASKS the Bureau of the Association to facilitate necessary arrangements so that such basis could be used, if desired, by neighbouring countries, to compare the results obtained by this process with those obtained by wires or tapes compared to the standards of the International Bureau of Weights and Measures