PRIMING AND LAGGING

by Henri BENCKER

A recent article by Mr. E.E. Mann in the well-known English review: « The Journal of the Institute Navigation », Forum, Vol. VII, No., 1, January 1954, which was accompanied by the comments of Dr. A.T. Doodson, the distinguished Director of the Tidal Institute of Liverpool, has once more turned the attention of the specialists in the field of navigation on the two terms: « priming » and « lagging », widely used during the last century by pilots and seafarers to describe, in colloquial use, the relation of the tide (high water) to the position of its generative celestial body.

In French tidal terminonoly, the expressions: « gagnant » - gain, « tidal advance » - gain de la marée, « the tides gain » - les marées gagnent, « revival » - revif (which denote an increase of intensity), « losing » - perdant, « the tides lose » - les marées perdent, « loss » - déchet (which denote the decrease in the intensity of tides), would appear to refer to the variation in the amplitude of the tide rather than to the moment of its arrival at a certain place. Therefore, they refer naturally, to the period from neap-tide to spring-tide and from spring-tide to neap-tide respectively within a single lunation.

The terms « regain » - gaining, « priming » - avance, and « loss » - déchet, « lagging » - retard, are very old and date back to what is now considered an out-fashioned conception of the tide in accordance with the theory developed by Daniel Bernoulli in 1740 and later put into practice after the empiric work undertaken by Sir John Lubbock (sr) using a complete period (18 years 6/10) of tidal observations carried on from 1809 to 1828 in the London Docks (cf.: « Philosophical Transactions of the Royal Society », London, 1832-1839).

The analysis of these observations let Mr. Dessiou, the calculator at the Nautical Almanac Office, to draw up several empirical tables supplying, on one

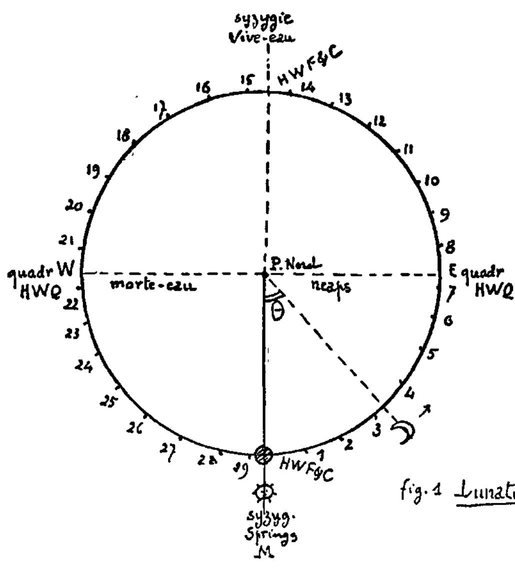


fig. 1 Lunation

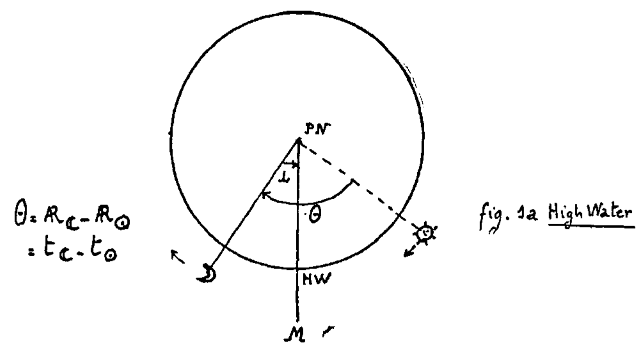
hand, the retardation of high water on the meridian transit of the Moon at a given place, and on the other hand, correction tables for taking into account respectively the values of the Declination and those of the parallax both for the Moon and the Sun during the lunation.

Figure 1 showns the Equator as seen from the North Pole.

If the Sun is assumed to be in a fixed position at meridian M, the figures from 1 to 29 in the diagram represent the equatorial positions of the Moon during a lunar month of 29 days. For a given position of the Moon (say from 1 to 29 days) the tide will occur with a time or different situation L in relation to the meridian transit of that body (see Fig. 1 a).

L is the « lunitidal interval » (HWI), i.e. the time elapsing from the meridian transit of the Moon to the local time of HW.

The angle θ represents the difference in the Right Ascensions (or in other terms, in the hour angles) of the Moon and the Sun.



☾ *Hour angle* (meridian transit) Angle horaire de la Lune (Passage au méridien)	*Hour*	*Correction of the Establishment* *minutes*	
springs *vive-eau*	0	0	priming (A)
	1	— 16	
	2	— 31	
	3	— 41	
	4	— 44	
	5	— 31	
neaps *morte-eau*	6	0	lagging (B)
	7	+ 31	
	8	+ 44	
	9	+ 41	
	10	+ 31	
springs *vive-eau*	11	+ 16	
	12	0	

In the course of a lunation, the quantity L varies from one day to another. Its average value occurs at spring-tide (conjunction or opposition) (HWF & C) (Establishment) and at neap-tide (HWQ) High Water Quadrature) showing only a slight variation at each.

With reference to Figure 1, the following table gives the establishment correction in terms of the time of the meridian transit of the Moon.

It will be seen that from the syzygy to the quadrature, the tide « primes », i.e. that it occurs earlier than when it is only under the influence of the Moon; from the quadrature to the syzygy, it « lags », i.e. it occurs later than when it is only under the influence of the Moon.

The tide, therefore, « gains » on the Moon from spring-tide to neap-tide as during this period the lunitidal interval is less than its average value; conversely, the tide « loses » on the Moon from neap-tide to spring-tide, as during this period the lunitidal interval is greater than its average value.

It can also be said, as in the « Admiralty Manual of Navigation », Vol. 1, 1928, that:

« When the Moon is in the first or third quarters, high water will be experienced before the Moon is on the meridian, in such cases the tide is said « to prime ».

« When the Moon is in the second or fourth quarters high water will be experienced after the Moon is on the meridian, and the tide is said « to lag ».

(See I.H.B. Special Publication No. 28: Vocabulary concerning tides.)

The above tables is based on the formula:

$$\text{Time of luni-solar HW} = \text{Time of lunar HW} - \frac{\varphi}{2}$$

$$\text{where } \tan \varphi := \frac{\sin 2\theta}{\dfrac{m}{s} + \cos 2\theta}$$

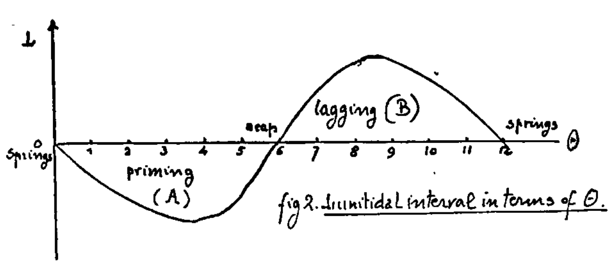
the angle φ representing what should be termed the phase inequality.

The ratio of the average actions of the Moon and the Sun, $\frac{m}{s}$ has been given the conventional value 2.73.

The tide « gains » when $0\text{ h.} < \theta < 6\text{ h.}$
$0° < 2\theta < 180°$
$6\text{ h.} < \theta < 12\text{ h.}$
$180° < 2\theta < 360°$

From this table, it will be seen that theoretically the minimum and maximum of L occurs towards 4 hours and 8 hours of the hour angle of the Moon.

If these data are shown by means of a curve, the following diagram will be arrived at and will show in terms of θ the variation of the *lunitidal interval* L in the course of the lunation (see Fig. 2).



But « definitions » (A) and (B) can be compared against others which can be found in the literature on tidal matters. For this reason, it is advisable, as Mr. Mann has so wisely observed, to come to some understanding on the subject under discussion before questioning the pupils or candidates.

The classics should therefore first be consulted, beginning with the « Oxford Dictionary », which is quoted by Dr. Doodson. In it, will be found:

Priming: The acceleration of the tide taking place from the neap to the spring tides.

Lagging: Lag of the tide: the interval by which the tide wave falls behind the mean time in the first and third quarters of the Moon. (This perhaps refers to L, but... it is rather imperfectly expressed. What happens during the 2nd and 4th quarters? What « mean time » is this?

In the very recent « Maritime Dictionary » by Mr. de Kerchove, a better definition will be found:

Priming: A shortening of the interval between the times of successive HW.

Lagging: The retardation in time of the occurrence of high and low water.

In the course of a lunation of 29.53 days:

The average tidal day lasts 24 h. 50 m.

The tidal day in syzygy last 24 h. 35 m. (i.e. 24 h. 17 m. at the perigee and 24 h. 33 m. at the apogee).

The tidal day in quadrature last 25 h. 25 m. (i.e. 24 h. 15 m. at the perigee and 25 h. 40 m. at the apogee).

Let us study this problem graphically, Figure 3 will give the position of the bodies on the day of syzygy when $\theta=0$ and $\theta'=L-S$.

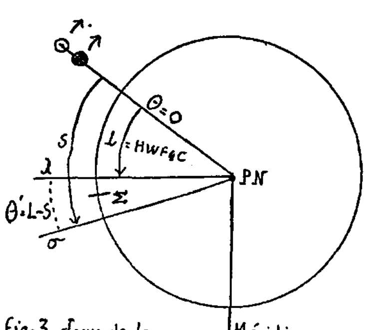


fig. 3 - Jour de la syzygie astronomique.

λ is the lunar tide (establishment L)
σ is the solar tide (establishment S)
Σ is the resulting tide

Figure 4 gives the position of the bodies on a syzygy tidal day, i.e. about two days after the astronomic syzygy due to the age of the tide, when $\theta=0$ and $\theta=L-S$.

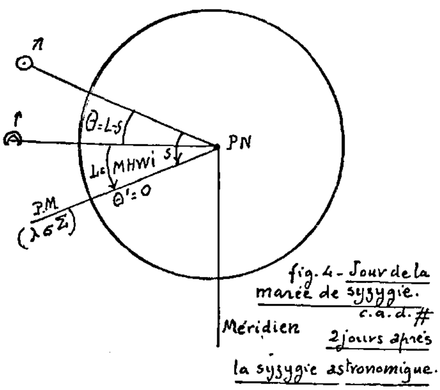
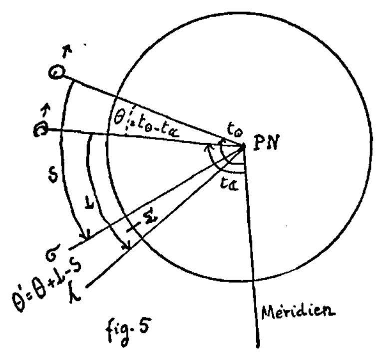


Figure 5 shows the position of the bodies on any day of the lunation when $\theta = t_{\odot} - t_{☾}$ and $\theta' = \theta + L - S$.



The tide « gains » in relation to the passage of the Moon when the angle θ' varies from 0° to 90°, i.e. between the syzygy and the quadrature tides.

Conversely, it « loses » (lags) when θ' varies from 90° to 180°, i.e. between the quadrature and the syzygy tides.

Moreover, it is known that spring-tides occur after the syzygy (new or full Moon) at intervals equal to the *age of the tide* and that neap-tides occur after the astronomic quadrature of the Moon (1st and 3rd quarters) at intervals equal to the *age of the tide.*

It follows therefore that the tide « advances » (primes) from spring-tide to neap-tide and that it « loses » (lags) from neap tide to spring tide.

Let us now pass to the analytical study of the question. The simple case of a single luni-solar semi-diurnal tide such as that of Brest will be considered and reference will be made to the developments to be found on this subject in classic works, such as « Etude et Prédiction des Marées » by Mr. Rollet de l'Isle, Hydrographic Engineer of the Hydrographic Service, Paris, 1905, « Tides and Tidal Streams » by Commander H.D. Warburg R.N., Cambridge, 1922, « Maree e Correnti di Marea », by Professor M. Tenani, Genoa, 1935, etc.

As it is well known, it is usual to relate the time of High Water to that of the meridian transit of the Moon. The time of luni-solar High Water in a given port is equal to the time of the transit of the Moon in the meridian of that port θ plus a constant period of time for that port (i.e. corrected establishment), plus a variable correction for taking into account the phase inequality.

$$\text{Time of HW} = \theta + E_c + C$$

The variable correction C is arrived at by the formula

$$\operatorname{tg} 2C = \frac{-\sin 2\theta}{\frac{30}{31}\alpha + \cos 2\theta}.$$

θ is the difference between the right ascensions of the Moon and of the Sun, the fraction $\frac{30}{31}$ is the value of $1 - \frac{d\theta}{dt}$ (dt being the variation of the hour angle), $\frac{d\theta}{dt} = \frac{50 \text{ min}}{24 \times 60} = \frac{1}{30}$ approximately, and α is the *ratio of the average actions of the Moon and of the Sun* $\frac{m}{S}$, and $\frac{30}{31}\alpha$ has an average value of 2.89 for Brest, and it varies from 1 to 3, for other ports of same tidal regime. E is the establishment of the port.

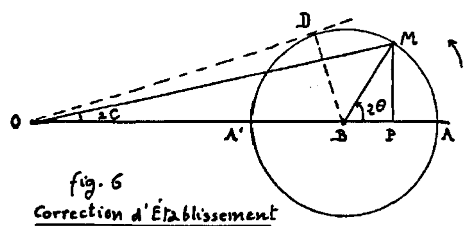


fig. 6
Correction d'Établissement

In Figure 6, let us assume OB = α = 2.89, BA = BM, the radius of the circle = 1. Then let us take ABM = 2θ and

$$\tan AOM = \frac{MP}{OP} = \frac{\sin 2\theta}{2.89 + \cos 2\theta}.$$

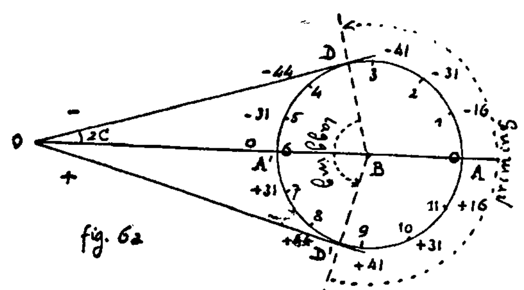
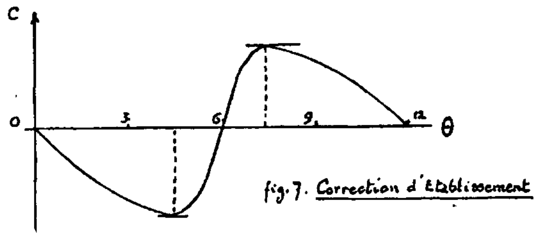
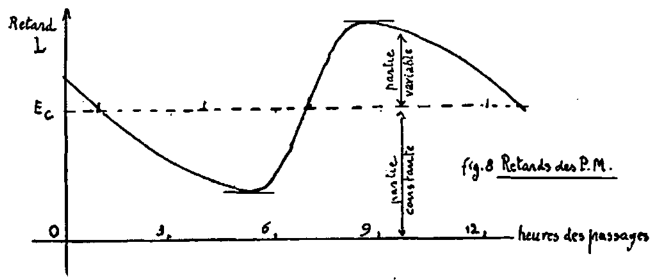


fig. 6a

Thus AOM = — 2C and this diagram supplies the values of C corresponding to each value of θ.

The variations in the times of High Water will therefore be arrived at by drawing the curve of the values of C in terms of those (see Figure 6) for which $\sin 2C = \frac{1}{2.89}$ i.d. C = 10° ou 0 h. 40 m. approximately.





The curve of the « lag » is the same as that for the values of C except that it is drawn towards the right of $\frac{1}{30}$th of the age of the Moon and is raised by a constant quantity equal to the corrected establishment (mean establishment as compared to HWF & C = vulgar establishment) (MHWI).

According to the values of α, C varies within the following limits:

when α = 1	C varies from 1 h. 22 m. to 3 h. 00 m.
when α = 2	C varies from 0 h. 36 m. to 1 h. 54 m.
when α = 3	C varies from 0 h. 24 m. to 1 h. 08 m.

The ratio of the average actions varies from 2 (at Gibraltar) to 7.8 (at Wilhemshafen).

This leads to the values of the average establishment correction which were given above empirically in connection with Figure 1.

The table given below, which is taken from the « Traité d'Hydrographie » of Germanin, Paris 1882, and was also reproduced in Commandant Houette's « Guide du Marin », is more exact in so far as it takes also into account, in terms of the parallax of the Moon, the variations in the average distance of the Moon from the Earth, but it does not take into account the variations in the declinations and of the solar parallax.

Establishment Corrections (Minutes)

Time of Moon's Meridian Transit.

Parallaxe ☾	Heure de passage de la Lune au Méridien											
	0 h 12 h	1 h 13 h	2 h 14 h	3 h 15 h	4 h 16 h	5 h 17 h	6 h 18 h	7 h 19 h	8 h 20 h	9 h 21 h	10 h 22 h	11 h 23 h
61' (périgée)	—4	—18	—32	—45	—56	—62	—56	—32	— 2	+14	+16	+ 8
60'	—3	—17	—32	—46	—58	—64	—58	—33	— 1	+15	+18	+10
59'	—2	—17	—33	—47	—59	—66	—60	—33	+ 1	+17	+20	+11
58'	—1	—17	—33	—49	—61	—67	—61	—34	+ 2	+19	+22	+12
57' (moyen. distance)	0	—17	—34	—50	—63	—69	—63	—34	+ 3	+21	+24	+14
56'	+2	—16	—35	—52	—66	—73	—66	—35	+ 5	+24	+27	+17
55'	+4	—16	—36	—56	—69	—77	—70	—36	+ 7	+27	+31	+20
54' (apogée)	+6	—15	—37	—57	—73	—81	—73	—37	+ 9	+32	+35	+24

On any day of the lunation, the true (or mean) time of High Water is equal to the true (or mean) time of the meridian transit of the Moon + E (establishmen) + Correction by means of the above table.

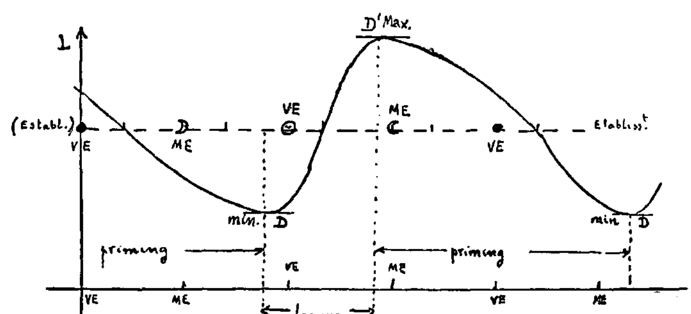


fig. 8 bis

Figure 8 bis illustrates this second way of defining the « advance » and the « losing » during a lunation.

Priming: Tidal period decreasing day-by-day, ie. : tide coming in faster.

Lagging: Tidal period increasing day-by-day, i.e. : tide coming in slower.

The curve drawn on the diagram (see figure 9) gives correction C (phase inequality) in terms of the time of the meridian transit of the Moon with the ratio $\frac{m}{s} = 2.73$ for the average actions.

If the age of the tide is 0, MHWI = HWF & C and the left-hand ordinate gives immediately the correction. The angle θ varies by 0.8 of an hour per lunar day; consequently, if the age of the tide is 2 1/2 days for the transit of the Moon at 0 hour, the phase inequality to be applied to the mean establishment

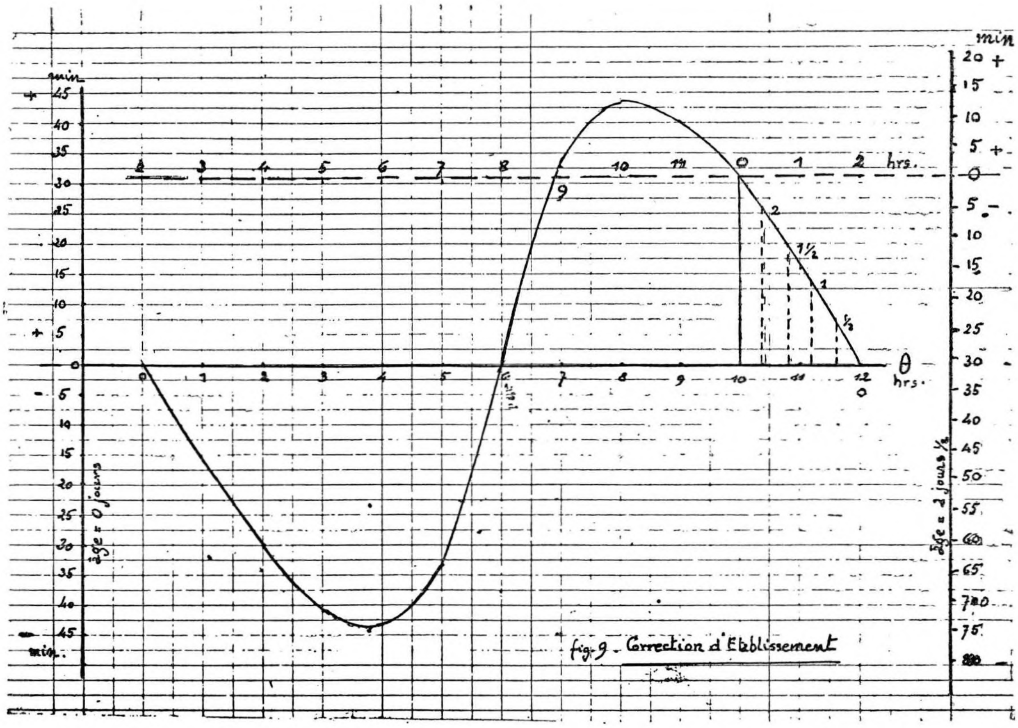
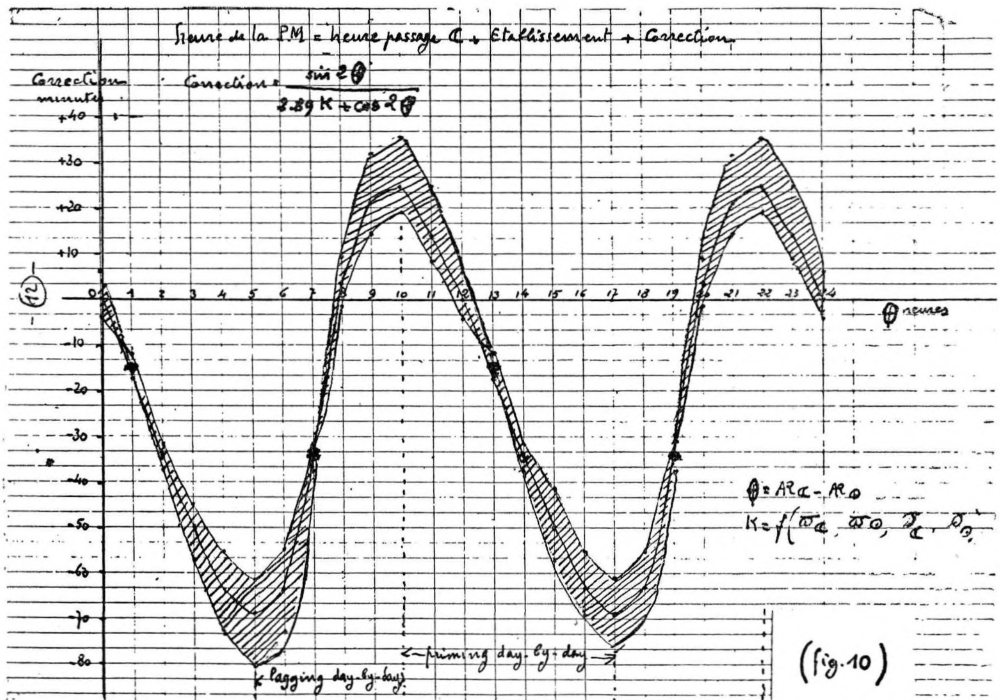


fig. 9 _ Correction d'Etablissement



(fig. 10)

(MHWI) will be the correction related to a meridian transit of the Moon occuring at 0 — (0.8 × 2 1/2) = 10 h. which can be found from the diagram by tracing accordingly a horizontal line passing through the point at which the curve intersects the ordinate of 10 hours and by reading the correction on the left-hand scale (see figure 9).

The following table allows the age of the tide to be taken into account.

Age of Tide	*Origin on diagram should be taken at point corresponding to ordinate:*
0 day	12 hours
1/2 —	11.6 —
1 —	11.2 —
1 1/2 —	10.8 —
2 —	10.4 —
2 1/2 —	10.0 —

Let us, for example, consult the Tide Table for Brest and select one specific lunation. It is easy to draw up, for each day of this lunation, a table giving the time of the transit of the Moon in the meridian of Brest, the value of the lunar parallax, the time of High Water taken from the table of tidal predictions, the lag of High Water on the meridian transit of the Moon and by substraction by the Establishment of Brest, i.e. (III hours 46) the daily establishment correction that can be compared with the theoretical correction for the parallax given by the above table quoted from the « Traité d'Hydrographie ».

When shown in graphical form, this correction supplies a curve which according to the variable value of the parallax in the course of a lunation, is contained in the shaded section of the diagram on Figure 10. This diagram, of which the minimum and maximum fall roughly on the ordinates of the hour angles of the Moon at 5 h., 10 h., 17 h., and 22 h. (i.e. on the 6th, 12th, 20.5th and 26.5th days of the Moon), allows for the determination of the days of the lunation on which time tide starts and ceases to « prime » and starts and ceases to « lag » on the Moon.

The following definition is arrived at:

Definition

« In regions where the semi-diurnal flow of the tide predominates, the tide « advances » (primes) in the course of a lunation, from the day of the syzygy tide to the day of the quadrature tide. »

« The tides « loses » (lags) from the day of the quadrature tide to the day of the syzygy tide ».

This exposé however is, as has already been mentioned, based on the somewhat exceptional character of the predominantly semi-diurnal tide found at Brest and in other European ports of the Atlantic.

As outlined above it cannot be generally applied to tides troughout the world and it is fully concurred with Dr. Doodson's comment that it is wiser not to try to define too rigorously what is meant by the terms « priming » and « lagging ». These terms have long since become obsolete in scientific treatises on the system of the tides, as now-a-days the navigator who learns to obtain the time of High Water with reference to the time of the meridian transit of the Moon and to the establishment of the port would seem to receive instruction in the methods as used formerly by his ancestors of the Eighteenth Century