SPECIAL CHART FOR DIRECT PLOTTING OF RADIOGONIOMETRICAL BEARINGS TAKEN FROM VESSELS OR AIRCRAFT

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Navigators are aware of Weir's azimuthal diagram as well as of the plotting process indicated in the *Pilot Charts* to obtain a great circle course. The idea of a geographical chart using the same lattice is no doubt of prior origin, (1) but no one has as yet drawn the attention of navigators and fliers to the use that can be made of such a chart for the rapid plotting of a ship's or plane's position where its radiocompass has given a bearing from a radio station. (2)

Let us recall briefly the characteristics of the chart :

Let there be two rectangular co-ordinate axes Ox and Oy

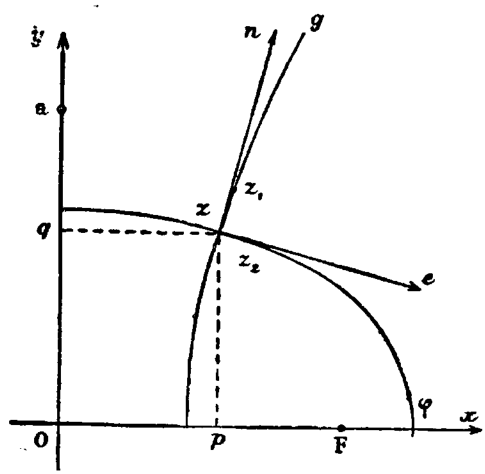

Fig. 1

representing respectively the equator and meridian G of the transmitting station which will be used as the longitude point of origin. A point Z of the geographical co-ordinates φ and G is shown on the chart by point z of the rectangular co-ordinates :

$$x = r \sin g \sec \varphi, \qquad y = r \cos g \tan \varphi,$$

in which unit r may be chosen arbitrarily and $g = G - G_0$ algebraically. The co-ordinates of image a of station A (φ_0, G_0) will specifically be :

$$x_0 = O, \; y_0 = r \tan \varphi_0.$$

The parallel of latitude φ is represented by the ellipse :

$$\frac{x^2}{r^2 \sec^2 \varphi} + \frac{y^2}{r^2 \tan^2 \varphi} = 1.$$

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(1) Littrow's projection.

(2) See Note, at end.

The meridian of longitude G is represented by the hyperbola :

$$\frac{x^2}{r^2 \sin^2 g} - \frac{y^2}{r^2 \cos^2 g} = 1.$$

The two curves are homofocal, their common focal half-length being r. They are at right angles at their intersection point z. Let us for instance consider a slight change of position from Z to Z_1 on the terrestrial meridian of Z corresponding to a variation $d\varphi$; the variations of the co-ordinates going from z to z_1 are :

$$(dx)g = r \sin g = \frac{\sin \varphi}{\cos^2 \varphi}\, d\varphi, \qquad (dy)g = r \cos g.\ \frac{1}{\cos^2 \varphi}\, d\varphi$$

and a slight change of position ZZ_2 on the parallel of Z corresponding to a variation in longitude dg ; the variations of the co-ordinates when going from z to z_2 are :

$$(dx)\,\varphi = r \sec \varphi \cos g\, dg, \qquad (dy)\,\varphi = - r \tan \varphi \sin g\, dg.$$

We have the relationship :

$$\left(\frac{dx}{dy}\right)_g = - \left(\frac{dy}{dx}\right)_\varphi = \tan g \sin \varphi$$

which proves that the tangents are perpendicular.

If, moreover, it is assumed that on the earth : $ZZ_1 = ZZ_2$,

$dg = \dfrac{d\varphi}{\cos \varphi}$ is obtained, and :

$$(dx)\, g = - (dy)\, \varphi, \qquad (dy)\, g = (dx)\, \varphi$$

therefore : $zz_1 = zz_2$, and the similarity of the small letters is retained on the chart ; it is a true representation. The lattice consists of two sets of ellipses and hyperbolae having a common focus and at right angles to each other, which are easy to construct.

Basic characteristic.

The locus of the plotted points z *of Points* Z *from which a constant bearing* Z *on a fixed station* A *on the meridian of origin is taken, is a straight line from the plotted* a *of the station which, together with the southern projection of the meridian, contains angle* Z.

Spherical triangle PAZ, where P is the pole of the hemisphere from station A, gives the relationship between φ and G when point Z moves along the arc containing angle Z constructed on PA :

$$\tan \varphi_0 \cos \varphi - \sin \varphi \cos g = \cot Z \sin g,$$

or

$$\tan \varphi_0 - \tan \varphi \cos g = \cot Z \sin g \sec \varphi.$$

On the chart, the differences of the co-ordinates of points a and z are :

$$\overline{qa} = y_0 - y = r\,(\tan \varphi_0 - \tan \varphi \cos g)$$

and

$$\overline{qz} = x = r \sin g \sec \varphi,$$

whence :

$$\cot \widehat{Oaz} = \frac{y_0 - y}{x} = \frac{\tan \varphi_0 - \tan \varphi \cos g}{\sin g \sec \varphi} = \cot Z.$$

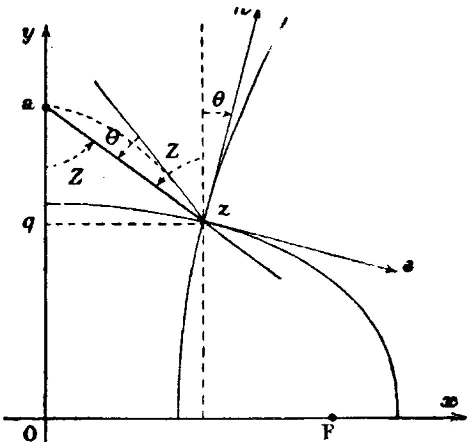


Fig. 2

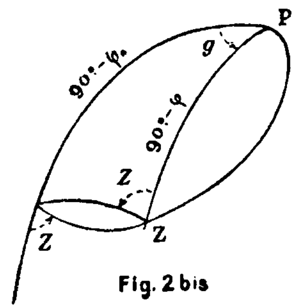


Fig. 2 bis

Both angles being between o and 180° : $\widehat{Oaz} = Z$.

The locus of point z on the chart is therefore a straight line from a, containing with aO angle Z. The chart changes the containing arcs into straight lines with their points of origin at any point of the first meridian. Moreover, the projection being true, the angle made by za with the northward projection of tangent zn to the meridian of z, is equal to the angle contained by arc ZA and meridian ZP on the sphere. It was seen above that :

$$(1) \qquad \left(\frac{dx}{dy}\right)_g = \tan g \sin \varphi = \tan \theta.$$

It is the trigonometric tangent of angle θ that zn makes with Oy and : $\widehat{azn} = Z + \theta$.

We drew up a formula in 1927 (1) that gives the direction of a radio bearing on a nautical chart. It may be noted that za representing great circle arc ZA makes angle θ with the straight line za.

Chart Construction.

The value of r in millimetres must first be settled. The scale must be chosen according to the desired approximation. Under present conditions, accurate bearings can only be obtained at relatively short distances, but technical achievements have already increased the effective range of bearings. However, in our estimation, the difference in longitude may be limited to 15° from the meridian of origin. In order to increase plotting accuracy, the largest scale consistent with the size of the sheet must be used, which requires a division of the chart along the meridian of origin. We suggest a size of 500 millimetres × 350 millimetres, which is not too cumbersome for flying purposes. Five charts can be constructed in this way by using a different value of r for each, with latitudes overlapping in such a way that a chart is always available with no given station at the division limit.

First chart : r = 1,400 mm., from 5° S. to 15° N, variation in longitude : about 14° ;

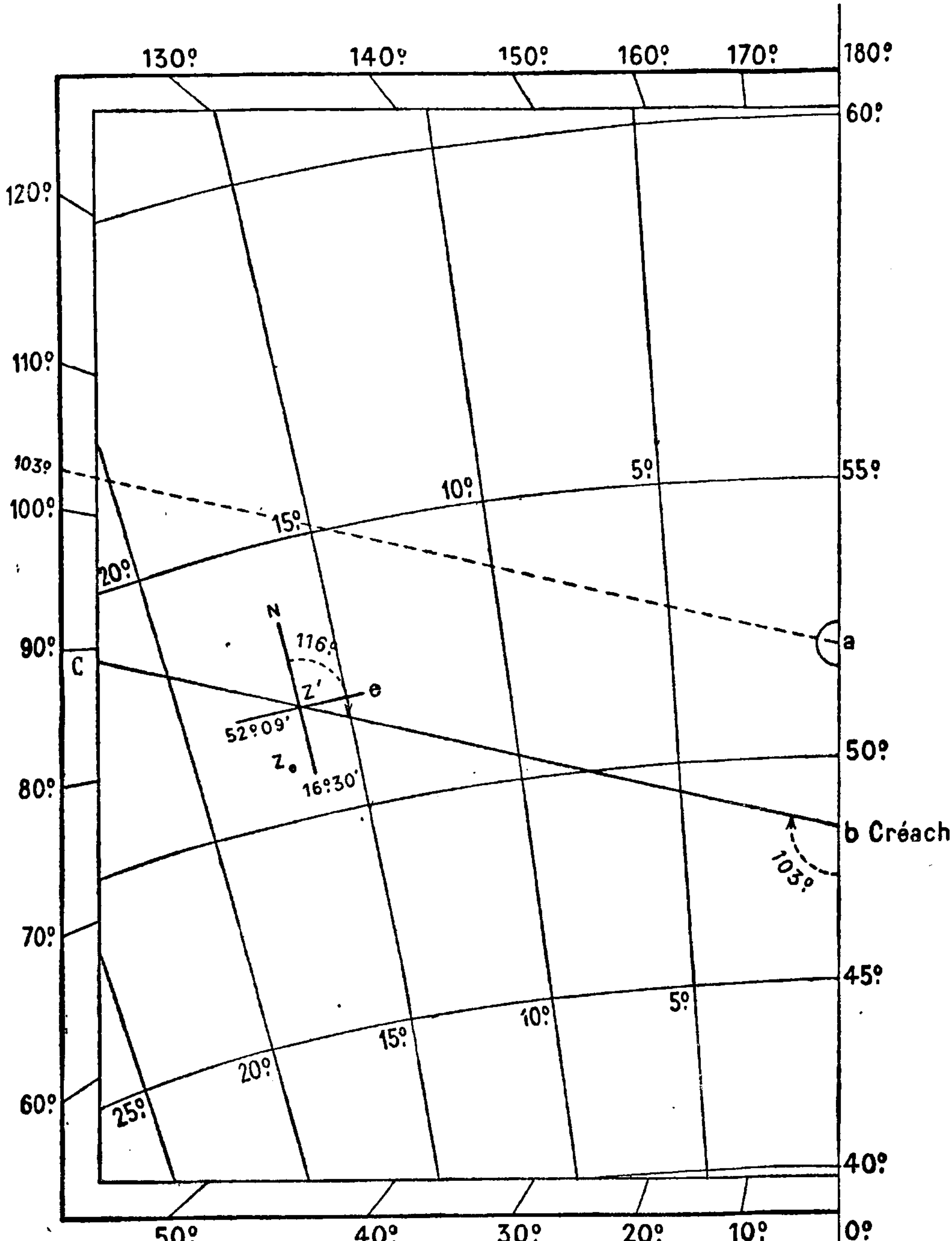
Second chart : r = 1,200 mm., from 10° N. to 30° N., variation in longitude : about 15° ;

Third chart : r = 900 mm., from 25° N. to 45° N., variation in longitude : about 15° ;

Fourth chart : r = 550 mm., from 40° N. to 60° N., variation in longitude : about 17° ;

Fifth chart : r = 210 mm., from 55° N. to 75° N., variation in longitude : about 20°.

A 3/10 scale reduction of the fourth map is appended, in which the lines of longitude are drawn in a westerly direction from the meridian of origin, which is more convenient in the case of observations off European coasts in the Atlantic. (Graph No. 1).



GRAPHIQUE 1

The easiest method of plotting is to compute first a table of the co-ordinates of the intersection points of two sets of meridians and parallels while varying the latitudes and longitudes degree by degree. The 10-minute intermediate curves can be plotted by interpolating graphically, or more accurately, by means of the space between them. By referring to previous remarks, it can be seen that :

$$zz^1 = r\ (1 - \sin^2 g \cos^2 \varphi)\ 1/2 \sec^2 \varphi\ d\ \varphi$$

$$zz^2 = r\ (1 - \sin^2 g \cos^2 \varphi)\ 1/2 \sec \varphi\ dg.$$

By substituting $d\ \varphi$ and dg by the radian value of 10 minutes, distances λ between two parallels and λ between two meridians that are adjacent are obtained :

$$\lambda = \frac{\pi r}{1080} (1 - \sin^2 g \cos^2 \varphi)\ 1/2 \sec^2 \varphi$$

$$\gamma = \frac{\pi r}{1080} (1 - \sin^2 g \cos^2 \varphi)\ 1/2 \sec \varphi = \cos \varphi.$$

All charts may be used indiscriminately by ships east or west of the station and for northern and southern latitudes owing to the symmetry of the curves with relation to the first meridian and equator of the charts.

Plotting is made easier if a hypothetical station *a* is plotted towards the middle of the latitude scale ; the margin is then divided by a scale of bearings computed starting at *a*, from 0° towards the equator to 180° northwards (or from 360° to 180° according to orientation).

Uses of chart.

1. *The position of a radio station having first been obtained, plot the locus of the ship and transfer it to the nautical chart.*

Plot point *b*, representing the station, on the meridian of origin. Correct the instrument position G*i* to obtain station bearing Z*v* :

$Zv = Gi + \delta + Cv$ (δ : radio-compass deviation ; Cv : true course).

Find the corresponding division on the margin of the chart and plot from the point found an equipollent vector to $\vec{ab}$, which gives a locus point *c*. The straight line *bc* represents the arc containing the angle on the chart. The co-ordinates of a certain number of points may be taken and transferred to the chart ; the longitude of the radiostation must of course be added to the longitudes computed from the first meridian. It can be seen that the ship's reckoning does not have to be known.

2. *Construct the radio bearing.* After plotting the bearing on the special chart as shown, the reckoned position *ze* is marked approximately, and a determining point *z'* is chosen near by, whose co-ordinates are taken, φ', $G' = g' + G_0$. The angle of *z' b* with the meridian of *z'* is measured at the same time, which makes it possible to find the direction of the bearing on the nautical chart upon which the determining point has been plotted. The quickest method is to make a transparent tracing of the meridian and parallel of *z'* as well as a section of *z' b* near *z'*. The tracing can be placed upon the nautical chart, and its meridian and parallel made to coincide with the transferred determining point. The radio bearing is then automatically in position. If the ship is east of the first meridian and the chart is constructed west, or vice versa, the tracing has to be turned over.

Illustration. — An airplane whose reckoned position is $\varphi e + 51°30'$ N, G*e* = 21°45' W takes a 103° bearing on the radiobeacon at Créach (Ushant). Construct the radio bearing, given the co-ordinates of Créach : $\varphi_0 = 48°27'6$ N, $G_0 = 5°07'8$ W. Créach is at latitude *b* ; An equipollent segment is drawn from the 103° margin division to *ab*, giving *c*. Bearing *bc* is the locus sought. Near *ze* (very approximately placed) determining point *z'* is taken on *bc* — on meridian *g'* = 16°30' for instance. The reading becomes $\varphi = 52°09'$ N and G' = 5°07'8 + 16°30' = 21°37'8 W. The meridian, parallel and the useful part of the line that together with the meridian of *z'* contains angle $Z + \theta = 116°$, are traced on transparent paper, and transferred to the nautical chart, where the determining point has been marked.

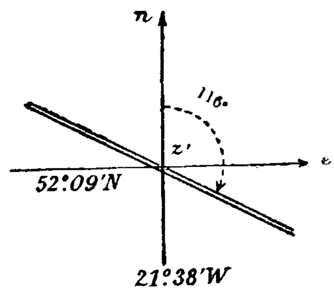


FIG. 3

Special charts for navigation by radio direction finder (Graph No. 2).

Following the same principle, actual geographical maps can be made adapted to the needs of navigation by radio direction finder, providing a direct solution to problems of position reckoning without recourse to auxiliary nautical charts. The application is interesting in cases of landfall and in areas where airlines are numerous. As an illustration a diagram of an English Channel and Bay of Biscay landfall chart is given. Projection is made by taking as first meridian that of the Villano radio-beacon, the axis of origin of bearings. The meridians have longitude divisions computed from the Greenwich meridian, there by avoiding conversions. Any radio-beacon, say Créach, is projected to *c*, following its parallel, on the first meridian ; the difference in longitude to the meridian is noted down next to *c*.

Supposing a ship has taken the following bearings :

Cape Villano : 150° ; Créach : 74°30' ; Mizen Head : 21°30'.

The first bearing, reversed, is plotted without correction from Villano ; it is a locus of the ship. The others are plotted without correction from the projected points of the stations on the first meridian. Each must then be shifted in the amount of longitude shown on the chart and in the direction bringing the projected points back to their respective stations. New loci of the ship are obtained and the radio direction position. It should be pointed out that this changes the rectilinear bearing into a curve passing through the station, but which need not be drawn entirely. The shifting of two points of the bearing on either side of the reckoned parallel or of the parallel of the determining point of the first bearing will suffice. Moreover, if the ship is a short distance away from the station, no serious error is made by plotting the uncorrected bearing from the station, but the nearest meridian should then be taken as reference.

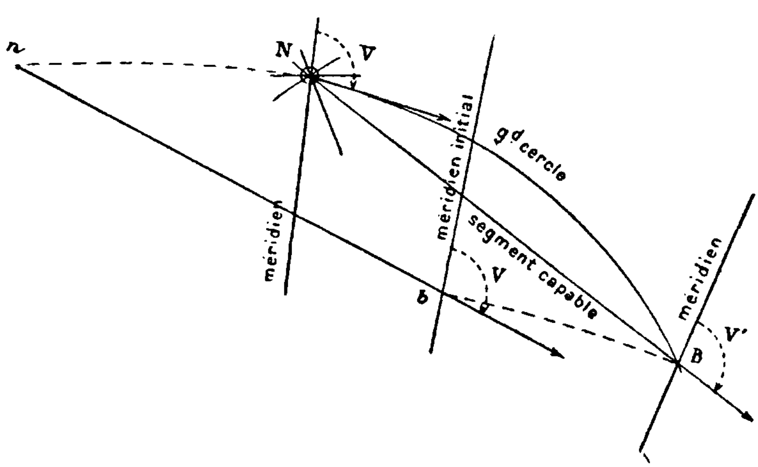
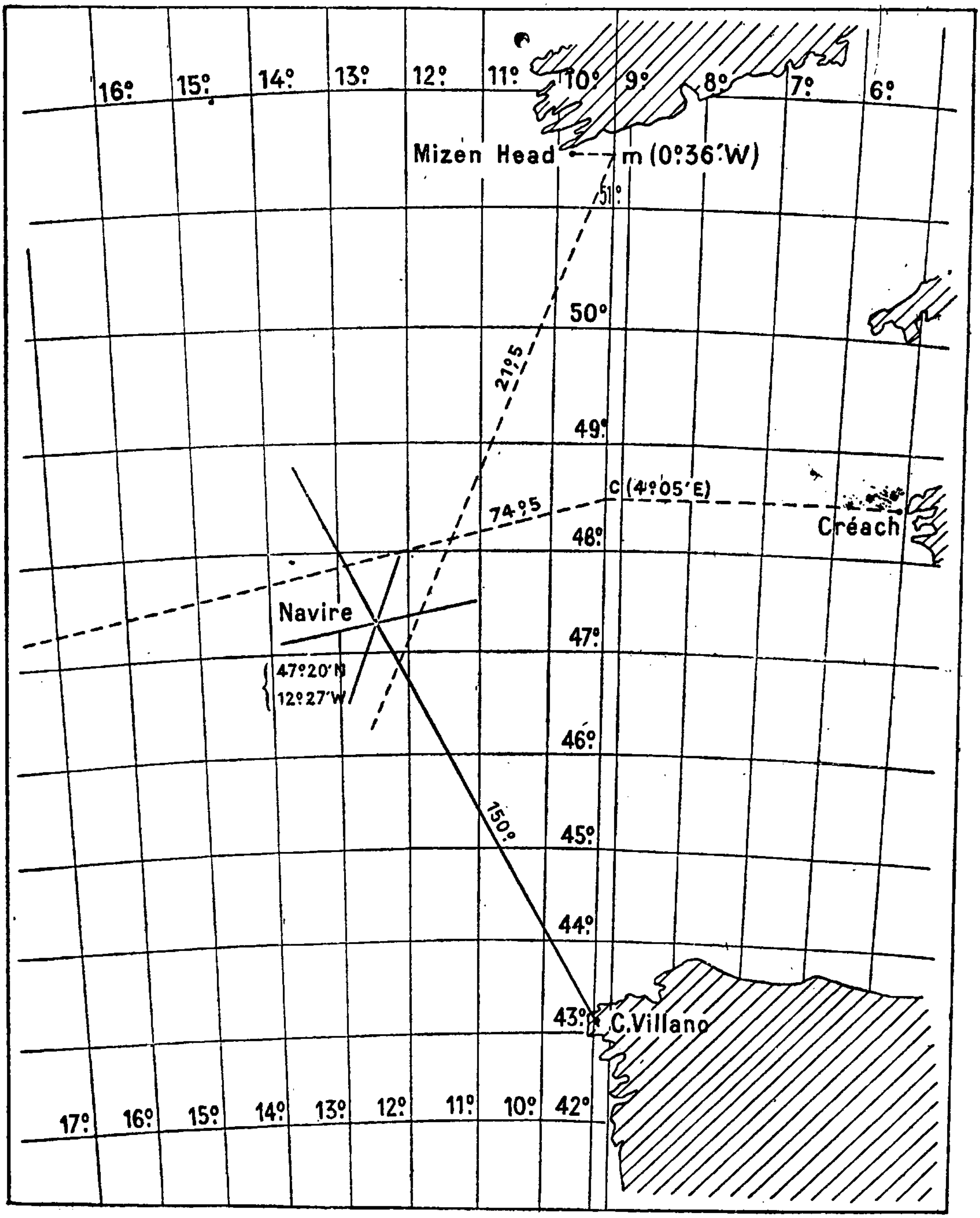


FIG. 4.

Determination of course to be taken to reach destination.

The chart makes it easy to solve course problems. Having first obtained position N of the ship (or airplane), a given point B must be made for, say Bordeaux in the



GRAPHIQUE 2

preceding illustration. It will be enough to join $\overrightarrow{NB}$ and to take angle V' made by the line with the nearest meridian to B. This is the *approximate great circle* course towards B. For arc NB, which is practically straight, and the meridian of B contain the same angle as great circle NB and the meridian of N. Strictly speaking, N should be moved along its parallel to *n* changing the longitude so that B is brought to *b* on the meridian of origin ; the angle contained by bearing *nb* and the meridian of origin is the angle sought, V.

NOTE. Several writers have suggested or constructed "charts of equal azimuth". H. Maurer's paper on the subject may be consulted in Vol. XXII 1945 of the *International Hydrographic Review*, and Professor Lecoq's article entitled : "The spherical angle-containing arc" in the *Annales hydrographiques* of 1933.

Among the solutions given is Littrow-Lambert's projection as Prüfer has termed it. Our only purpose was to point out its advantages in navigation and our paper may perhaps contain a few new ideas on the subject.