ASTRONOMY AND NAVIGATION

by D. H. Sadler.

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Introduction. — The principles of astronomical navigation must be familiar to all astronomers ; there are many excellent text-books, and almost every book on astronomy will cover the main features of the application of astronomy to navigation. The principles are so simple that there can hardly have been any new fundamental ideas in the last half-century. Moreover, its historical development — the curious, absorbing and stimulating mixture of patient endeavour, gradual improvement in technique, lucky accident and glorious achievement — has often been told ; it is a story worth re-telling, but this is not the place nor the time. What then is there to write about to interest astronomers ?

The main object of this article is to follow recent progress in the technique and application of astronomical navigation, developing a central theme of the constant interdependence of astronomy and navigation ; to view the whole subject in proper perspective it will be neccessary, however, briefly to review the principal high-lights of its history.

Part I. — BACKGROUND.

Brief general history. — Until comparatively recently astronomical navigation formed the only practical method of determining position on the Earth's surface other than by the recognition of physical features. Up to the seventeenth century navigational astronomy was crude : it is true that the astrolabe and the quadrant, used by the first famous navigators like Columbus and Vasco da Gama had been superseded by the cross-staff and the back-staff (Plate I), but no great accuracy was possible ; even so the accuracy was greater than the knowledge of the positions of the Sun, Moon, planets and stars ; there were no means of keeping time at sea and consequently, in the absence of both adequate instruments and almanacs, it was impossible to determine longitude by astronomical observations although the principle of lunar distance had been published by Regiomontanus as long ago as 1474. It is to this deficiency that we owe the Royal Observatory at Greenwich, founded in 1675 by Charles the Second for the purpose of improving our knowledge of the motions of the Moon and planets and the positions of the stars.

Three independent, but nearly contemporaneous, events revolutionised the theory and practice of navigation in the eighteenth century ; first, the invention of the sextant, generally associated with Hadley's paper to the Royal Society in 1731 ; secondly, the successful construction of the chronometer by Harrison leading to his famous fourth model in 1759 ; and finally, the publication by Tobias Mayer of his tables of the motion of the Moon in 1753. They heralded the enormous advances of this period, in which the foundations of modern navigation were firmly laid. The knowledge accumulated from the observations of the Royal Observatory, coupled with Mayer's tables, enabled Nevil Maskelyne to produce the *Nautical Almanac* for 1767 from which year the modern period of navigation can be said to date ; Maskelyne also published in 1766 the justly celebrated "Requisite Tables...", thus providing the seaman with the first complete astronomical and tabular equipment for the determination of position.

There followed a quiescent period, during which the enormous advances were consolidated ; sextants underwent gradual development, the positions of the heavenly bodies in the almanacs were improved, and chronometers became more plentiful. Little advance in the technique of presentation of the astronomical data or the reduction of observations was made ; the *Nautical Almanac* was in fact redesigned in 1834 in a form rather more suitable for astronomers than for navigators. Then came what must be a perpetual challenge to the astronomer — the discovery by a practical sea-captain, Captain T. H. Sumner, of the astronomical position line while approaching St. George's Channel in a storm on 18th December, 1837. This was followed in 1875 by the publication by Marcq. St. Hilaire of the principle of the intercept method. These methods are still not fully utilised in the practice of astronomical navigation at sea — such is the traditional conservatism of the mariner ! They were of comparatively little value while seamen were still so distrustful of their chronometers that they observed lunar distances as a check on their longitude, and on the time-keeping of the chronometers. The method of lunar distances ("lunars") must rank as the most splendid of astronomy's contributions to navigation ; only the diret necessity could have overcome the manifold difficulties, not the least of which is the excessively complicated reduction of the observations to be done by the navigator. It is indeed sad that nearly 500 years after the method was first proposed, it should still not be possible to predict the Moon's position for several years in advance to within the equivalent of one or two miles on the Earth. Lunars were given in the *Nautical Almanac* until 1908.

A special edition of the *Nautical Almanac, Abridged for the Use of Seamen* was issued for the first time for the year 1914 ; previously the only provision for the navigator was the separate issue since 1896 of the first part of the standard edition. The provision of special almanacs and the introduction of radio time-signals gave the necessary impetus for a more general introduction of the position-line method and the publication of reduction tables to facilitate the computation. This development is still in progress today.

The need for navigation in the air has in recent years led to rapid developments in instruments and techniques, though no new fundamental principles have been invoked. The integrating bubble sextant, special air almanacs and a veritable host of devices, instruments and tables have been introduced. The rapidity of the development in air navigation has compared markedly with the slow and deliberate progress of marine navigation, once again demonstrating that necessity is a powerful spur. The first almanac specially published for air use is generally regarded as the experimental edition of the American *Air Almanac* for 1933 and the first regular publication that of the French *Ephémérides Aéronautiques* introduced in 1936 ; but the Japanese "Abridged Nautical Almanac", stated to be for air use, was introduced in 1926. Most countries now publish air almanacs.

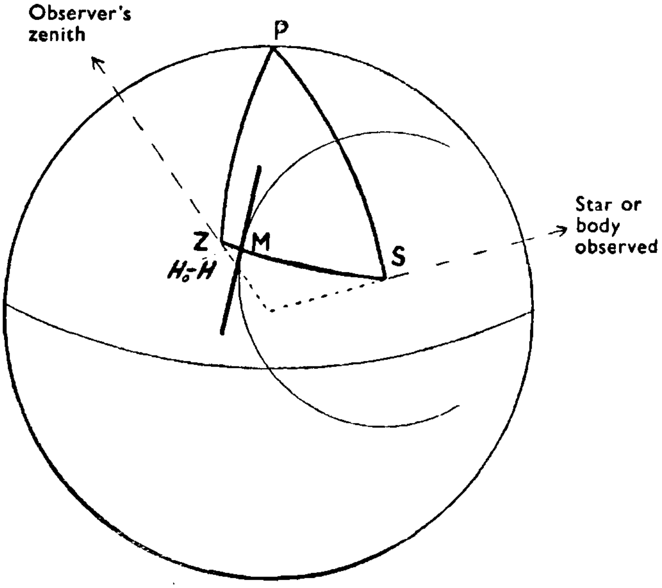


FIG. 1

The Marc St. Hilaire method of drawing an astronomical position line.

Fundamental problems. — Before going further it may be as well briefly to remind readers of the basic procedure. In modern astronomical navigation, an observation of the altitude of the Sun, Moon, planet or star is made with a sextant ; the horizontal is determined by the horizon, or the vertical by a bubble ; the time is obtained from a chronometer or watch which is checked by radio time-signals. The position of the observed body in the sky at the time of observation is obtained from an almanac, or other means, thus giving immediately the geographical position S of the "sub-stellar" point on the Earth. It is clear that the observer must lie on a small circle, centre S and angular radius 90°— corrected observed altitude, *Ho*. Near the observer, this position circle can be represented with sufficient accuracy by a straight line ; this is the Sumner line, or position line.

This position line can evidently be drawn in a number of different ways : as a straight line through two points at which the calculated altitude equals the observed altitude ; as a straight line through one such point in a given direction ; as a tangent to two circles ; and in several other ways. In general, however, it can be constructed from any point Z, sufficiently close to the true position of the observer, in the following manner — due to Marcq. St. Hilaire. (See Figure 1). Along ZS, which in the neighbourhood of Z can be regarded as a straight line in the direction of the azimuth of the observed body, mark off the "intercept" ZM equal to the difference between the calculated altitude, *H*, at Z and the observed altitude, *Ho* ; M is clearly on the position circle ; the position line is given by the tangent drawn through M perpendicular to ZM. To reduce an observation to give a position line it is thus necessary to have some simple method of calculating the altitude and azimuth of a heavenly body from a given point on the Earth at a given time : in other words to solve the navigational spherical triangle PZS, where P is the pole, PZ, PS, ZS are respectively 90° minus latitude, declination and altitude, ZPS is the local hour angle, and PZS is the azimuth. There are other methods of determining latitude and longitude directly, particularly when the observed body is near the meridian or on the prime vertical ; but they are all special cases of the one general principle.

The astronomer is primarily concerned with the presentation of the almanac and the design of the methods, tables or instruments to facilitate the reduction of the observations.

Navigation and the R. A. S. — It will be of interest to consider the special relationship between the Royal Astronomical Society and navigation. Reference to the *General Index to Monthly Notices* shows that up to 1931 only three papers are listed under the title of "Nautical Astronomy", though there are many references to the *Nautical Almanac*. From Hannyngton's paper in Vol. 23, there seems to be a complete gap to the publication in Vol. 79 of the now classical paper by our present President, W. M. Smart, on the position line in navigation. It was this paper which placed on a sound foundation the many so called "short" methods of constructing a position line from an "assumed" or "chosen" position — chosen so as to simplify the tabulation or calculation involved in the solution of the astronomical spherical triangle — by showing that the approximations used were adequate over the longer intercepts required.

Shortly afterwards, in Vol. 80, there is a paper by Herbert Bell proposing for the solution of this problem the construction of triple-entry tables with arguments latitude, declination and observed altitude, and with respondents hour angle, azimuth and parallactic angle ; there was to be no numerical interpolation, as a chosen position would be used and corrections for minutes of declination and observed altitude applied graphically on the chart. This paper has considerable interest in that the basic principles have actually been used in *Tables de lineas de Posicion de Altura* by Juan Garcia, of which Volume II (the only one so far issued) was published in Madrid in 1944 ; Garcia calls it "the involute method". Although these probably form the shortest and most compact tabular method for the solution of the position-line problem, they have not been well received, primarily because of the necessity for a change of method near the meridian — a necessity not fully appreciated by Bell.

Since 1931, there appears to be only the paper by J. H. Clarke in 1945, proposing a special method of determining an astronomical position line for fixed air routes, and a short note by Professor Smart in 1946 on the effect of the ellipticity of the Earth on the calculation of the "middle latitude" used in the determination of the difference in longitude in "plane sailing".

But it must not be thought that the interest of the Society in navigation has been confined solely to the publishing of original contributions ; until recent years the Society played a very large part in the design of the *Nautical Almanac*, being consulted by the Admiralty whenever questions of change arose. In the early days of its history in 1830, while in fact the Society was waiting for its charter, the Council was asked to advise on the reform of the Almanac, which had then fallen into disrepute ; the Council acted with energy and speed. It rejected the changes proposed by the Admiralty as altogether inadequate and submitted a detailed report recommending a radical revision which was, to the great credit of both the Admiralty and the Society, accepted immediately. While the principal object for which the *Nautical Almanac* had been originally formed, namely the advancement of nautical astronomy and navigation, was constantly kept in view, the recommendations included the extension of the ephemerides, both in accuracy and in scope, to make it equally useful for all purposes of practical astronomy. There is no record that the practical navigators were consulted about the change, though it was undoubtedly to their great advantage. This quite admirable report determined the general contents and arrangement of the *Nautical Almanac* for nearly 100 years — until, in fact, the recent revision for the year 1931, by which time separate provision for the navigator had been made.

In 1890 the Society took the initiative in suggesting a number of improvements, mainly concerning an increase in the number of stars, for which apparent places were given ; these suggestions were adopted by the Admiralty and incorporated into the *Nautical Almanac* for 1896. By a curious coincidence the first part of the almanac for that same year was reprinted separately, at the request of the Shipmasters' Society, for the special use of navigators ; even so it was far from ideal for the limited purpose, and the Admiralty approached the Society in 1910 suggesting that the provision for astronomical navigation could with advantage be still further simplified. In 1897 the Society became aware that the values of the fundamental constants adopted at the 1896 Paris Congress had been introduced into the *Nautical Almanac* without it having been consulted ; a dignified protest to the Admiralty brought forth the admission that a mistake had been made, and it was doubtless for that reason that the Society was asked, 13 years later, for its advice on the design of a purely navigational almanac. The recommendations of the Society for a separate publication were adopted and introduced in 1914 in the *Nautical Almanac, Abridged for the Use of Seamen* ; although there have been many changes of detail, and additions, the almanac is essentially the same today, showing that the Society planned well and wisely, even though without the advice of the practical navigators.

The last occasion upon which the Society was asked to determine a navigational matter was in 1917 when the Admiralty asked for the views of astronomers generally on the proposal, made by the Conference on Time-keeping at Sea, that the day should be shown in nautical publications as commencing at midnight instead of noon, and that the civil day should be substituted generally for the astronomical day ; the Society was also asked specifically whether it would be practicable to introduce this change into the *Nautical Almanac*, probably meaning the abridged edition but interpreted as the standard edition. The Society publicised this question as well as the circumstances of the war permitted, and rightly sought the views of foreign astronomers ; on the basis of the replies to a circular letter, the Council considered that it was practical and desirable to introduce the change into the *Nautical Almanac* commencing with the year 1925. It is a great pity that the question of terminology for mean time on the meridian of Greenwich (which has been the subject of discussion and resolution at practically every international congress since that date) and the definitions of hour angle were not specifically considered at this time.

It is thus seen that the Society played a leading part in the development of the "seaman's bible", though we find in the pages of *Monthly Notices* only a single unimportant paper during the whole of this period, with no reference to the practical developments of Sumner and St. Hilaire. With the rapid widening of astronomical knowledge and the increasing specialisation on navigation it is but natural that the Society should lose touch with the practical aspects of navigation ; it was not even consulted when *E* and *R* were introduced into the Abridged edition in 1929 as a belated consequence of the change of time reckoning ; the *Air Almanac* was introduced in 1937, and the *Abridged Nautical Almanac* has recently been radically redesigned — both without reference to, or protest from, the Society. The privilege of the Society of advising the Admiralty on matters of astronomical navigation has quite properly descended to the recently formed Institute of Navigation, which can co-ordinate from among its own members the views of the professional astronomer, the almanac maker and the practical user. As in the case of the Society, one of the earliest functions of the Institute has been to advise the Admiralty on the redesign of the almanac for surface navigation.

Part II. — MODERN DEVELOPMENTS.

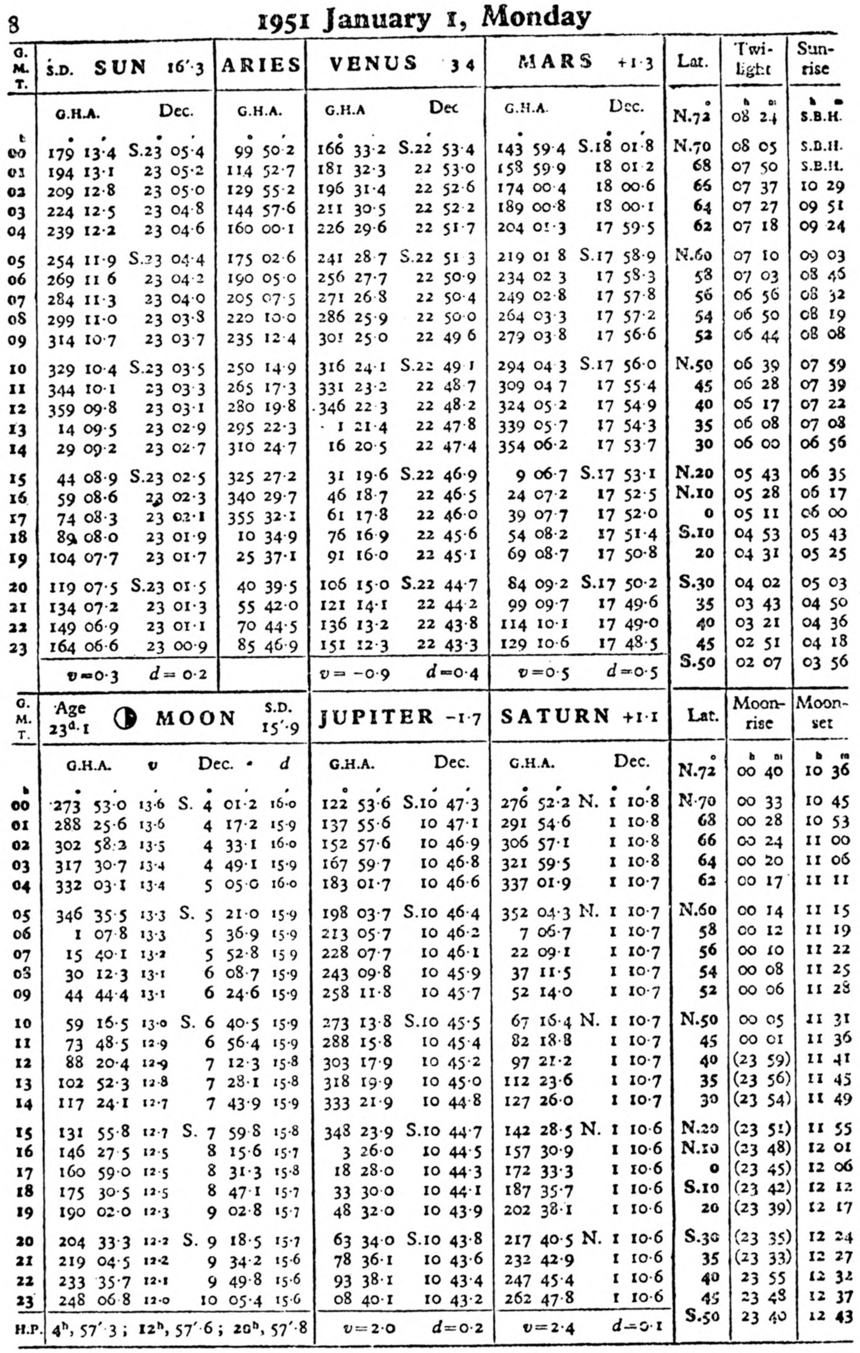
Need for simplification. — The speed of air travel, with the consequent necessity of determining position within the shortest possible time of making observations clearly demanded a new approach to astronomical navigation in the air ; but the speed of air travel has not increased sufficiently to make it essential to change the methods that have been in use so successfully for many years. Moreover, new and powerful radio methods of navigation have been developed which allow positions both on the sea and in the air to be determined by "reading" the co-ordinates of fixed "patterns" of radio lattices; in other words, an artificial topography is superposed on the Earth's surface recognisable through the medium of radio frequencies. To the echo-sounder which enables the ocean bed to be used for recognition purposes, has been recently added radar which can be used to extend "visibility" in both mediums — and which forms the basis of many other aids to air navigation. Clearly the demands on astronomical navigation, with its limitations of application and accuracy and with its dependence on careful observations and elaborate calculation, must be getting less ; why, therefore, the need for further development ?

Part of the answer lies in the wealth of new methods of, or aids to, navigation that have been recently developed ; the modern navigator must have a mastery of them all. The full understanding of the theory of astronomical navigation calls for a lengthy course in spherical astronomy ; the rapid and accurate reduction of observations (themselves a matter of no little difficulty as anyone can say who has tried to use a sextant on a rolling ship, or a bubble sextant in a cold astrodome) calls not only for knowledge of the theory but constant practice. The very existence of alternative methods discourages, or even forbids, that practice which is so necessary for the reliable application of astronomical navigation when it alone is available.

So the astronomer's task is to simplify : firstly, and perhaps the more important, to simplify the understanding of the principles ; secondly, to simplify and to speed up the actual application. It is to be hoped that the necessity will never arise when navigators will have to use "astro" without the basic astronomical knowledge — simplification can hardly extend so far, but the simpler the concepts and the procedure the easier they are to learn, to remember and to apply.

Almanacs. — The purpose of a navigational almanac is to provide, in the simplest and easiest manner and to the accuracy desired, the positions of the heavenly bodies used for navigation. Until recently the equatorial co-ordinates, right ascension and declination, were used universally except for the Sun for which the equation of time was given instead of right ascension. But the navigator wants positions referred to his horizon and is interested in the tabulated positions only as a means to that end. Altitude and azimuth can be tabulated directly for a given place at short (say 10 minutes) intervals of time, but anything like universal coverage is impossible. The Japanese "Altitude-Azimuth Almanac" introduced in 1947 actually tabulates these quantities for the Sun for the four latitudes N. 25°, N. 30°, N. 35°, N. 40°, and longitude E. 150° ; with special methods of plotting position lines, it provides the optimum data for its limited objective — navigation by the Sun for Japan's large coastal traffic. The same principle can be extended to Moon, planets and stars (and in fact was actually done for air navigation by the Japanese during the late war), but, except for the stars, the difficulties are even greater than with the Sun. In general the navigator must calculate altitude and azimuth, for which he needs local hour angle (L.H.A.) and declination ; now L.H.A. is formed directly from Greenwich hour angle (G.H.A.) by application of longitude, and G.H.A. and declination are thus the quantities he needs from the almanac.

It is clear that right ascension is an unsuitable co-ordinate from which to obtain G.H.A. unless sidereal time is immediately available, which is not the case at present. The practical alternatives are the tabulation of (i) G.H.A. directly, and (ii) G.H.A.— U.T. = E ; either can be in arc or in time. This is not the place to compare their merits, which have been widely discussed ; G.H.A. certainly scores on the question of simplicity of principle. The direct tabulation of G.H.A. was first introduced by Weems into the American "Lunar Ephemeris for Aviators" in 1929, leading in course of time to the experimental edition of the *Air Almanac* for 1933 ; it was introduced, as an addition to R. A., in the *American Nautical Almanac* for 1931, and was subsequently adopted in a half-hearted way by several surface almanacs. The British *Air Almanac* of 1937 was the first entirely G.H.A. (except for the stars) almanac to be published continuously, rapidly followed by the *American Air Almanac*



8

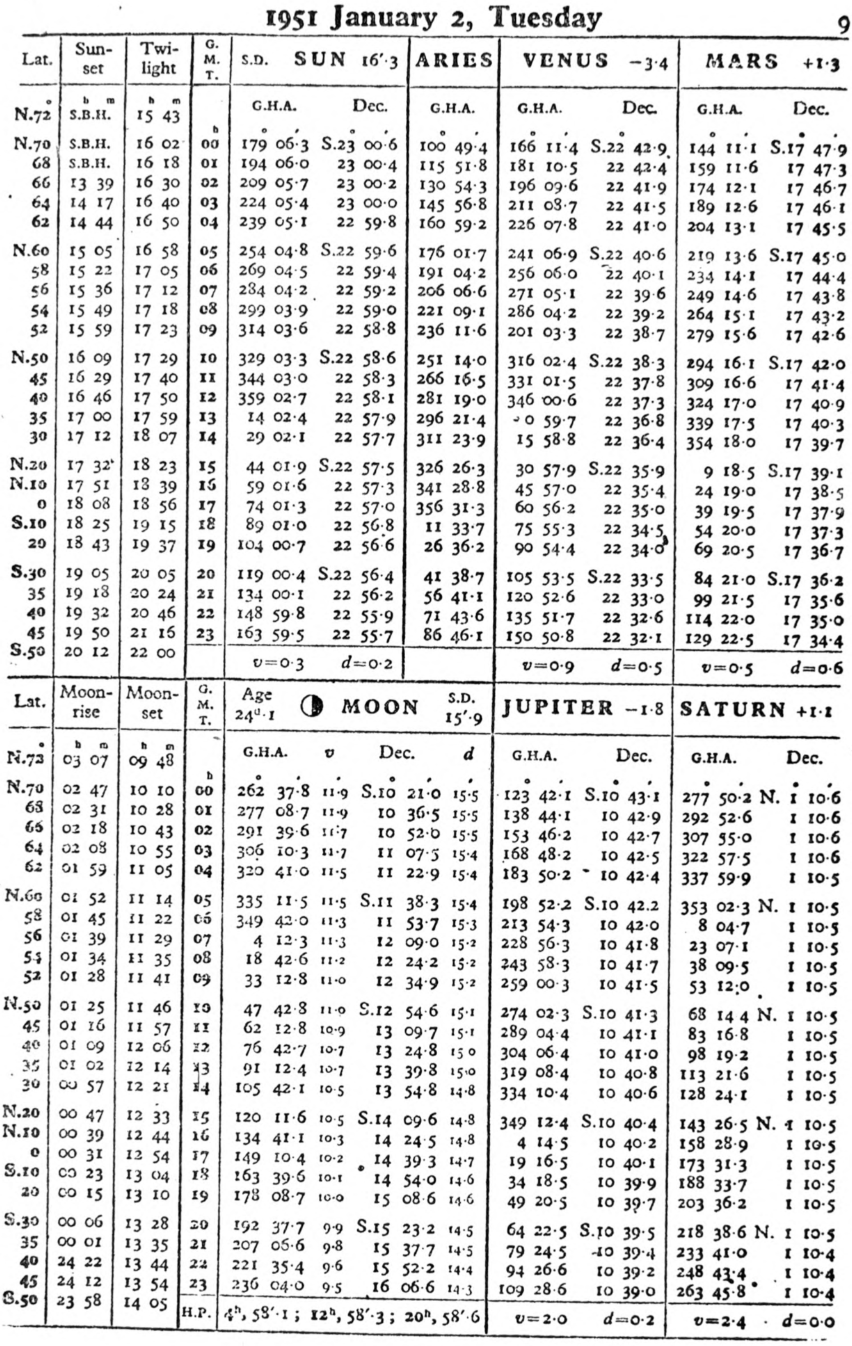
1951 January 1, Monday

G.M.T.	S.D. SUN 16′·3		ARIES	VENUS 3 4		MARS +1·3		Lat.	Twi-light	Sun-rise
	G.H.A.	Dec.	G.H.A.	G.H.A	Dec	G.H.A.	Dec.	N.72	08 24	S.B.H.
h	° ′	° ′	° ′	° ′	° ′	° ′	° ′			
00	179 13·4	S.23 05·4	99 50·2	166 33·2	S.22 53·4	143 59·4	S.18 01·8	N.70	08 05	S.B.H.
01	194 13·1	23 05·2	114 52·7	181 32·3	22 53·0	158 59·9	18 01·2	68	07 50	S.B.H.
02	209 12·8	23 05·0	129 55·2	196 31·4	22 52·6	174 00·4	18 00·6	66	07 37	10 29
03	224 12·5	23 04·8	144 57·6	211 30·5	22 52·2	189 00·8	18 00·1	64	07 27	09 51
04	239 12·2	23 04·6	160 00·1	226 29·6	22 51·7	204 01·3	17 59·5	62	07 18	09 24
05	254 11·9	S.23 04·4	175 02·6	241 28·7	S.22 51·3	219 01·8	S.17 58·9	N.60	07 10	09 03
06	269 11·6	23 04·2	190 05·0	256 27·7	22 50·9	234 02·3	17 58·3	58	07 03	08 45
07	284 11·3	23 04·0	205 07·5	271 26·8	22 50·4	249 02·8	17 57·8	56	06 56	08 32
08	299 11·0	23 03·8	220 10·0	286 25·9	22 50·0	264 03·3	17 57·2	54	06 50	08 19
09	314 10·7	23 03·7	235 12·4	301 25·0	22 49·6	279 03·8	17 56·6	52	06 44	08 08
10	329 10·4	S.23 03·5	250 14·9	316 24·1	S.22 49·1	294 04·3	S.17 56·0	N.50	06 39	07 59
11	344 10·1	23 03·3	265 17·3	331 23·2	22 48·7	309 04·7	17 55·4	45	06 28	07 39
12	359 09·8	23 03·1	280 19·8	346 22·3	22 48·2	324 05·2	17 54·9	40	06 17	07 22
13	14 09·5	23 02·9	295 22·3	1 21·4	22 47·8	339 05·7	17 54·3	35	06 08	07 08
14	29 09·2	23 02·7	310 24·7	16 20·5	22 47·4	354 06·2	17 53·7	30	06 00	06 56
15	44 08·9	S.23 02·5	325 27·2	31 19·6	S.22 46·9	9 06·7	S.17 53·1	N.20	05 43	06 35
16	59 08·6	23 02·3	340 29·7	46 18·7	22 46·5	24 07·2	17 52·5	N.10	05 28	06 17
17	74 08·3	23 02·1	355 32·1	61 17·8	22 46·0	39 07·7	17 52·0	0	05 11	06 00
18	89 08·0	23 01·9	10 34·9	76 16·9	22 45·6	54 08·2	17 51·4	S.10	04 53	05 43
19	104 07·7	23 01·7	25 37·1	91 16·0	22 45·1	69 08·7	17 50·8	20	04 31	05 25
20	119 07·5	S.23 01·5	40 39·5	106 15·0	S.22 44·7	84 09·2	S.17 50·2	S.30	04 02	05 03
21	134 07·2	23 01·3	55 42·0	121 14·1	22 44·2	99 09·7	17 49·6	35	03 43	04 50
22	149 06·9	23 01·1	70 44·5	136 13·2	22 43·8	114 10·1	17 49·0	40	03 21	04 36
23	164 06·6	23 00·9	85 46·9	151 12·3	22 43·3	129 10·6	17 48·5	45	02 51	04 18
	v=0·3	d=0·2		v=−0·9	d=0·4	v=0·5	d=0·5	S.50	02 07	03 56

G.M.T.	Age 23^{d}·1 ◑ MOON S.D. 15′·9				JUPITER −1·7		SATURN +1·1		Lat.	Moon-rise	Moon-set
	G.H.A.	v	Dec.	d	G.H.A.	Dec.	G.H.A.	Dec.	N.72	00 40	10 36
h	° ′	′	° ′	′	° ′	° ′	° ′	° ′			
00	273 53·0	13·6	S. 4 01·2	16·0	122 53·6	S.10 47·3	276 52·2	N. 1 10·8	N.70	00 33	10 45
01	288 25·6	13·6	4 17·2	15·9	137 55·6	10 47·1	291 54·6	1 10·8	68	00 28	10 53
02	302 58·2	13·5	4 33·1	16·0	152 57·6	10 46·9	306 57·1	1 10·8	66	00 24	11 00
03	317 30·7	13·4	4 49·1	15·9	167 59·7	10 46·8	321 59·5	1 10·8	64	00 20	11 06
04	332 03·1	13·4	5 05·0	16·0	183 01·7	10 46·6	337 01·9	1 10·7	62	00 17	11 11
05	346 35·5	13·3	S. 5 21·0	15·9	198 03·7	S.10 46·4	352 04·3	N. 1 10·7	N.60	00 14	11 15
06	1 07·8	13·3	5 36·9	15·9	213 05·7	10 46·2	7 06·7	1 10·7	58	00 12	11 19
07	15 40·1	13·2	5 52·8	15·9	228 07·7	10 46·1	22 09·1	1 10·7	56	00 10	11 22
08	30 12·3	13·1	6 08·7	15·9	243 09·8	10 45·9	37 11·5	1 10·7	54	00 08	11 25
09	44 44·4	13·1	6 24·6	15·9	258 11·8	10 45·7	52 14·0	1 10·7	52	00 06	11 28
10	59 16·5	13·0	S. 6 40·5	15·9	273 13·8	S.10 45·5	67 16·4	N. 1 10·7	N.50	00 05	11 31
11	73 48·5	12·9	6 56·4	15·9	288 15·8	10 45·4	82 18·8	1 10·7	45	00 01	11 36
12	88 20·4	12·9	7 12·3	15·8	303 17·9	10 45·2	97 21·2	1 10·7	40	(23 59)	11 41
13	102 52·3	12·8	7 28·1	15·8	318 19·9	10 45·0	112 23·6	1 10·7	35	(23 56)	11 45
14	117 24·1	12·7	7 43·9	15·9	333 21·9	10 44·8	127 26·0	1 10·7	30	(23 54)	11 49
15	131 55·8	12·7	S. 7 59·8	15·8	348 23·9	S.10 44·7	142 28·5	N. 1 10·6	N.20	(23 51)	11 55
16	146 27·5	12·5	8 15·6	15·7	3 26·0	10 44·5	157 30·9	1 10·6	N.10	(23 48)	12 01
17	160 59·0	12·5	8 31·3	15·8	18 28·0	10 44·3	172 33·3	1 10·6	0	(23 45)	12 06
18	175 30·5	12·5	8 47·1	15·7	33 30·0	10 44·1	187 35·7	1 10·6	S.10	(23 42)	12 12
19	190 02·0	12·3	9 02·8	15·7	48 32·0	10 43·9	202 38·1	1 10·6	20	(23 39)	12 17
20	204 33·3	12·2	S. 9 18·5	15·7	63 34·0	S.10 43·8	217 40·5	N. 1 10·6	S.30	(23 35)	12 24
21	219 04·5	12·2	9 34·2	15·6	78 36·1	10 43·6	232 42·9	1 10·6	35	(23 33)	12 27
22	233 35·7	12·1	9 49·8	15·6	93 38·1	10 43·4	247 45·4	1 10·6	40	23 55	12 32
23	248 06·8	12·0	10 05·4	15·6	08 40·1	10 43·2	262 47·8	1 10·6	45	23 48	12 37
H.P.	4^{h}, 57′·3 ; 12^{h}, 57′·6 ; 20^{h}, 57′·8				v=2·0	d=0·2	v=2·4	d=0·1	S.50	23 40	12 43

FIG

Showing the arrangement of two facing specimen
The data for all bodies other than the positions
the positions of the brighter stars are printed
reduced in reproduction ; the c



1951 January 2, Tuesday

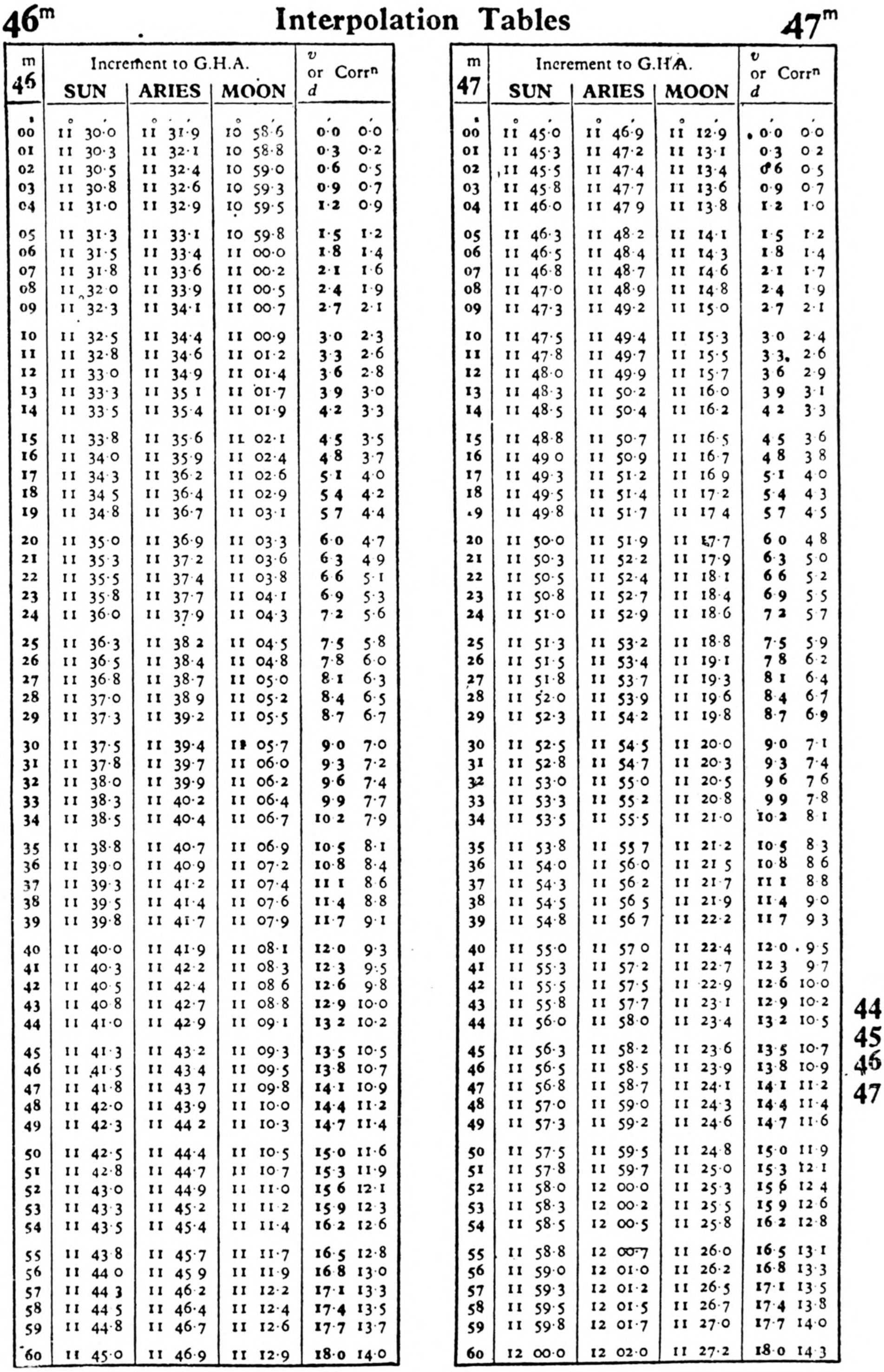
9

Lat.	Sunset	Twilight	G. M. T.	S.D. SUN 16′·3		ARIES	VENUS −3·4		MARS +1·3	
N.72	S.B.H.	15 43		G.H.A.	Dec.	G.H.A.	G.H.A.	Dec.	G.H.A.	Dec.
N.70	S.B.H.	16 02	00	179 06·3	S.23 00·6	100 49·4	166 11·4	S.22 42·9	144 11·1	S.17 47·9
68	S.B.H.	16 18	01	194 06·0	23 00·4	115 51·8	181 10·5	22 42·4	159 11·6	17 47·3
66	13 39	16 30	02	209 05·7	23 00·2	130 54·3	196 09·6	22 41·9	174 12·1	17 46·7
64	14 17	16 40	03	224 05·4	23 00·0	145 56·8	211 08·7	22 41·5	189 12·6	17 46·1
62	14 44	16 50	04	239 05·1	22 59·8	160 59·2	226 07·8	22 41·0	204 13·1	17 45·5
N.60	15 05	16 58	05	254 04·8	S.22 59·6	176 01·7	241 06·9	S.22 40·6	219 13·6	S.17 45·0
58	15 22	17 05	06	269 04·5	22 59·4	191 04·2	256 06·0	22 40·1	234 14·1	17 44·4
56	15 36	17 12	07	284 04·2	22 59·2	206 06·6	271 05·1	22 39·6	249 14·6	17 43·8
54	15 49	17 18	08	299 03·9	22 59·0	221 09·1	286 04·2	22 39·2	264 15·1	17 43·2
52	15 59	17 23	09	314 03·6	22 58·8	236 11·6	201 03·3	22 38·7	279 15·6	17 42·6
N.50	16 09	17 29	10	329 03·3	S.22 58·6	251 14·0	316 02·4	S.22 38·3	294 16·1	S.17 42·0
45	16 29	17 40	11	344 03·0	22 58·3	266 16·5	331 01·5	22 37·8	309 16·6	17 41·4
40	16 46	17 50	12	359 02·7	22 58·1	281 19·0	346 00·6	22 37·3	324 17·0	17 40·9
35	17 00	17 59	13	14 02·4	22 57·9	296 21·4	0 59·7	22 36·8	339 17·5	17 40·3
30	17 12	18 07	14	29 02·1	22 57·7	311 23·9	15 58·8	22 36·4	354 18·0	17 39·7
N.20	17 32	18 23	15	44 01·9	S.22 57·5	326 26·3	30 57·9	S.22 35·9	9 18·5	S.17 39·1
N.10	17 51	18 39	16	59 01·6	22 57·3	341 28·8	45 57·0	22 35·4	24 19·0	17 38·5
0	18 08	18 56	17	74 01·3	22 57·0	356 31·3	60 56·2	22 35·0	39 19·5	17 37·9
S.10	18 25	19 15	18	89 01·0	22 56·8	11 33·7	75 55·3	22 34·5	54 20·0	17 37·3
20	18 43	19 37	19	104 00·7	22 56·6	26 36·2	90 54·4	22 34·0	69 20·5	17 36·7
S.30	19 05	20 05	20	119 00·4	S.22 56·4	41 38·7	105 53·5	S.22 33·5	84 21·0	S.17 36·2
35	19 18	20 24	21	134 00·1	22 56·2	56 41·1	120 52·6	22 33·0	99 21·5	17 35·6
40	19 32	20 46	22	148 59·8	22 55·9	71 43·6	135 51·7	22 32·6	114 22·0	17 35·0
45	19 50	21 16	23	163 59·5	22 55·7	86 46·1	150 50·8	22 32·1	129 22·5	17 34·4
S.50	20 12	22 00		v=0·3	d=0·2		v=0·9	d=0·5	v=0·5	d=0·6

Lat.	Moonrise	Moonset	G. M. T.	Age $24^{\mathrm{d}}{\cdot}1$ ◐ MOON S.D. 15′·9				JUPITER −1·8		SATURN +1·1	
N.72	03 07	09 48		G.H.A.	v	Dec.	d	G.H.A.	Dec.	G.H.A.	Dec.
N.70	02 47	10 10	00	262 37·8	11·9	S.10 21·0	15·5	123 42·1	S.10 43·1	277 50·2	N. 1 10·6
68	02 31	10 28	01	277 08·7	11·9	10 36·5	15·5	138 44·1	10 42·9	292 52·6	1 10·6
66	02 18	10 43	02	291 39·6	11·7	10 52·0	15·5	153 46·2	10 42·7	307 55·0	1 10·6
64	02 08	10 55	03	306 10·3	11·7	11 07·5	15·4	168 48·2	10 42·5	322 57·5	1 10·6
62	01 59	11 05	04	320 41·0	11·5	11 22·9	15·4	183 50·2	10 42·4	337 59·9	1 10·5
N.60	01 52	11 14	05	335 11·5	11·5	S.11 38·3	15·4	198 52·2	S.10 42.2	353 02·3	N. 1 10·5
58	01 45	11 22	06	349 42·0	11·3	11 53·7	15·3	213 54·3	10 42·0	8 04·7	1 10·5
56	01 39	11 29	07	4 12·3	11·3	12 09·0	15·2	228 56·3	10 41·8	23 07·1	1 10·5
54	01 34	11 35	08	18 42·6	11·2	12 24·2	15·2	243 58·3	10 41·7	38 09·5	1 10·5
52	01 28	11 41	09	33 12·8	11·0	12 34·9	15·2	259 00·3	10 41·5	53 12·0	1 10·5
N.50	01 25	11 46	10	47 42·8	11·0	S.12 54·6	15·1	274 02·3	S.10 41·3	68 14·4	N. 1 10·5
45	01 16	11 57	11	62 12·8	10·9	13 09·7	15·1	289 04·4	10 41·1	83 16·8	1 10·5
40	01 09	12 06	12	76 42·7	10·7	13 24·8	15·0	304 06·4	10 41·0	98 19·2	1 10·5
35	01 02	12 14	13	91 12·4	10·7	13 39·8	15·0	319 08·4	10 40·8	113 21·6	1 10·5
30	00 57	12 21	14	105 42·1	10·5	13 54·8	14·8	334 10·4	10 40·6	128 24·1	1 10·5
N.20	00 47	12 33	15	120 11·6	10·5	S.14 09·6	14·8	349 12·4	S.10 40·4	143 26·5	N. 1 10·5
N.10	00 39	12 44	16	134 41·1	10·3	14 24·5	14·8	4 14·5	10 40·2	158 28·9	1 10·5
0	00 31	12 54	17	149 10·4	10·2	14 39·3	14·7	19 16·5	10 40·1	173 31·3	1 10·5
S.10	00 23	13 04	18	163 39·6	10·1	14 54·0	14·6	34 18·5	10 39·9	188 33·7	1 10·5
20	00 15	13 10	19	178 08·7	10·0	15 08·6	14·6	49 20·5	10 39·7	203 36·2	1 10·5
S.30	00 06	13 28	20	192 37·7	9·9	S.15 23·2	14·5	64 22·5	S.10 39·5	218 38·6	N. 1 10·5
35	00 01	13 35	21	207 05·6	9·8	15 37·7	14·5	79 24·5	10 39·4	233 41·0	1 10·4
40	24 22	13 44	22	221 35·4	9·6	15 52·2	14·4	94 26·6	10 39·2	248 43·4	1 10·4
45	24 12	13 54	23	236 04·0	9·5	16 06·6	14·3	109 28·6	10 39·0	263 45·8	1 10·4
S.50	23 58	14 05	H.P.	4^{h}, 58′·1 ; 12^{h}, 58′·3 ; 20^{h}, 58′·6				v=2·0	d=0·2	v=2·4	d=0·0

2.

of the redesigned *Abridged Nautical Almanac*.
e stars, for each day are given on one page ;
e book-mark. The pages have been slightly
type-area is 6×9 ½ inches.



46ᵐ **Interpolation Tables** 47ᵐ

m 46	Increment to G.H.A. SUN	ARIES	MOON	v or d	Corrⁿ
s	° ′	° ′	° ′	′	′
00	11 30·0	11 31·9	10 58·6	0·0	0·0
01	11 30·3	11 32·1	10 58·8	0·3	0·2
02	11 30·5	11 32·4	10 59·0	0·6	0·5
03	11 30·8	11 32·6	10 59·3	0·9	0·7
04	11 31·0	11 32·9	10 59·5	1·2	0·9
05	11 31·3	11 33·1	10 59·8	1·5	1·2
06	11 31·5	11 33·4	11 00·0	1·8	1·4
07	11 31·8	11 33·6	11 00·2	2·1	1·6
08	11 32·0	11 33·9	11 00·5	2·4	1·9
09	11 32·3	11 34·1	11 00·7	2·7	2·1
10	11 32·5	11 34·4	11 00·9	3·0	2·3
11	11 32·8	11 34·6	11 01·2	3·3	2·6
12	11 33·0	11 34·9	11 01·4	3·6	2·8
13	11 33·3	11 35·1	11 01·7	3·9	3·0
14	11 33·5	11 35·4	11 01·9	4·2	3·3
15	11 33·8	11 35·6	11 02·1	4·5	3·5
16	11 34·0	11 35·9	11 02·4	4·8	3·7
17	11 34·3	11 36·2	11 02·6	5·1	4·0
18	11 34·5	11 36·4	11 02·9	5·4	4·2
19	11 34·8	11 36·7	11 03·1	5·7	4·4
20	11 35·0	11 36·9	11 03·3	6·0	4·7
21	11 35·3	11 37·2	11 03·6	6·3	4·9
22	11 35·5	11 37·4	11 03·8	6·6	5·1
23	11 35·8	11 37·7	11 04·1	6·9	5·3
24	11 36·0	11 37·9	11 04·3	7·2	5·6
25	11 36·3	11 38·2	11 04·5	7·5	5·8
26	11 36·5	11 38·4	11 04·8	7·8	6·0
27	11 36·8	11 38·7	11 05·0	8·1	6·3
28	11 37·0	11 38·9	11 05·2	8·4	6·5
29	11 37·3	11 39·2	11 05·5	8·7	6·7
30	11 37·5	11 39·4	11 05·7	9·0	7·0
31	11 37·8	11 39·7	11 06·0	9·3	7·2
32	11 38·0	11 39·9	11 06·2	9·6	7·4
33	11 38·3	11 40·2	11 06·4	9·9	7·7
34	11 38·5	11 40·4	11 06·7	10·2	7·9
35	11 38·8	11 40·7	11 06·9	10·5	8·1
36	11 39·0	11 40·9	11 07·2	10·8	8·4
37	11 39·3	11 41·2	11 07·4	11·1	8·6
38	11 39·5	11 41·4	11 07·6	11·4	8·8
39	11 39·8	11 41·7	11 07·9	11·7	9·1
40	11 40·0	11 41·9	11 08·1	12·0	9·3
41	11 40·3	11 42·2	11 08·3	12·3	9·5
42	11 40·5	11 42·4	11 08·6	12·6	9·8
43	11 40·8	11 42·7	11 08·8	12·9	10·0
44	11 41·0	11 42·9	11 09·1	13·2	10·2
45	11 41·3	11 43·2	11 09·3	13·5	10·5
46	11 41·5	11 43·4	11 09·5	13·8	10·7
47	11 41·8	11 43·7	11 09·8	14·1	10·9
48	11 42·0	11 43·9	11 10·0	14·4	11·2
49	11 42·3	11 44·2	11 10·3	14·7	11·4
50	11 42·5	11 44·4	11 10·5	15·0	11·6
51	11 42·8	11 44·7	11 10·7	15·3	11·9
52	11 43·0	11 44·9	11 11·0	15·6	12·1
53	11 43·3	11 45·2	11 11·2	15·9	12·3
54	11 43·5	11 45·4	11 11·4	16·2	12·6
55	11 43·8	11 45·7	11 11·7	16·5	12·8
56	11 44·0	11 45·9	11 11·9	16·8	13·0
57	11 44·3	11 46·2	11 12·2	17·1	13·3
58	11 44·5	11 46·4	11 12·4	17·4	13·5
59	11 44·8	11 46·7	11 12·6	17·7	13·7
60	11 45·0	11 46·9	11 12·9	18·0	14·0

m 47	Increment to G.H.A. SUN	ARIES	MOON	v or d	Corrⁿ
s	° ′	° ′	° ′	′	′
00	11 45·0	11 46·9	11 12·9	0·0	0·0
01	11 45·3	11 47·2	11 13·1	0·3	0·2
02	11 45·5	11 47·4	11 13·4	0·6	0·5
03	11 45·8	11 47·7	11 13·6	0·9	0·7
04	11 46·0	11 47·9	11 13·8	1·2	1·0
05	11 46·3	11 48·2	11 14·1	1·5	1·2
06	11 46·5	11 48·4	11 14·3	1·8	1·4
07	11 46·8	11 48·7	11 14·6	2·1	1·7
08	11 47·0	11 48·9	11 14·8	2·4	1·9
09	11 47·3	11 49·2	11 15·0	2·7	2·1
10	11 47·5	11 49·4	11 15·3	3·0	2·4
11	11 47·8	11 49·7	11 15·5	3·3	2·6
12	11 48·0	11 49·9	11 15·7	3·6	2·9
13	11 48·3	11 50·2	11 16·0	3·9	3·1
14	11 48·5	11 50·4	11 16·2	4·2	3·3
15	11 48·8	11 50·7	11 16·5	4·5	3·6
16	11 49·0	11 50·9	11 16·7	4·8	3·8
17	11 49·3	11 51·2	11 16·9	5·1	4·0
18	11 49·5	11 51·4	11 17·2	5·4	4·3
19	11 49·8	11 51·7	11 17·4	5·7	4·5
20	11 50·0	11 51·9	11 17·7	6·0	4·8
21	11 50·3	11 52·2	11 17·9	6·3	5·0
22	11 50·5	11 52·4	11 18·1	6·6	5·2
23	11 50·8	11 52·7	11 18·4	6·9	5·5
24	11 51·0	11 52·9	11 18·6	7·2	5·7
25	11 51·3	11 53·2	11 18·8	7·5	5·9
26	11 51·5	11 53·4	11 19·1	7·8	6·2
27	11 51·8	11 53·7	11 19·3	8·1	6·4
28	11 52·0	11 53·9	11 19·6	8·4	6·7
29	11 52·3	11 54·2	11 19·8	8·7	6·9
30	11 52·5	11 54·5	11 20·0	9·0	7·1
31	11 52·8	11 54·7	11 20·3	9·3	7·4
32	11 53·0	11 55·0	11 20·5	9·6	7·6
33	11 53·3	11 55·2	11 20·8	9·9	7·8
34	11 53·5	11 55·5	11 21·0	10·2	8·1
35	11 53·8	11 55·7	11 21·2	10·5	8·3
36	11 54·0	11 56·0	11 21·5	10·8	8·6
37	11 54·3	11 56·2	11 21·7	11·1	8·8
38	11 54·5	11 56·5	11 21·9	11·4	9·0
39	11 54·8	11 56·7	11 22·2	11·7	9·3
40	11 55·0	11 57·0	11 22·4	12·0	9·5
41	11 55·3	11 57·2	11 22·7	12·3	9·7
42	11 55·5	11 57·5	11 22·9	12·6	10·0
43	11 55·8	11 57·7	11 23·1	12·9	10·2
44	11 56·0	11 58·0	11 23·4	13·2	10·5
45	11 56·3	11 58·2	11 23·6	13·5	10·7
46	11 56·5	11 58·5	11 23·9	13·8	10·9
47	11 56·8	11 58·7	11 24·1	14·1	11·2
48	11 57·0	11 59·0	11 24·3	14·4	11·4
49	11 57·3	11 59·2	11 24·6	14·7	11·6
50	11 57·5	11 59·5	11 24·8	15·0	11·9
51	11 57·8	11 59·7	11 25·0	15·3	12·1
52	11 58·0	12 00·0	11 25·3	15·6	12·4
53	11 58·3	12 00·2	11 25·5	15·9	12·6
54	11 58·5	12 00·5	11 25·8	16·2	12·8
55	11 58·8	12 00·7	11 26·0	16·5	13·1
56	11 59·0	12 01·0	11 26·2	16·8	13·3
57	11 59·3	12 01·2	11 26·5	17·1	13·5
58	11 59·5	12 01·5	11 26·7	17·4	13·8
59	11 59·8	12 01·7	11 27·0	17·7	14·0
60	12 00·0	12 02·0	11 27·2	18·0	14·3

44
45
46
47

FIGURE 3.

Showing a typical page of the interpolation tables in the redesigned *Abridged Nautical Almanac*. The page has been slightly reduced in reproduction; the type area is 6 × 9 ½ inches.

and many others ; it is interesting to note that the French *Ephémérides Aéronautiques*, introduced in 1936 and using "l'ascension verse" throughout, has since 1948 changed over completely to the American model. The latest recruit to this same pattern is the Spanish *Almanaque Aeronáutico* which follows the British model closely ; it was previously very like the war-time German almanac. It has the distinction of being the only almanac to measure hour angle from the lower meridian — a logical consequence of the 1925 change. All the above almanacs (and several others — Brazil, Argentine, etc.), deviate from the G.H.A. principle in the case of the stars, for which 360°— R.A. (in arc) is tabulated under various names ; this deliberate departure from principle is made for practical convenience. The name Sidereal Hour Angle (S.H.A.) for this quantity, introduced through the *Air Almanac*, has been severely criticised by some astronomers, but it serves its purpose well and it is not easy to think of a better.

This same principle is now being introduced fully into the surface almanacs ; the American, British, French and Spanish almanacs are all shortly to be published in the now form, although the detailed presentation will differ. G.H.A. (with S.H.A. for the stars) is now firmly established for both surface and air almanacs. Figures 2* and 3* show specimen pages (slightly reduced in reproduction) of the redesigned *Abridged Nautical Almanac*. It will be noted that the data for all bodies except the stars for two days are given on one opening ; the brighter stars will be printed on a loose book-mark. G.H.A. increases by about 15° in each hour so that interpolation is heavy ; however, really comprehensive interpolation tables enable G.H.A. to be obtained for any body in two openings and the addition of a main entry (day and hour), increment (minute and second) and correction (minute and v). v is the excess of the actual hourly difference over the adopted hourly motions : 15°00'.0 for the Sun and planets, 14°19'.0 for Moon and 15°02'.4...... for the stars. The declination is interpolated at the same opening. The same interpolation tables have been adopted by the *American Nautical Almanac*, which will be issued in the new form for 1950. In order to allow adequate notice of the change to mariners, the British almanac will not be introduced until 1952.

A few almanacs, notably the Japanese, continue to tabulate E in time as defined above ; its chief advantage is that the unit (1^s) is much more realistic of the attainable accuracy at sea than the comparable arc unit of 0'.1.

Reduction Tables. — The calculation of altitude and azimuth required for the reduction of an observation to give a position line is a problem of no little difficulty ; no really satisfactory solution has yet been devised to the nominal accuracy of 0'.1 required in surface navigation. Hundreds of different methods have been proposed and nearly as many tables have been published. In surface navigation the scale used in plotting is such as to discourage the use of the long intercepts arising from a working position chosen to simplify the calculation ; the general consensus of opinion among navigators is that the position line must be constructed from the D.R. (dead reckoning) position.

There are three classes of available methods : direct, mixed and tabular. In the direct method, the spherical triangle is solved directly from the fundamental formulæ of spherical trigonometry, rearranged to avoid ambiguities, using only single-entry tables of trigonometrical functions. Many ingenious variations of the "cosine" formula have been used — and are in fact still appearing — but it is unlikely that any are, or will be, superior to the "cosine-haversine" method for the computation of the altitude, which was introduced in its present form by P. L. H. Davis of the Nautical Almanac Office in 1905. Among the direct methods we can include those which solve the spherical triangle by splitting it into two right-angled triangles, but still using single-entry tables. Two of these are well-known and in common use : Ageton's and Aquino's "log tan ± log sec" methods. Although both may be used to find altitude and azimuth from the D.R. position, great care is necessary to avoid large errors in certain circumstances. All direct methods involve a comparatively large number of table entries and, to a greater or lesser extent, are artificial in that the procedure cannot be directly related to the geometry ; a similar artificiality occurs in the mixed methods. The extreme case of this occurs in Martelli's tables, first published in 1873 but still being reprinted, where each table has some constant multiplier and additive constant included and is often expressed in unnatural units — all without explanation. It will thus be seen why this class of solution is unsuitable

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(*) Reproduced from the *Admiralty Notices to Mariners* No. 308, Feb. 1949, by permission of the Hydrographer of the Navy, and the Controller of H. M. Stationery Office. Some minor changes have since been made to these specimens.

for air navigation, where speed is all important, but is well established for surface navigation where time is of less importance than convenience in plotting and where a routine can be adopted and followed.

The mixed methods all arise in the solution of the navigational spherical triangle by division into two right-angled triangles ; they are "mixed" because they combine tabulated solutions (double-entry) of one triangle with single-entry (or, in a few cases double-entry) solutions for the second triangle. Although these methods of solving spherical triangles have been known for centuries, the first navigation tables to utilise these principles appear to be Souillagouet's of 1891. They were rapidly followed by many others, with a recrudescence of table-making activity for the simpler problem of air navigation. The navigational triangle can be divided by a perpendicular to the opposite side from either the zenith or the observed body ; either division can be used with tabular solutions for both triangles, or tabular solution for the first and direct calculation for the second.

Admiral Radler de Aquino, probably the greatest authority and the most prolific writer on such methods, has concentrated primarily on tabular solutions. The latest edition of his tables *Tabuas Nauticas e Aéronauticas* (Rio de Janeiro, 1943) is a model of ingenuity and careful planning, presented in an admirable form ; it is a worthy culmination of nearly 50 years of published work in this field. It suffers from the disadvantage inhérent in this form of solution — elaborate interpolation can only be avoided by the use of a chosen position, not readily acceptable to the surface navigator. It does not suffer from the theoretical inaccuracies that are so difficult to avoid in combining tabular solutions of two triangles. Most methods, however, are content to avoid these difficulties by replacing the solution of the second triangle by a direct (logarithmic) solution requiring single-entry tables alone. Our President, with Commander Shearme, was responsible for one of the earliest tables of this form with the well-known Smart and Shearme "sine-method". The most carefully compiled tables are those by Comrie in Hughes' *Sea and Air Navigation Tables* ; these are typical of the best of these methods, and have the added advantage that they can, if necessary, be used with a D.R. position. However, it is for air navigation that the method can be simplified to give short, easily used tables ; and many such specially designed tables are in use.

The third class of methods is the triple-entry tabulation of altitude and azimuth, with arguments latitude, local hour angle and declination. Simplicity of principle is attained at the expense of bulk, and if working from the D.R. position, by tricky interpolation. Such tables have been available for limited ranges for many years — Ball's tables published in 1907 were computed at the Royal Observatory, while P. L. H. Davis of the Nautical Almanac Office produced over many years his well-known *Alt.-Azimuth Tables* ; but the tabular method did not come into its own until the publication from 1937 onwards of the magnificiently conceived *Tables of Computed Altitude and Azimuth* published by the U. S. Hydrographic Office as H.O. 214. These tables, which occupy nine large volumes each of about 265 pages crowded with figures, give altitude to 0'.1, with variations for changes in declination and hour angle, and azimuth to 0°.1, for each degree of latitude, each degree of hour angle and each half-degree of declination up to 29° and thereafter for declinations to cater for the navigational stars. When used with a position chosen to give an integral degree of latitude and hour angle, altitude and azimuth can be taken out with the one interpolation for declination. Interpolation is required in all three arguments, however, if the position line is to be plotted from the D.R. position. This is a difficult problem for the table-maker as the second-order terms in the altitude (i. e. second differences) are appreciable at intervals of 1° ; errors due to their neglect can be reduced to an acceptable magnitude, only if the method of interpolation is carefully planned ; unfortunately, insufficient attention was given to this aspect of H.O. 214 and (even with the smaller interval in declination) interpolation errors are larger than they need be. Criticism has also been levelled at the presentation, which is definitely crowded ; but more space means more pages or more volumes ! However, if the experience of American navigators (who have had access to H. O. 214) can be regarded as representative, it can be safely said that, in spite of interpolation difficulties, the surface navigator of the future will use altitude-azimuth tables in preference to other methods.

In the air, the British *Astronomical Navigation Tables*, originally prepared for the R.A.F. in a restricted edition but now on sale to the public, led the way. Similar in concept to H.O. 214, they differ in detail and in presentation ; altitudes are given to the nearest minute and azimuths to the nearest degree, while provision is made for interpolation for declination only ; for the stars, use is made of the approximate constancy of the declinations to simplify interpolation. This must be one of the earliest tables in which the effect of refraction was included in the altitudes ; the inclusion is of doubtful value. These tables, covering all latitudes to 79° in fifteen volumes, have been photographically reproduced in America as H.O. 218, and have been largely copied in the French *Table de Hauteur et d'Azimut* ; other countries have similar tables. Bulk and weight in the air are even more serious than in a ship, but the price seems to be paid to avoid calculation.

With diminishing need, both almanacs and reduction tables are increasing in size ; it is almost a case of "the more one has, the more one wants" !

Stars. — By reason of their approximate fixity, the stars offer the possibility of special treatment, especially in air navigation where the number of stars tabulated can be limited to about 30 and where the effects of nutation and aberration can be ignored. Perhaps the first to take full advantage of this, were Commander T. Y. Baker and Professor L. N. G. Filon in their suggested use of precomputed star-altitude curves, superimposed by transparencies on a chart. This principle was utilised most successfully in the Astrograph (Plate II) during the late war ; precomputed star-altitude curves, on a film, were projected from a point source directly on the plotting chart. Orientation and scale were easy to adjust, while the time-scale (actually graduated in mean time due to an ingenious arrangement) could be brought to any time simply by turning the film. An observation of altitude immediately gave a position line on the chart ; two altitudes (only two stars and *Polaris* could be put on the film) gave a fix. The instrument had several disadvantages, but it was the supreme example of simplicity, which enabled many to use "astro" without the necessity of a full understanding of spherical astronomy.

Earlier, Weems had introduced his printed *Star Altitude Curves* ; these still provide one of the quickest methods of navigation, especially if a watch keeping sidereal time is available.

In 1942, Commander C. H. Hutchings, U. S. Navy, proposed the construction of star altitude and azimuth tables, with (effectively) sidereal time as argument ; for each range of argument the stars would be chosen according to position in the sky and arranged in order of azimuth. Thus, in addition to having no interpolation, there would be no need for selection of suitable stars to observe or for a planisphere to find or identify them. Previously star identification for the navigator had, of course, been a problem demanding help from the astronomer. Since then, several tables using this principle have been published, the outstanding one being the American H. O. 249, *Star Tables for Air Navigation* (designed by Hutchings himself) in 1947. This truly magnificent volume of 330 pages gives for each degree of north and south latitude, and for each degree (2° near the poles) of local sidereal time, the computed altitude (corrected for refraction) and azimuth for the six most "suitable" stars for that time. One apparent disadvantage is that precession will necessitate recalculation and reprinting every few years ; it has been pointed out, however, by A. H. Jessell, that this is unnecessary since the resulting fix can be corrected simply for the effect of precession from the epoch of the tables. This has prompted an investigation into the possibility of correcting for nutation and aberration in a similar way and thus using similar tables for surface navigation ; the result is that it is not difficult to do so (though probably too complicated for actual use) but that proper motion would limit the applicability !

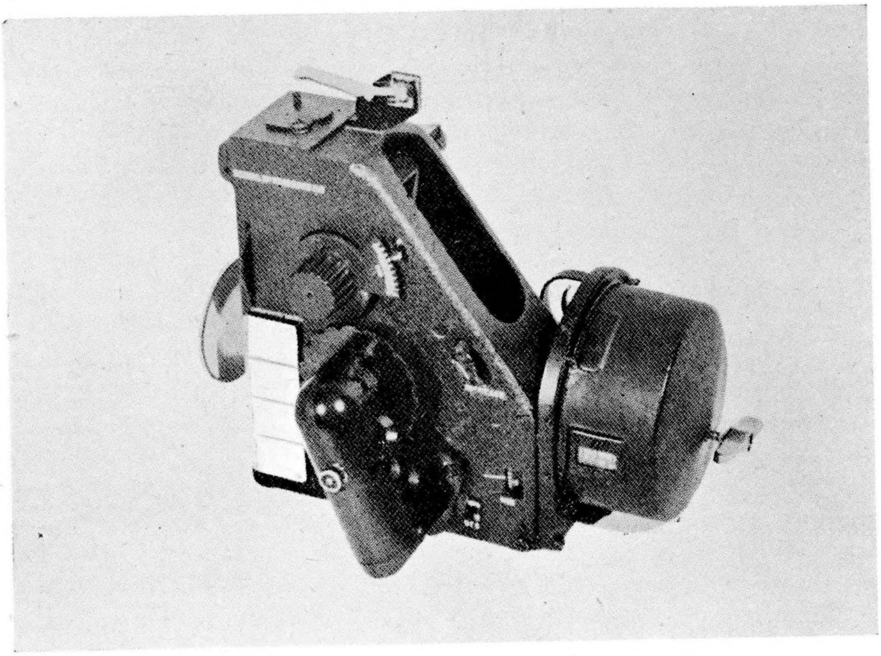
In an experimental edition of similar tables in 1943 in this country, a mean time scale was used in conjunction with a special overprint on the chart. Similar arrangements to avoid the use of an almanac have been used in conjunction with various graphical and slide-rule methods, which, however, introduced no new principles.

A by-product of these tables has been a complete survey of the altitudes and relative bearings of the brighter stars at all times in all latitudes ; the choice of stars for the astrograph, or for tables of the Hutchings' type, is a fascinating game requiring considerable judgment.

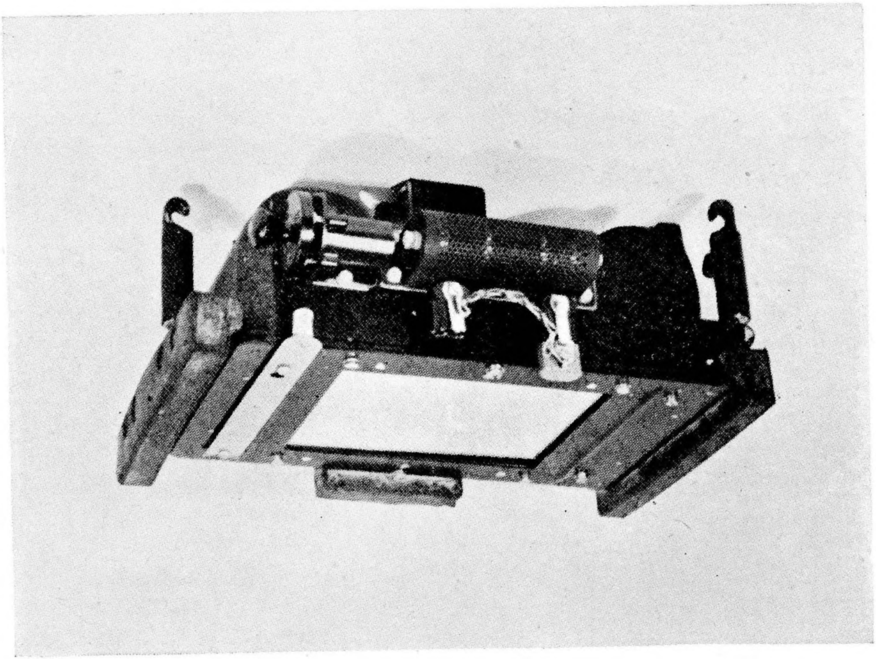
Polar Navigation. — By appealing jointly to the special features of polar astronomy and polar geography, it is possible to provide simplified techniques in polar regions. Tables prepared by Lincoln Ellsworth use the pole itself as an assumed position and give means of correcting for the curvature of position lines drawn with long intercepts ; similar tables can be used from other assumed positions and were so used by the Germans for marine navigation in 1916. Using these correction tables, or a simple curved ruler, it is possible to provide reduction tables for both polar regions (within 10° of the poles) for all bodies in a single-page table. Since altitude is practically independent

PLATE I.

Boy using a Back-staff to observe the altitude of the Sun. Taken from "A new Systeme of the Mathematicks" by Sir Jonas MOORE, 1681 and reproduced by permission of the Director of the National Maritime Museum.



A Modern Air Sextant. — The horizontal is determined by a bubble. The observed object is kept continuously in the centre of the bubble for a period of two minutes, during which time clockwork mechanism automatically integrates and averages the indicated altitude. In later models a period of one or two minutes may be chosen.



The Astrograph. — The photograph shows the rear view of the instrument as suspended ; the film passes under the glass plate in the centre, and the star curves are projected from a point source of light directly on the chart beneath.

of right ascension, while corrections to declination enter with their full weight, it is also possible to make permanent tables for the Sun and stars in a very small compass independently of any almanac.

Graphical and Instrumental solutions. — A vast number of graphical and instrumental methods have been devised for the solution of the astronomical spherical triangle. Generally the accuracy obtainable is insufficient for surface navigation, except in regard to the azimuth for which several graphical methods are in common use, in conjunction with direct methods for the altitude. Most graphical methods for the altitude are based on the combination of the solutions of two right-angled triangles ; interpolation is usually easier than with tables, since it can be made in the optimum direction, but the methods are unlikely to replace tabular ones.

It would seem not too difficult to devise an instrument, either a computer or an analogue machine, capable of giving the altitude to an accuracy suitable for air navigation. Several such instruments are in existence, but none (so far as is known) can be regarded as entirely satisfactory. The Fairchild machine actually produces the altitude on dials after the latitude, declination and hour angle have been fed in ; but the feeding in is slow and two solutions are required for a position line. Of a number of mechanisations of the "cosine" formula the Matthews computer, which utilises the potentiometer principle, is the most promising.

The direct analogue machines are of more interest to the astronomer. In their usual form, for instance in the Hagner Position Finder and the original Willis Navigating Machine, they consist of five graduated arcs — one for each of latitude, hour angle and declination, altitude and azimuth. Setting any three quantities will in general give the other two. In later models of the Willis machine, and in other instruments, the five arcs have been replaced by three — and Herrick has designed a two arc instrument. All of these instruments, beautiful in design and execution though some of them are, suffer from the difficulties inherent in setting or reading accurately on small circles.

Several suggestions have been made for extending the scope of the principle of the astrograph to project position lines directly on the chart for bodies other than the stars. The latest one of these, devised by Herrick, combines the basic principle of McMillen's Spherographical System (essentially actual construction with precision scales on a sphere) with the projective principle of the astrograph ; but there are clearly many practical difficulties to be overcome.

One of the most successful instruments is the German ARGI , developed during the late war from an original French design ; it is based, like the ancient astrolabes, essentially on a stereographic projection (that useful projection on which all circles on the sphere project into circles on the plane). A small eyepiece is positioned so that the finely engraved graticule reads hour angle and declination ; it is then rotated, relative to the graticule, through an angle equal to the latitude when altitude and azimuth can be read off from the graticule.

Observations. — So far little mention has been made of the actual observations of altitude without which the almanacs, tables and instruments would be useless. The marine sextant, although vastly improved in detail and with a tangent screw instead of vernier, is still the same basic instrument. Most advance has been made with bubble sextants (Plate II), which have now reached a high level of mechanical and optical efficiency. Whereas the accuracy of an observation with a marine sextant is limited by the exact knowledge of the dip of the horizon, that of the air observation is limited by errors due to accelerations, which combine with gravity to give a false vertical. In the latest sextants, observations are automatically integrated over periods of one or two minutes to smooth out random accelerations due to the motion of the aircraft. There is, however, a coriolis acceleration due to the rotation of the Earth which, being systematic, cannot be smoothed out in this way and must be corrected for by special tables. The altitude correction for coriolis is very sensitive to the exact path flown during the course of the observations, and there is still some uncertainty about how it should be calculated in certain circumstances ; for instance, if in the course of a two-minute sight an aircraft travelling at 300 knots deviates from a true great circle course by 1° (less than the discrimination of most compasses) the apparent zenith will be shifted by as much as 8'. The determination of the true vertical in an aircraft is a problem of real difficulty — to which neither the Hagner device of dropping shot nor the gyro stabilisers can give a really satisfactory solution.

Part III. — THE FUTURE.

Standards of time and frequency. — Until a few months ago the astronomer could watch with equanimity the encroachment upon his one-time monopoly of the increasing number of radio aids to navigation ; he knew that basically all these methods relied upon a standard of frequency, which could only be determined by an accurate knowledge of time obtained by astronomical observations. If now the atomic clock, locked to the microwave frequency standard of the ammonia molecule, proves to be a superior time-keeper over long periods to the Earth the astronomer will be robbed of his satisfaction, and would appear, at least to the proponents of radio and radar systems, to be rapidly falling out of the navigational business. But this assumes that the radio aids are one hundred per cent. perfect and available, which is unlikely to be the case for many years : there will still be a place for conventional "astro" for some time yet.

Artificial satellites. — When we read in the daily press of statements by responsible statesmen of "Earth Satellite Vehicle Programmes" it is time that the astronomer began to take a serious interest in the new science of astronautics. Artificial satellites have been considered ; one conception of such a space-station is of a structure 200 feet in diameter, weighing 2,000 tons, staffed by 24 people and revolving with a period of about 24 hours (thus remaining stationary over one spot on the Earth) at a mean distance of about 22,000 miles — well outside Roche's limit. Leaving aside for the present the astronautical implications, it is of interest to enquire whether artificial satellites (unmanned) could assist astronomical navigation on the Earth as we know it. As we have seen, one of the great difficulties in air navigation is the determination of the vertical, upon which determination of position by the normal methods is completely dependent ; it is clear, however, that observations of an artificial satellite, of sufficiently large parallax, relative to the background of fixed stars would immediately give position of the Earth's surface without the necessity of a terrestrial plane of reference. A very approximate calculation shows that the expected magnitude, at opposition, of a man-made satellite of high albedo of diameter 100 metres at a distance of 10,000 kms. will be about 0 ; if the diameter is 100 d metres, and the distance 10,000 r kms., the magnitude will be 5 log (r/d), the angular diameter $2\,d/r$ seconds of arc, and the period about 3 $r^{3/2}$ hours. Roche's limit for the Earth is about 15,000 kms. but presumably a man-made satellite could withstand tide disrupting forces much closer to the Earth. Now the parallax of such a satellite would be 40° or more, so it would provide a perfect means of determining position — apart from providing a new and powerful time and distance standard. There would, of course, have to be at least two such satellites to provide continuous coverage especially as they would be subject to frequent and prolonged eclipses ; day-time observations would still provide difficulties unless the satellite were sufficiently bright to be observed in daylight.

The almanac maker of the era of these artificial satellites would have his full share of problems — both in celestial mechanics and in tabulation.

Astronautics. — Fom the idle speculative calculations of a few years ago there has rapidly grown up a new science of the navigation of space-ships under the combined forces of gravitational attraction and rocket propulsion. The subject is a large one and not even the broad outlines can be given here. But in the future cannot we visualise the navigator of a space-ship, an astronomer skilled in celestial mechanics beyond our present abilities and equipped with a "Space Almanac" and an electronic computing machine — for by no other means could he hope to keep control of the elaborate calculations — directing the propulsive units of his ship ! How the computers of cometary or planetary orbits would envy his ability to change his velocity to avoid — or to encompass — a close approach to Jupiter !

Conclusion. — This review has, rather naturally, emphasised the connections between astronomy and navigation with which the author has been most concerned. But there are many other indirect connections, where a particular subject — such as optics, instrument design, geodesy (long range radio navigation methods depend on accurate geodetic connections between continents, depending in turn on astronomical observations), etc. — is common to the two ; in these fields astronomy, both in general experience and through the work of particular astronomers, has been able to assist the science of navigation where apparently there is no such dependence. Ignoring then the "idle speculations" of the last two paragraphs, we still see there remain many problems in ordinary astronomical navigation for which satisfactory solutions are still required, and that Navigation, although rapidly out-growing its old dependence, cannot yet dispense with the help of its guide and mentor, Astronomy.