REFRACTIVE TEMPERATURE

by Arnold COURT

Climatologist, U.S. Government, Washington, D.C.

(Reprinted from *Journal of the Franklin Institute*, Vol. 247, No. 6, June, 1949)

ABSTRACT

Refraction tables used in navigation and surveying are almost all derived from those of Bessel, whose computations, based on English star observations, assume that atmospheric temperature decreases with height at the rate of about 0.6 *F.° per* 1,000 *ft. Neither this assumption nor that of any subsequent investigator is entirely in accord with present-day knowledge of atmospheric structure, which generally has a temperature decrease of around* 3 *F.° per* 1,000 *ft. to the tropopause, with widely varying conditions above it. The Bessel tables in general use cannot be extrapolated for surface air temperatures much different from* 50° *F.; in particular, for extreme cold conditions it is desirable to use, in place of the actual air temperature, the* refractive temperature, *which is that temperature for which the refraction correction given in the tables equals the correction which is actually required, and which may be estimated from general climatic considerations.*

INTRODUCTION

Refractive temperature, rather than actual temperature, should be used in determining the correction for atmospheric refraction for any astronomical observation for position, if this correction is to be taken from any of the tables commonly used by surveyors and navigators. The difference between these two temperatures is especially important for observations at zenith distances greater than 70° or 75°, when the actual temperature is unusually low and under certain other meteorological conditions.

As here proposed, *refractive temperature* is that temperature for which the refraction correction given in the tables equals the correction which is actually required. It applies primarily to the calculation of position observations made with theodolites, transits, and sextants, and not to the more elaborate computations of astronomers, although in such computations also the effect of weather conditions and various climatic regimes must be considered.

Actually, the temperature correction often is negligible, being usually smaller than the probable error of most navigation or azimuth observations. But since refraction tables include a temperature correction table, and use of such corrections is widespread, their basis should be known. If a temperature correction is applied to the refraction computation, it should be as accurate as possible, and therefore should be based not on the actual air temperature at the time of observation, but on the *refractive temperature.*

In this paper, the need for refraction corrections and their magnitude are discussed, the origins and assumptions of refraction tables in current use are explained, examples are given of errors involved in use of tables under all weather conditions, and a method is suggested for approximating the *refractive temperature.* Because navigators and surveyors work chiefly in English units, such units are used here.

GENERAL

Refraction corrections must be applied to astronomical observations because light rays passing through atmospheric layers of differing densities are bent toward the denser layer. In the atmosphere, the density always decreases with height (except in the few inches above a strongly heated surface such as a pavement or roof on a hot day), but at a rate which depends primarily on the temperature distribution.

On the average, the temperature of the air grows colder at increasing heights up to the "tropopause", at 5 to 10 miles. Above the tropopause, at which the temperature is between — 50° F. and — 130° F., the temperatures decrease or increase only slightly, for another 10 miles or so, then increase rapidly to about + 160° F. at heights of 30 to 40 miles. Above this level temperature falls again, then rises at heights above 50 miles (9),(1) but at these levels the atmosphere is so rare that differences in its density have no appreciable effect on incoming light rays. The density (temperature) distribution in only the lowermost 40 miles need be considered in computing the refraction (21).

These, however, are average conditions. The thermal structure of the atmosphere varies widely from day to night, season to season, and place to place. The diurnal range of temperature at the surface, from afternoon heat to pre-dawn chill, diminishes rapidly with height, and is only a degree or two above 2,000 or 3,000 ft. In summer the rate of temperature decrease, known as the lapse rate, is greater than it is in winter; it is also greater in tropical regions than in high latitudes. In the tropics, the tropopause is around 10 miles up, and about —120° F.; in polar regions, it remains at around 6 miles but varies from —50° F. in summer to —80°F. in winter, and may even vanish (1).

Most serious variation, from the standpoint of practical refraction computations, is the variation in the temperature distribution in the lowermost 2,000 ft. or so of the atmosphere. On any clear night the radiative cooling of the ground cools the adjacent air so that an *inversion* is created; that is, instead of the usual decrease of temperature with height, the temperature increases upward for a few hundred or thousand feet. During winter in cold continental regions, such inversions can become most pronounced. Wexler (10) has determined that the theoretical maximum limit to such an inversion is about 50 F.° when the surface temperature is 32° F., but is only 20 F.° when the surface temperature is —94° F.

Inversions of another sort, but equally important in the computation of atmospheric refraction, persist along seacoasts where the water is much colder than the land, and winds blow onshore. (California is an example.) Such inversions usually are at 3,000 to 5,000 ft., and may amount to as much as 20 F.° (6). Below them, the lapse rate shows the usual variation between day and night conditions, and above them a more or less "normal" lapse rate exists.

During stormy weather, the atmospheric structure may be extremely complicated, but at such times astronomical observations usually cannot be made, so the refraction computation is unimportant. Occasionally, however, during bad weather a mariner makes a hurried "shot" of the sun near sunrise or sunset; he should realize that there is strong likelihood of unusual refraction conditions rendering the results of such a sight, no matter how carefully and exactly made, less trustworthy than one in fair weather, when the atmosphere is stable and more closely approaches the conditions assumed by his refraction table.

Detailed knowledge of atmospheric structure, outlined above, is an accomplishment of only the last decade or two. Our refraction formulae and tables, however, are generally more than a century old, and while they involve some rather keen assumptions as to the atmospheric structure of western Europe, they are woefully inadequate to serve in all parts of the world in all seasons.

There is a need today for a system of refraction computation which permits corrections for various types of atmospheric structure, but none has yet appeared. Unfortunately, those who use refraction tables, the surveyors and navigators, usually are ignorant of the source and limitations of such tables; in turn, the astronomers who should provide better tables are uninformed on the latest knowledge of atmospheric structure and its variations.

Were such tables available, they could be applied on the basis of either average conditions for the locality, or of the actually observed conditions. With the increasing use of the radiosonde, in many cases the actual atmospheric structure at the time and place of the astronomical observation is known; this is particularly true of scientific exploring parties. With such more detailed information available, and improved tables, it should be possible to use observations of bodies near the horizon with far more reliability than is possible at present(2).

---

(1) The numbers in parentheses refer to the references appended to this paper.

(2) Since the preparation of this paper, three recent publications bearing on the subject have been noted :

"Celestial Navigation Computation Tables", Publication No. 601 of the Hydrographic Office of Japan (Tokyo, 1942), contains tables of corrections to observed altitudes described by Dr. Charles H. Smiley ("Mathematical Tables and Other Aids to Computation", Vol. 3, No. 23, July 1948, p. 195) as unusual because "they include corrections to be applied to altitudes less than 6°; special corrections for temperature, barometric pressure, and difference between air and water temperatures are given". The brief explanation of these tables is in Japanese, and the only clue to the nature of this temperature correction for refraction is that the name of Radau appears in the text

"Corrections to be applied to the dip of the horizon when the air is... warmer or colder than the water" are contained, according to Smiley ("Ibid.", No. 25, January 1949, pp. 369-71), in H.A. Goldhammer's "Nautisk Tabelsamling" (Copenhagen, J. Jorgensen & Co., 1946) ;

BESSEL

Almost all refraction tables used by surveyors and navigators in the United States today are derived from one basic source: the work of the great Friedrich Wilhelm Bessel. Early in the 19th century he re-analyzed the many star observations made from 1750 to 1762 at the Greenwich observatory by Bradley (19), and deduced from them a formula (13) for atmospheric refraction R :

$$R = {}_{\alpha}B^{A}T^{\lambda} \tan z,$$

where α, A and λ are functions of the zenith distance, z, for which he prepared tables; B is the barometric pressure, and T is the outside air temperature.

This formula, and the revised tables computed from it (14), soon became standard, and were incorporated into handbooks (18, 34), astronomy textbooks (20), and observatory manuals (22, 24). From these sources, or Bessel's original publications, were derived the tables used by navigators (16) and surveyors (26, 30).

The only U. S. government survey agency whose refraction tables are not wholly Besselian is the Coast and Geodetic Survey (17), which used from 1898 until 1948 tables published by Hayford (28). These were obtained by the uncritical method of arithmetically averaging refraction tables given in Doolittle's Practical Astronomy, which in turn were based on Bessel's tables, with those given in the *Connaissances des Temps* for 1897, which "depend upon other observations and upon a different theory—that of Laplace". Use of these tables has recently been superseded by a new publication (29), whose new tables are discussed later.

Because of this general dependence on Bessel's work, its basic assumptions must be analyzed thoroughly to understand why those tables, valid as they may have been for star observations at Greenwich, are not necessarily applicable to field work in different climates. According to Chauvenet (20), Bessel's derivation assumes that

$$1 - E(T_x - T_0) = e^{-as/h},$$

where

E is the coefficient of thermal expansion of air $= 0.0036438$,
T_x and T_0 are the temperatures at height x and at the surface,
e is the base of natural logarithms $= 2.71828$,
a is the radius of the earth $= 6{,}372{,}970$ meters,
$s = x/(a + x)$, and
h is an empirical constant $= 227{,}775.7$ meters.

(Values cited are those used by Bessel; more accurate determinations of E and a do not affect the computation appreciably.)

From this relation, the temperature distribution assumed by Bessel is found to be that of a lapse rate of around 0.6 F.° per 1,000 ft., decreasing very slightly with height : for the first 1,000 ft., a drop of 0.64 F.° is required; for 40 miles, the total decrease is 121 F.°, or an overall lapse rate of 0.57 F.° per 1,000 ft. This gradient is only about a fifth of that obtaining in the English troposphere, but since it does not recognize the stratosphere, the assumed distribution is colder than actual conditions at heights greater than 20 miles.

In computing his tables based on this temperature distribution Bessel used mean conditions as pressure 29.60 in., temperature 50° F. These are still acceptable mean annual values for London (11). To compensate for surface conditions differing from these means, Bessel's formula corrects the "normal" refraction by T^{λ}, where T is the air temperature and λ varies slightly with zenith distance; it is 1.000 from the zenith down to 40°, and at greater zenith distances becomes (15) :

z:	45°	50°	55°	60°	65°	70°	75°	80°	85°
λ:	1.0018	1.0023	1.0031	1.0046	1.0068	1.0111	1.0197	1.0420	1.1229

---

however, the introduction and explanation of the 174-page tables is limited to a single page, with no indication of the source of these corrections.

A bibliographical reference has been noted to I. Ya. Tantor, "Calculation of Atmospheric Refraction During Aerological Observations", "Izvestia Akademiia Nauk SSSR", Ser. Geog. Geofiz, Vol. 12, No. 4, but the publication has not been studied.

Since publication of this paper, two articles have appeared on the subject :

"Atmospheric Refraction at College, Alaska, during the Winter, 1947-1948". Pierre St. Amand and Harold Cronin. Transactions, American Geophysical Union, Vol. 31, No. 2, part I, April 1950, pp. 161-164.

"Atmospheric Refraction at Low Angular Altitudes in the Tropics". Charles H. Smiley. Navigation, Vol. 2, No. 5, March 1950, pp. 110-113.

Both are empirical studies, without consideration for atmospheric structure.

Magnitude of this adjustment may be seen from the following tabulation of the total refraction correction, for various temperatures and zenith distances, computed from Bowditch's version (16) of Bessel's tables :

Zenith	Air Temperature (Fahrenheit)			
Distance	—10°	+20°	+50°	+80°
60°	1'54"	1'47"	1'41"	1'35"
70°	3 00	2 48	2 39	2 30
75°	4 02	3 47	3 34	3 22
80°	6 02	5 40	5 19	5 01
85°	11 21	10 34	9 52	9 16

Actually, the effect of Bessel's correction for temperature is to alter the assumed lapse rate, increasing it for warmer temperatures and decreasing it for cooler conditions, with the temperature "at the top of the atmosphere" remaining constant. In turn, this implies that in warm weather the density decreases with height less rapidly than in cold weather. While such an assumption is generally in accord with meteorological conditions, Bessel's quantitative expression for it is valid only for climatic conditions such as exist in the British Isles.

MODIFICATIONS

Need for improving or modernizing Bessel's refraction formula and tables has been cited by many investigators. Bessel himself arbitrarily increased his original values, derived for England, in the ratio of 1.003282 (20) to fit them to observations at Königsberg, where the continental climate has a steeper lapse rate than the marine climate of England.

California's marine inversion makes the net lapse rate smaller than that of England (7), so that Crawford (23) found "from observations made at the Lick Observatory that, not only are Bessel's refractions too large, but that the Pulkowa refractions are also". But Crawford did not know the cause of this difference, and his proposed refraction tables are not based on meteorological data.

An improved formula for computing refraction at zenith distances greater than 75°, applicable even for zenith distances greater than 90° (that is, object below horizon) is offered by Esclangon (25), claiming it to be simpler than Radau's. It involves an exponential decrease of the index of refraction, and a lapse rate of 2.75 F.° per 1,000 ft. up to 30,000 ft., with an increasingly steep lapse rate still higher. Above 50,000 ft., Esclangon says, the correspondence between his model and observed temperatures ceases, "but these layers are of negligible importance in refraction". He does not offer tables based on his formula, which contains an exponential constant depending on temperature.

Two other systems of refraction computation, offered in recent years by Willis (33) and Garfinkel (27) profess to use modern knowledge of atmospheric structure. Actually they rely on obsolete mean pressure-temperature-height data given by Humphreys (3), which are based on 416 central European soundings made between 1900 and 1912.

Garfinkel follows the theoretical approach of Ivory in his proposed tables, which extend to zenith distances as great as 116°. His development involves a "polytropic index", defined as

$$n = (g/aR) - 1,$$

where g is the acceleration of gravity, a the lapse rate, and R the gas constant. Since the lapse rates obtained from Humphreys' tables vary from 5.6 to 5.9 C.° per km. (3.1 to 3.2 F.° per 1,000 ft.), the values of n range from 4.8 to 5.1, and a mean value of $n = 5.0$ is adopted. This corresponds to a lapse rate of 5.8 C.° per km. or 3.2 F.° per 1,000 ft., which is rather too small for the United States, although it is generally correct for western Europe.

The formula also contains an additive correction, significant ($>$ 1") only at zenith distances greater than 86°, for the existence of the stratosphere, which is assumed to be isothermal, at 0.778 of the surface temperature (A). The correction is tabulated as a function of zenith distance and observer's height; for 90° zenith distance it varies from 2 seconds of arc at the ground to 20 seconds at 10 km. (6.2 miles), the upper limit of the table.

There is also a correction for deviation of the lapse rate from its assumed value, but "in the absence of sufficient information about the daily and seasonal fluctuations of the temperature gradient, as well as its geographical distribution, this correction is rather uncertain." Fortunately it is negligible "for zenith distances of 90° or less," Garfinkel declares. One purpose of the present paper is to point out the availability of detailed knowledge of diurnal, seasonal, and geographical variations in the lapse rate, and that the effects of these variations, especially at extreme zenith distances, are far from negligible.

Willis' " system for the computation of astronomical refraction, perhaps entirely adequate to satisfy the needs of practical astronomy, without involving star-observations themselves ", includes 12 tables incorporating the latest values for the variation with temperature and wave length of the refractive indices of dry air, water vapor, and carbon dioxide. He expresses Humphreys' data as a power series,

$$T/T_0 = 0.670 + 0.5925\ (p/p_0) - 0.2625\ (p/p_0)^2.$$

From Humphreys' pressure data, this series indicates a lapse rate with a maximum of more than 3 F.° per 1,000 ft. at 10,000 ft., above which it decreases steadily with height.

No method is offered to allow for wholesale departure from this lapse rate. Willis declares that " The diurnal variation of temperature (is) ... of the order of a fraction of 1 C° at a few km. " While the average *diurnal* variation may be of this order, the *day-to-day* variation is very much larger. Ratner (8) has shown that, for instance, at 3 km. (10,000 ft.) over Washington, D.C., a range of about 15 C.° (27 F.°) is necessary to account for 80 per cent of the temperatures observed in January, and about half that much in July. According to Willis' Table 7, " Correction for departure of local temperature from that of standard atmosphere ", at a temperature of —4° F. the refraction is increased by only 0.002 seconds of arc for zenith distances of 70° and by 0.108 seconds for zenith distances of 80°. Here, however, " the local temperature is assumed to exist in a thin layer ".

U.S.C.G.S.

Willis' formula for refraction has been used to compute two independent sets of refraction tables for the recently published " Manual of Geodetic Astronomy " of the Coast and Geodetic Survey (29). Because of the recency of this publication, and the fact that its tables are likely to be copied uncritically into other publications, the meteorological assumptions of these tables require complete discussion. In general, it may be said that just as Willis in developing the original basis for these tables did not avail himself of the latest and most complete knowledge of variations in the state of the atmosphere, so the compilers of these tables did not utilize modern methods of determining meteorological variables, particularly atmospheric pressure.

Although the two sets of refraction tables are both based on Willis' formula, no discussion is given of the relative merits and differences of the two methods, nor of the types of observations for which each is best suited. The " simplified method " comprises a table for basic refraction and two tables of correction coefficients, one for departure of pressure from 760 mm. (29.921 in.), the other for departure of temperature from 10° C. (50° F.). Arguments of these tables are given in inches and degrees Fahrenheit. This arrangement is identical with the form of most of the commonly-used tables derived from Bessel's formula, but gives refractions about 2" less at zenith distances of 80°. For zenith distances greater than 85°, the tables are computed from the Pulkowa Observatory Refraction Tables (1930), which agree with Willis' tables between 83° and 85°. Because this system has no corrections for lapse rate, but only for surface temperature, *refractive temperature* should be used.

The second method involves the use of Willis' complete formula :

$$\log_{10} R = \log_{10} \tan z + \alpha + \beta + \gamma + (\lambda - 1)\ (\gamma + 0.1\beta).$$

Here z is the zenith distance, α and $(\lambda - 1)$ are functions of z alone and are given in a single table, and β and γ are corrections for departures of air pressure and temperature from the assumed mean values of 750 mm. (10 mm. less than in the simplified form) and 10° C. (50° F.).

The correction for temperature, γ, is simply $\log_{10}\ (T/283)$, and is given in a single table, but without regard to any deviations from the west European lapse rate used by Willis. Although it is given for each degree from —30° C. to +44° C. (—22° F. to + 111° F.), it probably is valid over less than half this range unless the *refractive temperature* is used.

The pressure correction, β, is unnecessarily complicated and confused, involving both instrumental corrections and physical considerations, some of them not pertinent. It is given as :

$$\beta = \beta_0 + \Delta\beta_t + \Delta\beta_g + \Delta\beta_w.$$

Here β_0 is the actual correction for deviation of air pressure from 750 mm., and the other three terms are corrections for, respectively, the temperature of the barometer, variation in gravity, and the wave length of the light used in the observation. The confusion is heightened by neglect to explain that

$$\beta_0 = \log_{10} (B + eC) - \log_{10} 750,$$

where B is the uncorrected barometer reading, 750 is the assumed normal pressure of dry air in mm., e is the atmospheric vapor pressure in mm., and C a factor varying directly with the wave length of the light used. The correction, eC, " is equivalent to the correction of the refractive power because of the water vapor content of the air ", Willis (33) explains. It changes the actual pressure to a sort of virtual refractive pressure, that is, the pressure of a mass of dry air which has the same refractive properties as the moist air actually prevailing. Logically, this correction should be applied to the true barometric pressure, that is, the barometer reading as corrected for scale errors, capillarity, temperature, and gravity, and not before such corrections are made.

Such corrections, $\Delta\beta_t$ and $\Delta\beta_g$, actually do not belong in this formula at all. A century ago it may have been permissible to include instrumental corrections in the same computation as physical variables, but such is no longer customary. Meteorologists have long considered that determining the atmospheric pressure includes the application of all necessary corrections to the reading of the mercurial barometer. A refraction formula should merely call for the use of true atmospheric pressure, corrected as required by standard practice as outlined in the Smithsonian Meteorological Tables (4) or observing manuals of the Weather Bureau or the Army or Navy weather services. Users of the Manual are left to their own devices to determine the vapor pressure, and barometric pressure should be treated similarly.

The table given for $\Delta\beta_t$ is inadequate anyhow, since it does not specify the type of barometer for which it applies. Fixed cistern barometers (used on shipboard and often in field work) require different corrections from those with movable cisterns, and the corrections differ again if the barometers are graduated in English or metric units, because then the zero points of the scales are different (4, 5).

Although not explained as such, the correction for gravity, $\Delta\beta_g$, is simply the logarithm of the ratio of the value of gravity at the place of observation to the value assumed in the tables. This assumed value is " for the latitude and elevation of the U.S. Naval Observatory in Washington, D. C. " and is equivalent to that " at sea level in latitude 38°37'20". " No justification is offered for reducing the atmospheric pressure to a standard of gravity other than that used by meteorologists throughout the world, which is the value of gravity at sea level in latitude 45°. It would be preferable to follow current practice and compute the basic tables for this " standard gravity ", and omit all corrections for gravity, as well as instrument temperature, from these tables.

The last correction to the barometer reading, $\Delta\beta_w$, has nothing to do with pressure at all, but is inserted in this computation for convenience. It merely increases or decreases the total refraction accordingly as the wave length of the light observed is shorter or longer than that of the yellow light, 0.578 microns, used in the basic computation. For violet light, the refraction is almost 2 per cent greater, and for red light about 0.4 per cent less.

All told, the new Coast and Geodetic tables are probably a slight improvement on the classical Besselian tables. But they could have been made far superior, had their compilers availed themselves of the latest knowledge concerning the structure of the atmosphere, and followed modern practice in the reduction and use of meteorological observations. As it is, they require, just as much as the Besselian tables, the use of *refractive temperature* for any computation involving atmospheric structure differing from that of western Europe.

EXAMPLES

While observers make every effort to observe bodies at small zenith distances, so as to avoid the large refraction corrections, it is often necessary in navigation and surveying, because of clouds, to obtain readings relatively close to the horizon. This is especially true of sun and moon observations in polar regions, where these bodies are quite low in the sky and, in the summer half-year, are the only ones observable. And it is precisely in polar regions that strong temperature inversions, with their abnormal temperature gradients, are most common, even in summer, rendering an accurate computation of the refraction correction highly essential.

Recently, summertime geodetic observations above the Arctic Circle (31) were based entirely on solar observations. The zenith distances, corrected according to standard practice (17), were consistently in error by an average of 5.4 seconds of arc. Since the thirteen sets of observations were well distributed in azimuth, averaging them cancelled this error, due to the refraction correction and uncertainty as to the gravity anomaly.

The authors conclude that "the point in favor of the polar regions is, of course, the fact that observations can be taken on the sun in all azimuths. The sun varies little in altitude and the atmosphere is quite uniform so that tables of refraction apply with constant precision". Actually, the taking of observations in all azimuths permits cancelling out of the relatively constant error in the refraction tables.

A wintertime example of errors arising from use of refraction corrections based on extremely low temperatures for observations at large zenith distances may be cited from the observations made at Little America III, Antarctica, by Leonard M. Berlin (12), cadastral engineer of the General Land Office and surveyor of the U. S. Antarctic Expedition of 1939-41.

Ten solar observations during January and February, 1940, gave the latitude as 78°29'06.5" S. with a standard deviation of 9.7". Surface temperatures at these times ranged from 6 to 19° F., with probably only a slight inversion (observations of upper air data did not begin for several months). On 8 July, the zenith distance of Sirius at upper transit was observed as 61°49'00", which, using the refraction correction of 118" computed from the tables (30) for the observed temperature of —10° F. and pressure of 28.85 in., gave a latitude of 78°29'07" ; on the same day the zenith distance of Canopus at upper transit was 25°48'45", giving a latitude of 78°29'06".

On 18 July, the same zenith distance was obtained again for the upper transit of Sirius. But since the temperature during the observation was —55° F., with a pressure of 28.51 in., an extrapolated value of 130" for the refraction correction was used, resulting in a latitude of 78°29'16"; the correction of 120", based on a *refractive temperature* of —15° F., deduced from radiosonde ascents (2), yielded a latitude of 78°29'06", exactly in agreement with previous determinations.

In this example, the error in latitude due to failure to use the *refraction temperature* is only 10 seconds of arc, or about 1,000 ft. However, the inversion in this case is far from extreme, and the zenith distance of 62° is not unusually great. Had the zenith distance of the sun at upper transit been observed on 5 September, when the surface temperature was —75° F. and the *refractive temperature* only —20° F., an error in latitude of about 100 seconds of arc, or nearly 2 miles, would have resulted from failure to use the proper *refractive temperature.*

APPROXIMATIONS

Unless a simple method for computing refraction corrections for various observed or assumed lapse rates becomes available, the present Besselian tables probably will continue in use for many years. Consequently, any rules for determining *refractive temperature,* even though crude and general, should improve materially the accuracy of observations at large zenith distances through atmospheres whose structure differs materially from that of England.

Actually, for adaptation to world-wide use, Bessel's tables require two corrections. First, the entire table should be altered for differences between the average picture of the atmosphere over the place of observation and the average British atmosphere; Bessel made such a correction for the Königsberg atmosphere in multiplying his original values by 1.003282. Second, the deviation of the atmospheric structure from the average must be expressed as some function of the actual difference of surface air temperatures.

Neither of these corrections can be developed rigorously, in view of the empirical nature of Bessel's original formula. Within the limits of necessary accuracy, however, both corrections may be combined. The following rules, based on study of individual atmospheric soundings and means for all parts of the world, is offered as a step toward improving computations.

The refractive temperature *is the same as the actual air temperature at the time of observation, except that it should not be more than* 10 *F°. less than the average temperature for the preceding* 24 *hr., nor* 25 *F°. less than the mean temperature of the month. It should never be less than* 0° *F. except in polar regions, where it may be one-fourth as far below* 0° *F. as the actual temperature.*

For any one locality, a series of accurate observations at large zenith distances, well distributed in azimuth, should reveal the amount of error involved in strict use of Bessel's refraction tables, and permit deduction of more accurate rules for determining the *refractive temperature.* Such rules, of course, will best be determined by collaboration among astronomers, meteorologists, and geodesists. Only by use of the latest techniques and information of astronomy and meteorology can the geodesist hope to attain improved refraction tables which will not require such rather gross corrections as that for *refractive temperature.*

REFERENCES

Meteorological

(1) ARNOLD COURT, "Tropopause Disappearance during the Antarctic Winter", *Bulletin of the American Meteorological Society,* Vol. 23, No. 5, pp. 220-238 (May, 1942).

(2) ARNOLD COURT, "Meteorological Data for Little America III", *Monthly Weather Review,* Supplement No. 48. In press.

(3) W. J. HUMPHREYS, "Physics of the Air", New York, McGraw-Hill Book Co., 1929.

(4) CHARLES F. MARVIN, "Smithsonian Meteorological Tables", Fifth Revised Edition, Washington, 1931.

(5) W. E. KNOWLES MIDDLETON, "Meteorological Instruments", Toronto, University of Toronto Press, 1943.

(6) MORRIS NEIBURGER, "Temperature Changes during Formation and Dissipation of West Coast Stratus", U.S. Department of Commerce, Weather Bureau, Research Paper No. 19 (July, 1944).

(7) BENJAMIN RATNER, "Upper Air Average Values of Temperature, Pressure, and Relative Humidity over the United States and Alaska", U.S. Department of Commerce, Weather Bureau, May, 1945.

(8) BENJAMIN RATNER, "Temperature Frequencies in the Upper Air", U.S. Department of Commerce, Weather Bureau, January, 1946.

(9) CALVIN N. WARFIELD, "Tentative Tables for the Properties of the Upper Atmosphere", National Advisory Committee for Aeronautics, Technical Note No. 1200. January 1947. (Extensive list of recent papers on temperature, etc., at extreme heights.)

(10) HARRY WEXLER, "Cooling in the Lower Atmosphere and the Structure of Polar Continental Air", *Monthly Weather Review,* Vol. 64, No. 4, p. 127 (April, 1936).

(11) (Anon.) "Temperature of the Upper Air over England", British Air Ministry, Meteorological Office, 1941.

Refraction

(12) LEONARD BERLIN, "Astronomical Observations at West Base, U. S. Antarctic Service", MS in National Archives.

(13) FRIEDRICH WILHELM BESSEL, "Fundamenta Astronomiae pro anno MDCCLV deducta ex observationibus viri incomparabilis James Bradley in Specula Astronomica Grenovicensi per annus 1750-62 institutis", Königsberg, 1818.

(14) FRIEDRICH WILHELM BESSEL, "Tabulae Regiomontanae reductionum observationum astronomicarum ab anno 1750 usque ad annum 1850 computatae", Königsberg, 1830.

(15) FRIEDRICH WILHELM BESSEL, "Astronomische Untersuchungen", Vol. I, Königsberg, 1841.

(16) NATHANIEL BOWDITCH, "The American Practical Navigator", Washington, 1938.

(17) WILLIAM BOWIE, "Determination of Time, Longitude, Latitude, and Azimuth", Special Publication No. 14, U. S. Department of Commerce, Coast and Geodetic Survey, 1931.

(18) KARL BREMIKER, "Vega's Tables of Logarithms", 40th ed., Berlin, 1856. (Also 40th ed., translated by W.L.F. Fisher, Berlin, London, and New York, 1859.)

(19) C. BRUHNS, "Die Astronomische Strahlenbrechung in ihrer historischen Entwickelung dargestellt", Leipzig, 1861.

(20) WILLIAM CHAUVENET, "A Manual of Spherical and Practical Astronomy", Philadelphia, 1863.

(21) ARNOLD COURT, "Insolation in the Polar Atmosphere", *Journal Franklin Institute,* Vol. 235, No. 2, pp. 169-78 (Feb., 1943).

(22) P.H. COWELL, "Refraction Tables Arranged for Use at the Royal Observatory", Greenwich Observations, Appendix I, 1898.

(23) RUSSELL TRACY CRAWFORD, "On Astronomical Refraction", Publication of the Lick Observatory, Vol. VII, part 6. Sacramento, 1915.

(24) J. R. EASTMAN, "Refraction Tables", U.S. Naval Observatory. Washington, 1887.

(25) ERNEST ESCLANGON, "Sur la réfraction astronomique", *Comptes Rendus,* Vol. 216, No. 2, pp. 100-103 (11 January, 1943).

(26) SAMUEL STINSON GANNETT, "Geographical Tables and Formulas", U. S. Department of the Interior, Geological Survey, Bull. 650, Washington, 1908.

(27) BORIS GARFINKEL, "An Investigation in the Theory of Astronomical Refraction", *The Astronomical Journal,* Vol. 50, pp. 169-79 (February, 1944).

(28) JOHN F. HAYFORD, "A Text-Book of Geodetic Astronomy", New York, 1898.

(29) ALBERT J. HOSKINSON and J.A. DUERKSEN, "Manual of Geodetic Astronomy", U.S. Department of Commerce, Coast and Geodetic Survey, Special Publication No. 237. Washington, 1947.

(30) ARTHUR D. KIDDER, "Standard Field Tables and Trigonometric Formulas", U.S. Department of the Interior, General Land Office, Washington, 1942.

(31) G. LEIFSON and W. T. THERRELL, "Geographical Positions by Solar Observation", *Trans. American Geophysical Union,* Vol. 28, No. 5, pp. 680-682 (October, 1947).

(32) SIMON NEWCOMB, "A Compendium of Spherical Astronomy", New York, 1906.

(33) JOHN E. WILLIS, "A Determination of Astronomical Refraction from Physical Data", *Trans. American Geophysical Union,* part II, pp. 324-338 (1941).

(34) R. S. WOODWARD, "Smithsonian Geographical Tables", Washington, 1894.

* * *