NOTE ON DIFFRACTION OF SWELL IN PERPENDICULAR INCIDENCE

by *Ingénieur hydrographe principal* Henri LACOMBE

The Service Hydrographique *of the French Navy recently published as No.* 1363 *of the* Annales Hydrographiques *an investigation by* Ingénieur hydrographe principal *H. Lacombe concerning the diffraction of swell in normal incidence. This note is a contribution to the study of diffraction of swell in that it offers complementary solution of problems discussed, in particular, by A. Putnam and R.S. Arthur :* Diffraction of Water Waves by Breakwaters, *Trans. Amer, Geophys. Union, Vol.* 29, *No.* 4, *August,* 1948 ; *H. Gridel :* Annales des Ponts et Chaussées, *Jan-Febr. and May-June*, 1946 ; *and A. Nizery :* Rapports et Travaux de la Société Hydrotechnique de France : La Houille blanche, 1948/*A.*

A short summary of this article follows :

....

In his introduction Mr. Lacombe points out that the investigation of this problem may be approached either from the physical point of view (Huyghens' Principle, on condition that it not be applied in the immediate vicinity of the elementary sources, and specifically no closer than one wave-length, or from a mathematical point of view by applying limiting conditions to the unlimited equations of hydrodynamics.

The development of his theme is based on the application of Huyghens' principle, and Mr. Lacombe takes care to point out that, contrarily to what occurs for the classical theory of optical diffraction and the simplified solutions deduced from the correct solution of hydrodynamics, it takes into account no other approximation than that inherent in the Huyghens principle itself.

GENERAL REMARKS ON THE DIFFRACTION OF SWELL IN PERPENDICULAR (NORMAL) INCIDENCE, BASED ON HUYGHENS' PRINCIPLE

1. **Elementary Motion.**—Mr. Lacombe demonstrates that the amplitude of the elementary motion starting at M is in inverse proportion to the square root of the distance and that, if the phase which the incident swell would have at the point M in the absence of obstacle is taken as phase origin, total motion at M is expressed by :

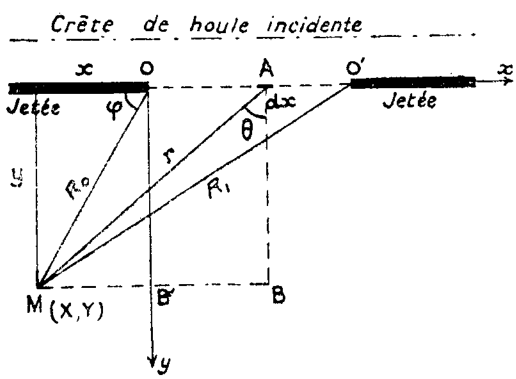


FIG. 1

(a) *In the case of no directional property of the elementary sources :*

(1) $$A = \left[I(\rho_1) - I(\rho_0)\right] \cos \sigma t + \left[J(\rho_1) - J(\rho_0)\right] \sin \sigma t$$

taking : $r - Y = \frac{\rho^2}{4}$

$$I(\rho) = \int_0^\rho \sqrt{\frac{\rho^2 + 4Y}{\rho^2 + 8Y}} \cos \frac{\pi}{2} \cdot \rho^2 . d\rho$$

$$J(\rho) = \int_0^\rho \sqrt{\frac{\rho^2 + 4Y}{\rho^2 + 8Y}} \sin \frac{\pi}{2} \rho^2 . d\rho$$

The absolute value being at least :

$$\rho_0 = 2\sqrt{R_o - Y} = 2\sqrt{\sqrt{X^2 + Y^2} - Y}$$

$$\rho_1 = 2\sqrt{R_1 - Y} = 2\sqrt{\sqrt{(X - D)^2 + Y^2} - Y}$$

the unit of length being the wave-length.

(b) *If directivity of the elementary sources with respect to cos* θ *is assumed,* the expression of the motion becomes :

(2) $$B = \left[K(\rho_1) - K(\rho_0)\right] \cos \sigma t + \left[L(\rho_1) - L(\rho_0)\right] \sin \sigma t$$

with :

$$K = \int_0^\rho \frac{d\rho}{\sqrt{\left(1 + \frac{\rho^2}{4Y}\right)\left(2 + \frac{\rho^2}{4Y}\right)}} \cos \frac{\pi}{2} . \rho^2$$

$$L = \int_0^\rho \frac{d\rho}{\sqrt{\left(1 + \frac{\rho^2}{4Y}\right)\left(2 + \frac{\rho^2}{4Y}\right)}} \sin \frac{\pi}{2} \rho_2$$

2. **Finite Motion.**—FIRST SOLUTION : *No directional property of the elementary sources :*

Movement reaching point X, Y is in proportion to :

$$A = b \cos(\sigma t - \psi)$$

with :

$$b = \sqrt{[I(\rho_1) - I(\rho_0)]^2 + [J(\rho_1) - J(\rho_0)]^2}$$

$$\operatorname{tg} \psi = \frac{J(\rho_1) - J(\rho_0)}{I(\rho_1) - I(\rho_0)}$$

The expression equivalent to b shows that in a system of rectangular co-ordinates such as :

$$\Delta X = I(\rho_1) - I(\rho_0) \text{ and } \Delta Y = J(\rho_1) - J(\rho_0),$$

this b value is the hypotenuse of a right-angled triangle constructed on ΔX, ΔY and the phase lag ψ is given by : $\operatorname{cotan} \psi = \frac{\Delta X}{\Delta Y}$.

This shows the advantage of plotting the curve, as Cornu has done, in terms of parameter Y :

$$x = I(\rho) = \int_0^\rho \sqrt{\frac{\rho^2 + 4Y}{\rho^2 + 8Y}} \cos \frac{\pi}{2} \rho^2 \, d\rho$$

$$y = J(\rho) = \int_0^\rho \sqrt{\frac{\rho^2 + 4Y}{\rho^2 + 8Y}} \sin \frac{\pi}{2} \rho^2 \, d\rho$$

where b is the length of the line connecting the points relative to the values ρ_0 and ρ_1.

Mr. Lacombe has computed a table giving the values of the integrals x and y in terms of ρ, for five values of Y : 1/4, 1, 5, 10, $+\infty$, varying from 0.1 to 3.0 by tenths.

Plates I, II and III represent the spirals relating to these Y values, plotted on the same plate in series of two in order to emphasize discrepancies.

It may be seen that for a value of Y greater than 5 with $\rho \leq 3.0$, the spirals practically no longer depend on parameter Y.

In addition, Mr. Lacombe has computed a table of ρ values for different X and Y values.

This table is advantageously completed by the curves ρ = constant of plate IV.

2nd Solution.—*In the case of directional property of the elementary sources with regard to cos θ, the relative amplitude is :*

$$C = \sqrt{[K(\rho_1) - K(\rho_0)]^2 + [L(\rho_1) - L(\rho_0)]^2}$$

$$\text{and } \tan \psi = \frac{L(\rho_1) - L(\rho_0)}{K(\rho_1) - K(\rho_0)}$$

for ρ small and Y large, we have approximately :

$$K \simeq \frac{1}{\sqrt{2}} G \qquad L \simeq \frac{1}{\sqrt{2}} F.$$

F and G being the *Fresnel* integrals. The results obtained should therefore be comparable to those given by the integrals I and J.

Mr. Lacombe applies these results to three particular cases :

I.—First particular case. Jetty of undefined length in one direction (perpendicular incidence) :

A.—No Directional property of elementary sources

Agitation behind Jetty ($Y > O$). For different Y values, amplitudes are given on the spirals by the distance between point Ω of a spiral and the point of the same spiral the parameter of which is $\rho_0 = 2\sqrt{R_0 - Y}$.

Plate V illustrates for the different Y values the amplitude variation curve in terms of X, and the envelope of fringes outside the sheltered area. A few curves ρ = constant in the sheltered area have been plotted.

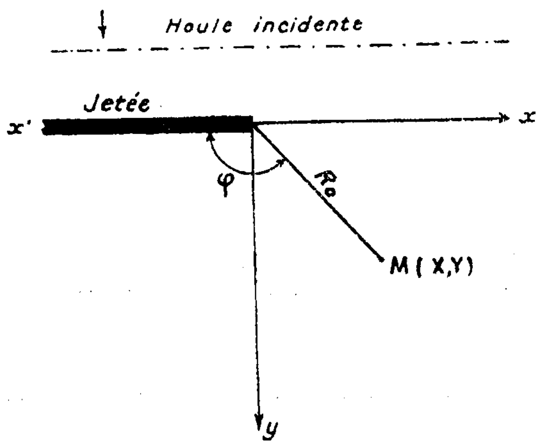


Fig. 2.

Comparison of these results with those of optics (Fresnel integrals) shows that they are practically the same only at the edges of line X = O, giving the geometrical shadow, i.e. by analogy with luminous rays very slightly deflected from their initial direction. Within the sheltered area the "optical" amplitude is less ; in general it is greater in the unsheltered area, except for very large values of ρ with finite values of Y.

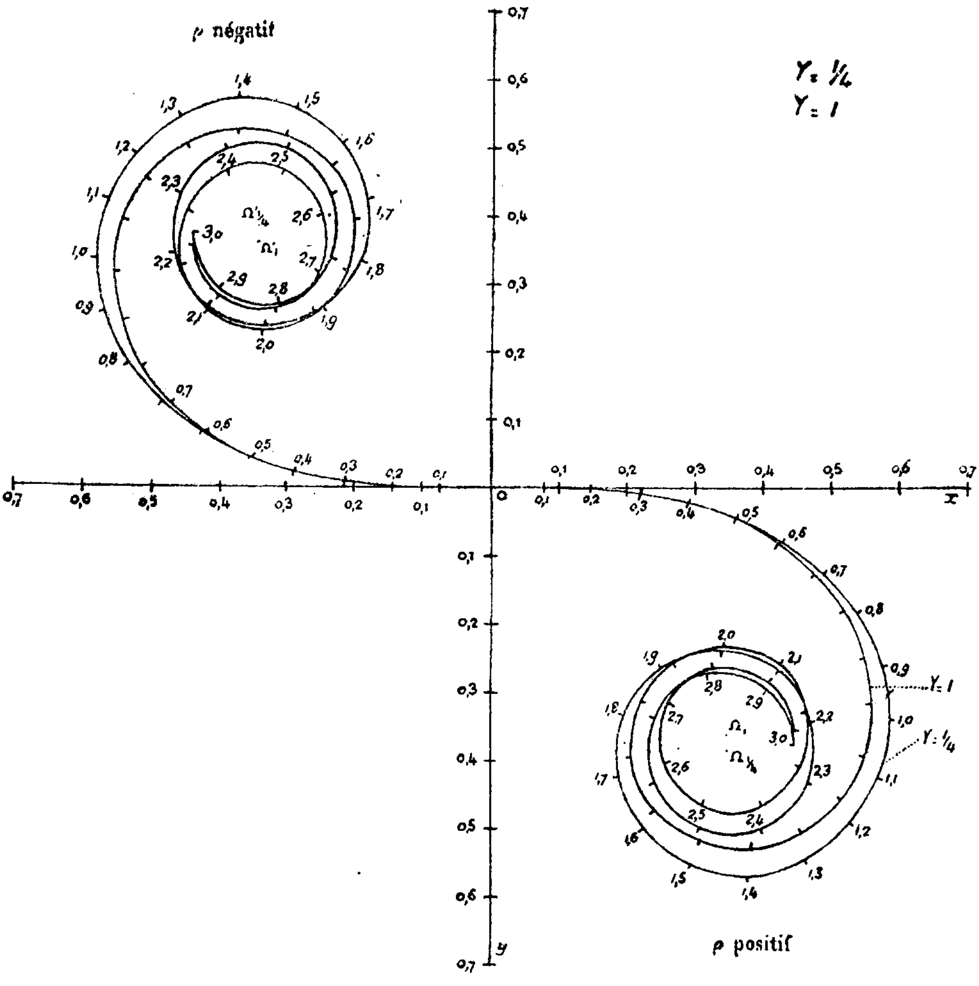


Planche I.

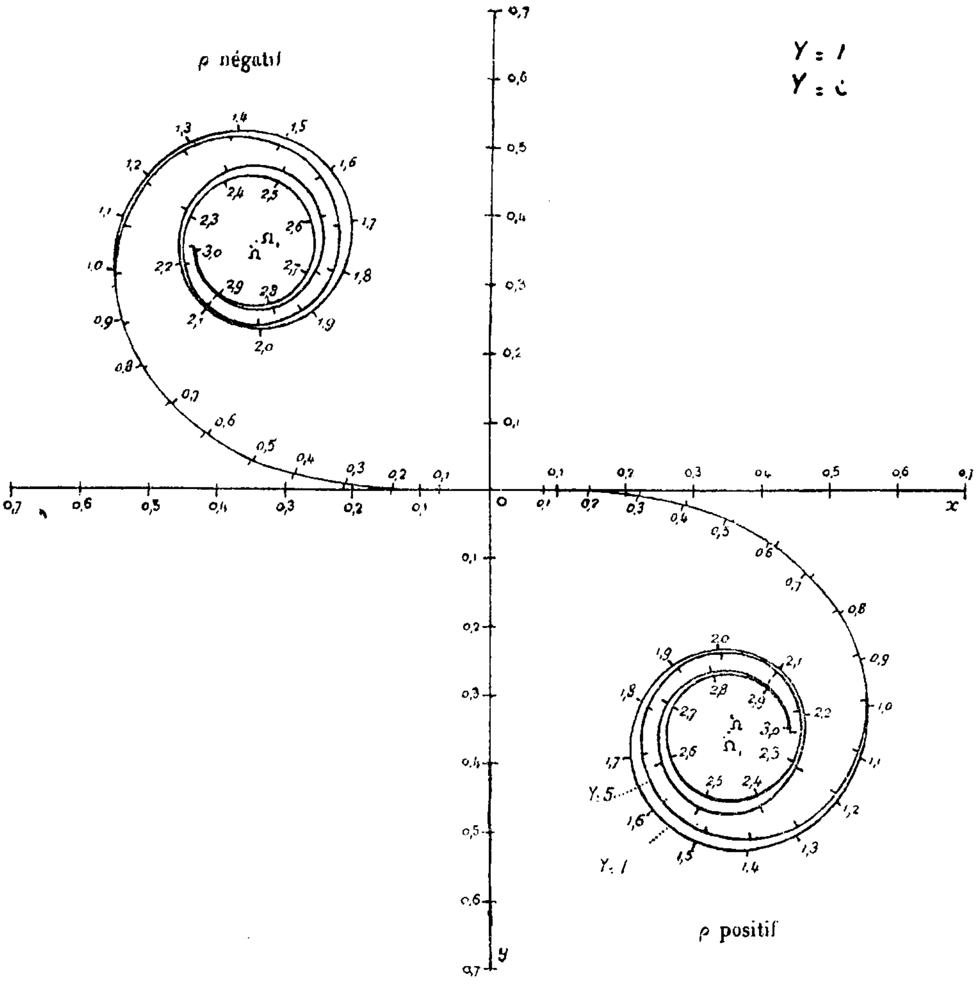


Planche II

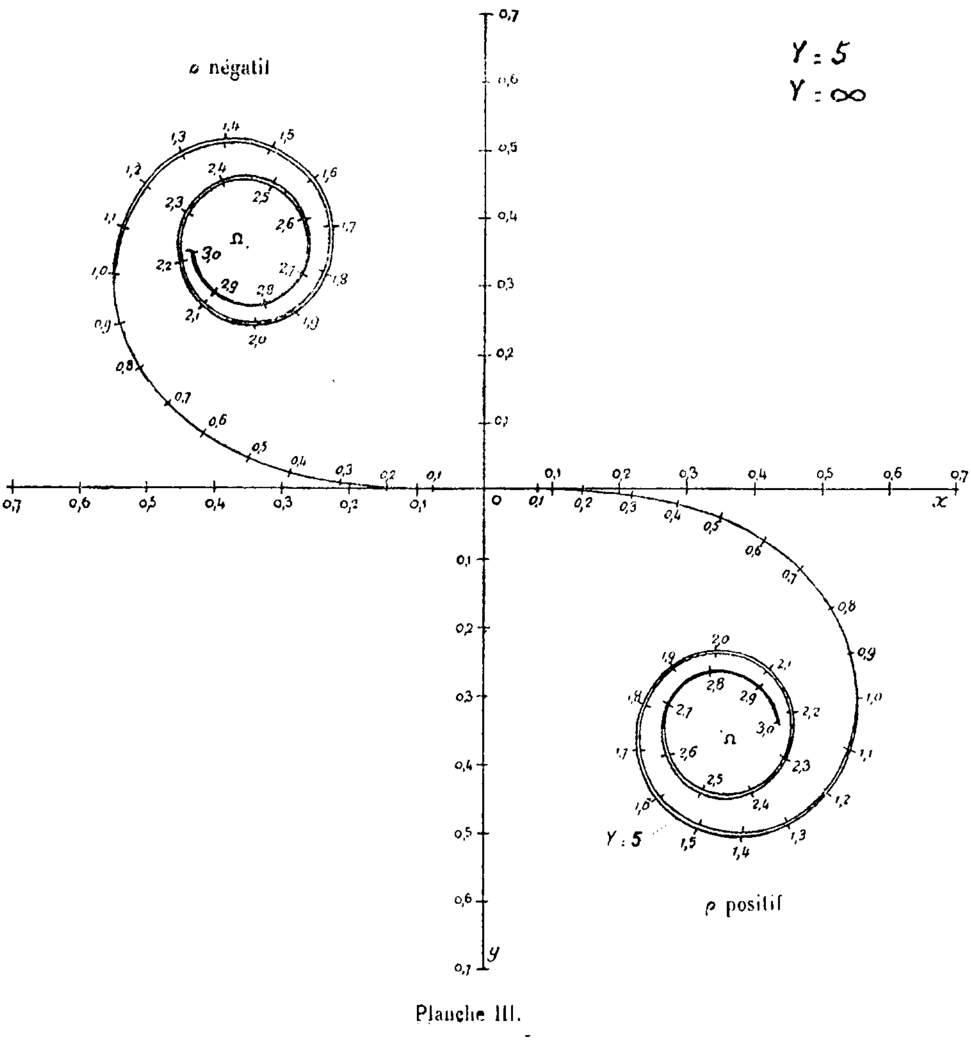
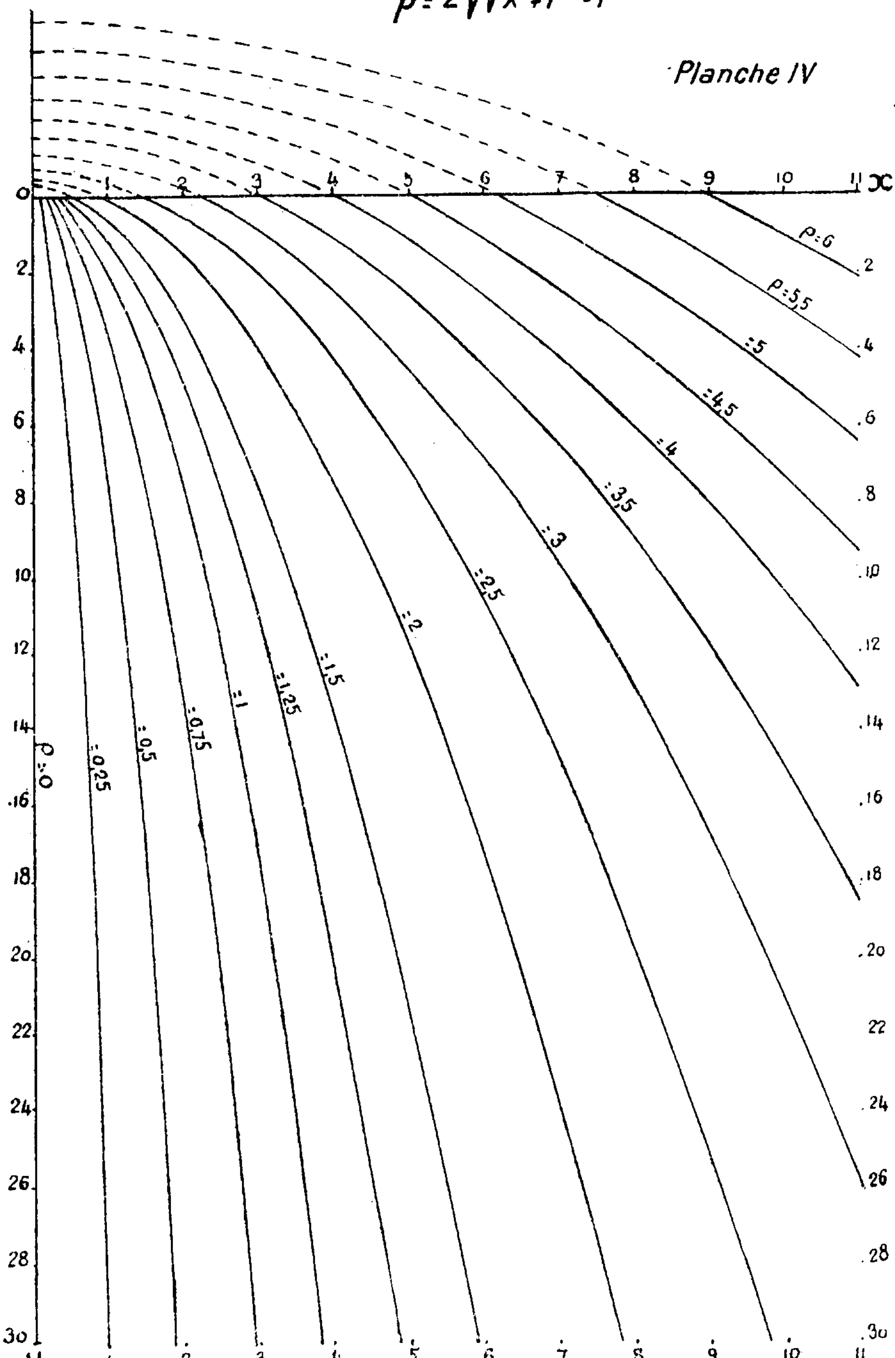


Planche III.

Graphique donnant les valeurs du paramètre

$$\rho = 2\sqrt{\sqrt{X^2+Y^2}-Y}$$



L'échelle des longueurs verticales est la moitié de l'échelle des longueurs horizontales.

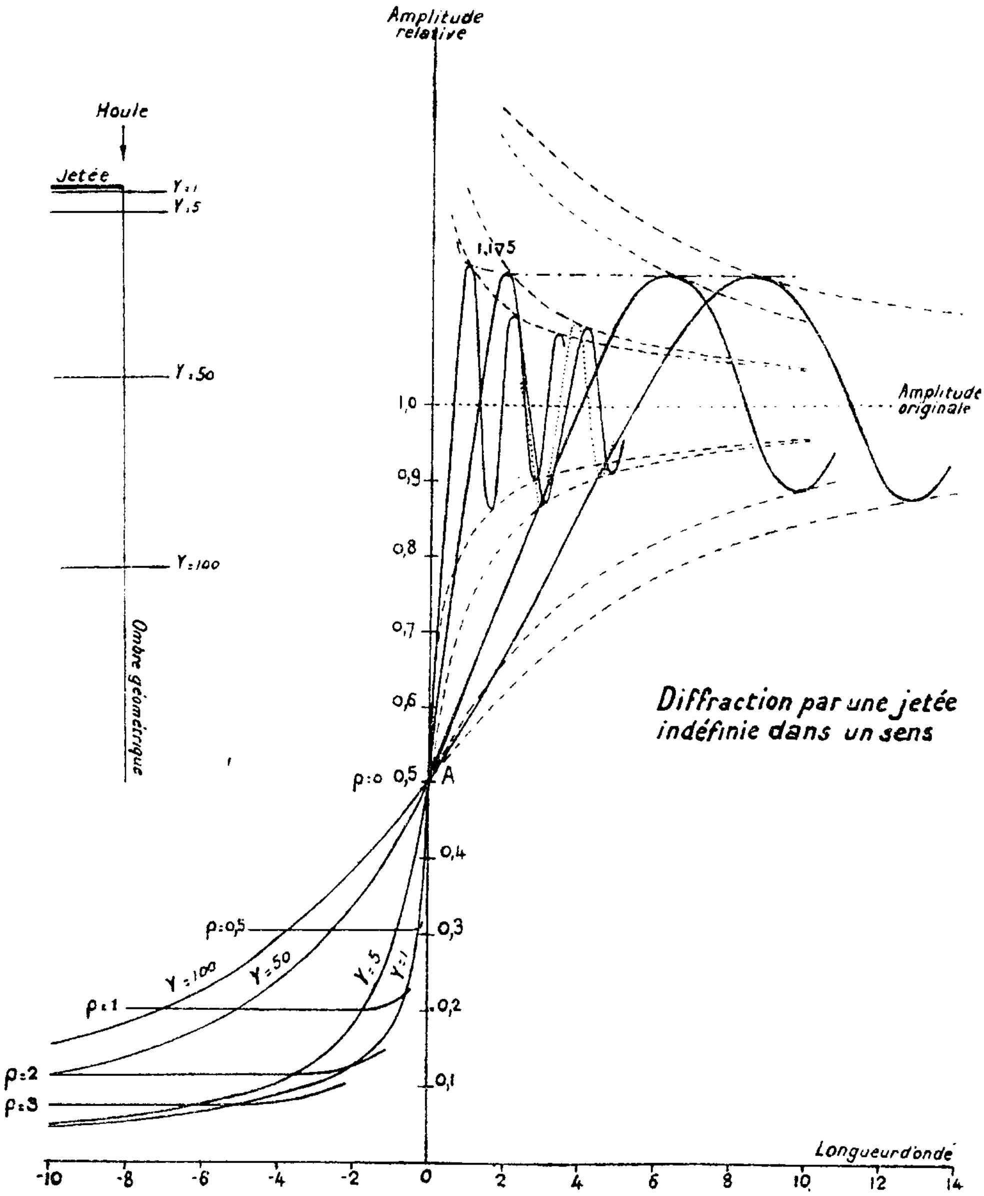

Amplitude relative
Houle
Jetée
Y:1
Y:5
Y:50
Y:100
Ombre géométrique
1,175
1,0
0,9
0,8
0,7
0,6
ρ:0 0,5 A
0,4
0,3
0,2
0,1
Amplitude originale
Diffraction par une jetée indéfinie dans un sens
ρ:0,5
ρ:1
ρ:2
ρ:3
Y:100
Y:50
Y:5
Y:1
Longueur d'onde
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14

Planche V.

Curves of Equal Amplitude.—Plate VI. shows lines of equal amplitude, of maxima and minima, and of shape of crests.

Considering, from the point of view of approximations which may be made in them, the three zones of figure 3, Mr. Lacombe shows that :

In zone 1 : the spirals do not depend on Y ; the constant amplitude curves are parabolae.

$$Y = \frac{2\,X^2}{\rho_0^2} - \frac{\rho_0^2}{8}$$

the origin of which is the focus, and the director straight line $Y = -\frac{\rho_0^2}{4}$, and to which different b amplitude values correspond according to whether the sheltered or unsheltered areas are involved (opposite values of ρ).

The maxima and minima of the amplitudes in the unsheltered area correspond to :

$$\frac{\pi}{2}\rho_0^2 = \frac{3\pi}{4} + n\,\pi$$ (maxima for even value of n) (minima for uneven value of n)

For the sheltered area the value of the amplitude is given by the spiral.

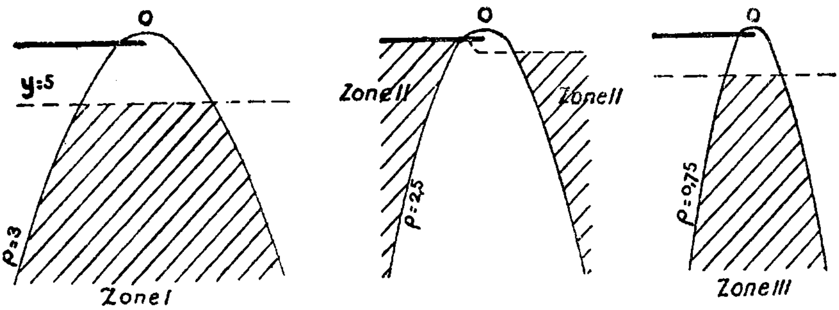


Fig. 3

In zone 2 : In the sheltered area, amplitude is obtained by the radius of curvature

$$\mathcal{R} = b = \frac{1}{2\pi \cos\varphi \sqrt{R_o}}$$

The b curves (b = constants) are then the curves :

$$R_o = \frac{1}{4\,\pi^2\, b^2 \cos^2\varphi}$$

for all points in zone 2.

It will be noted that the curve b = 0.05 diverges sharply from the path of the parabola corresponding to $\rho \approx 4.5$ approximately.

The loci of the maxima and minima outside the sheltered area are still parabolae which are not plotted through the edge of the pier-head (origin).

In zone 3 : the curves of equal amplitude are deduced from the spirals by plotting each point.

With regard to the *phase lag of the swell and form of the crests* :

1. In that half of zone 2 for which X is negative, with reference to the motion existing in the prolongation of the jetty the phase lag is :

$$\psi_0 = 2\,\pi\,R_0 + \text{constant}$$

which means that the crests in this zone are centred at the end of the pier-head.

2. Throughout the whole of the unsheltered area, i.e. lying to the right of the geometric shadow, the phase lag oscillates in the vicinity of $\frac{\pi}{4}$, a value which it actually has for values of ρ_0 equal to 0, and for values of ρ_0 giving minima and maxima amplitudes.

B.—Directional property of the elementary sources with regard to cos θ.

It is observed that there are, *outside the sheltered area,* fringes for the same values of ρ, therefore for the same values of X, Y being known. Their amplitude, however, is somewhat smaller, especially when the distance from the pier-head increases.

In the vicinity of the geometric shadow, the amplitude variation is practically the same.

In the area sheltered by the jetty the amplitude is systematically less.

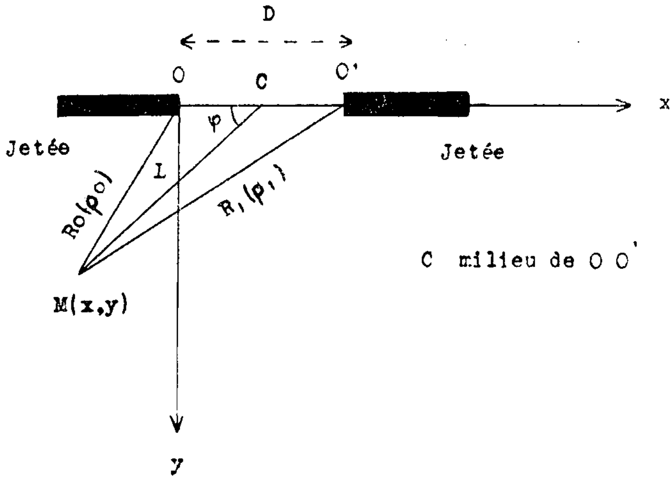


Fig. 7

II.—Second particular case. Limited opening in a jetty (perpendicular incidence) :

A.—No Directional property of elementary sources

Analysis of the Amplitudes.—1. When distance L from the opening is equal to or greater than three or four times its width :

Mr. Lacombe develops a very simple approximate formula for the amplitude :

$$b = \frac{D \sin(\pi . D \cos \varphi)}{\sqrt{L}. \ \pi D \cos \varphi}$$

i.e. according to the axial direction ($\cos \varphi = O$)

$$b = \frac{D}{\sqrt{L}}$$

He also gives, on plate IX for D = 3, a representation of the equal-amplitude lines. (Plate IX.)

2. When the distance is approximately $L < 4 D$:

(a) *Along the axis,* a minimum of the minima in the neighbourhood of 0.73 for a known value of approximately $\rho_0 = 1.9$, then a maximum in the vicinity of 1.36 for the value $\rho = 1.22$ are found.

Plate No. VIII represents amplitude variation along the axis for D = 3 obtained from the spirals, and as pecked lines, according to the approximate formula : $\frac{D}{\sqrt{L}}$.

(b) *Outside the axis,* for small values of L, the amplitude varies and may be subject to maxima and minima with a maximum value for maxima with respect to certain values of Y. This maximum value may be greater than the maximum amplitude value of maxima along the axis (for different values of Y).

Plate VII represents, for D = 3, amplitude variation curves over sections Y = Constant.

Analysis of Phase Lag of the motion.—Where L is large with regard to D, Mr. Lacombe finds approximate values of :

$$\psi = 2\pi(L - Y) - \frac{\pi}{2}$$

i.e. by referring this phase lag to the one existing at the straight line Y when there is no jetty.

$$\psi_0 = 2\pi L + \text{constant}$$

which means that the *crests are, quite far from the opening, centred on the axis of the latter.*

B.—**Directional property of the elementary sources with regard to cos θ.**

By similar deduction, Mr. Lacombe establishes, for a sufficient distance from the opening, the following expression for amplitude :

$$C = \frac{D \sin\varphi \sin(\pi D \cos\varphi)}{\sqrt{L}.\ \pi D \cos\varphi}$$

which gives the same directions for the cancellation of motion with, in addition, $\varphi = O$. Amplitude maximum has the same value, $\frac{D}{\sqrt{L}}$. The other maxima are given by :

$$\tan(\pi.D\cos\varphi) = \pi.D\tan\varphi\sin\varphi$$

which differ from the values obtained in the preceding case in ratio to the distances from the axis of the passage.

SHELTER OFFERED BY DETACHED JETTY AND GENERAL DEVELOPMENT PERPENDICULAR (NORMAL) INCIDENCE

The two components I and J of swell amplitude at point X, Y are obtained by :

$$I = I(\rho_1) - I(-\infty) + I(+\infty) - I(\rho_0)$$

$$J = J(\rho_1) - J(-\infty) + J(+\infty) - J(\rho_0)$$

obtained by integration through x' to O' then through O to x.

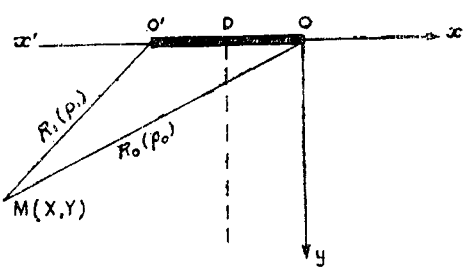


Fig. 9.

When Y exceeds a few units, we have :

$$I = I(\rho_1) - I(\rho_0) + \frac{1}{\sqrt{2}}$$

$$J = J(\rho_1) - J(\rho_0) + \frac{1}{\sqrt{2}}$$

For any number whatever of openings affected perpendicularly by the sea, and of whatever width, the components of motion are obtained by calculation of integrals I and J between the successive heads.

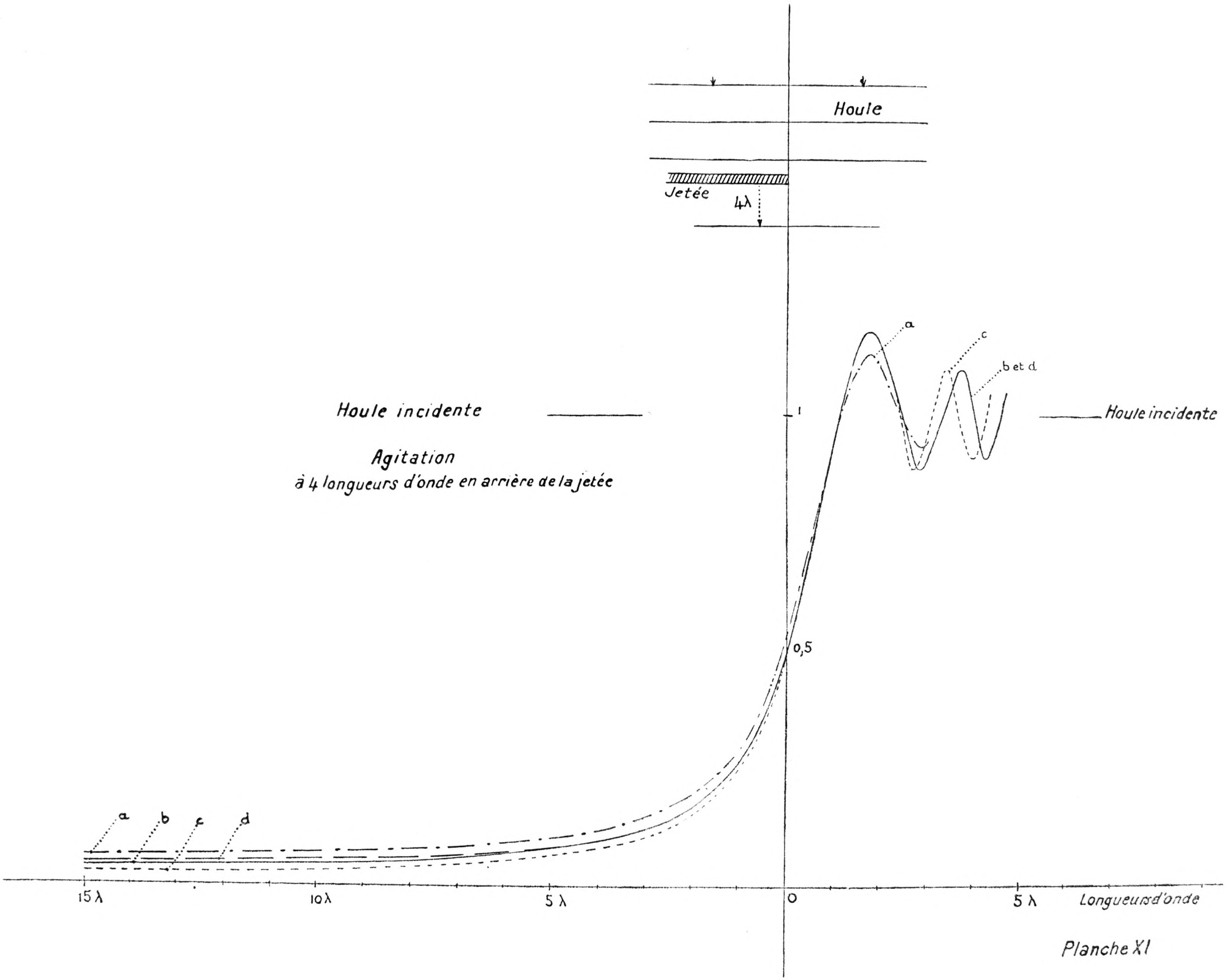

Houle
Jetée
4λ
Houle incidente
Agitation
à 4 longueurs d'onde en arrière de la jetée
a
c
b et d
1
Houle incidente
0,5
a
b
c
d
15 λ
10 λ
5 λ
0
5 λ
Longueurs d'onde

Planche XI

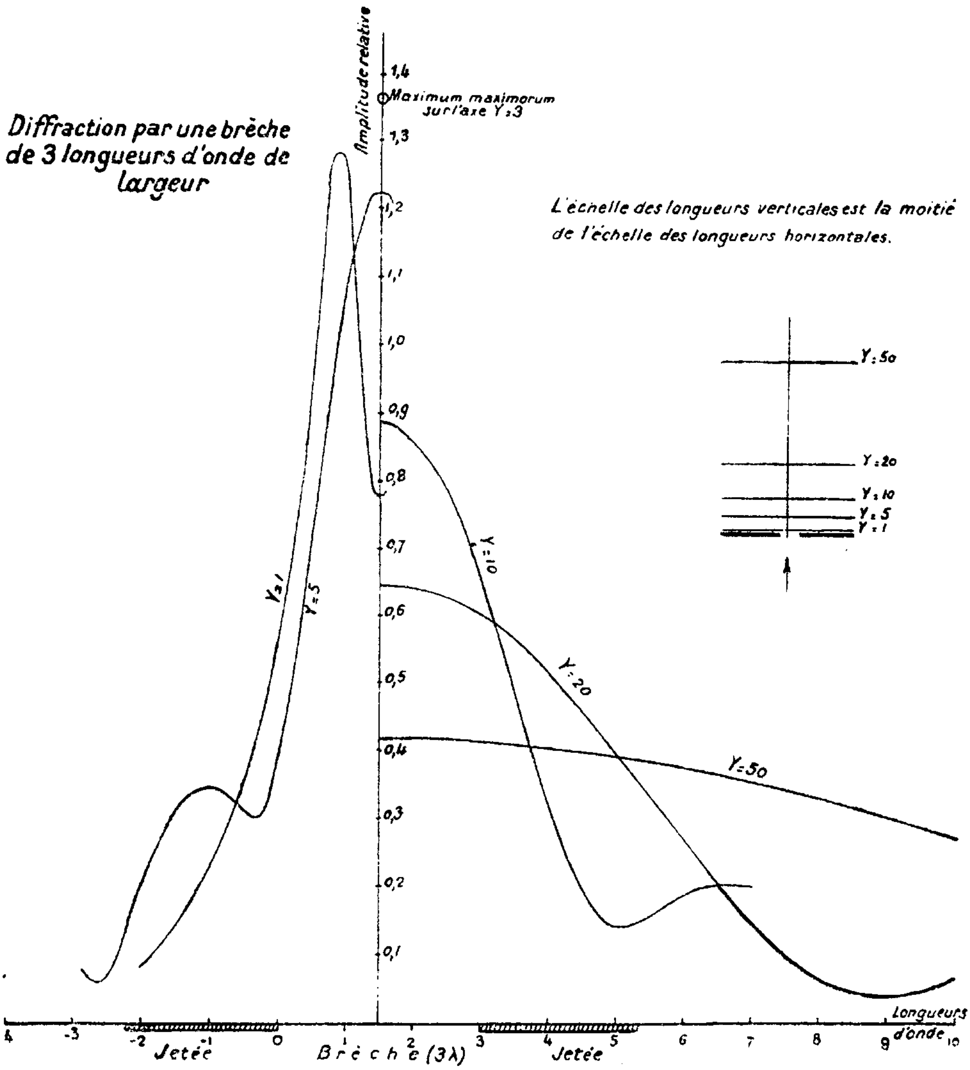

Diffraction par une brèche de 3 longueurs d'onde de largeur
Amplitude relative
Maximum maximorum sur l'axe Y:3
L'échelle des longueurs verticales est la moitié de l'échelle des longueurs horizontales.
1,4
1,3
1,2
1,1
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
Y:50
Y:20
Y:10
Y:5
Y:1
Y=1
Y=5
Y=10
Y=20
Y=50
Longueurs d'onde
4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
Jetée
Brèche (3λ)
Jetée

Planche VII.

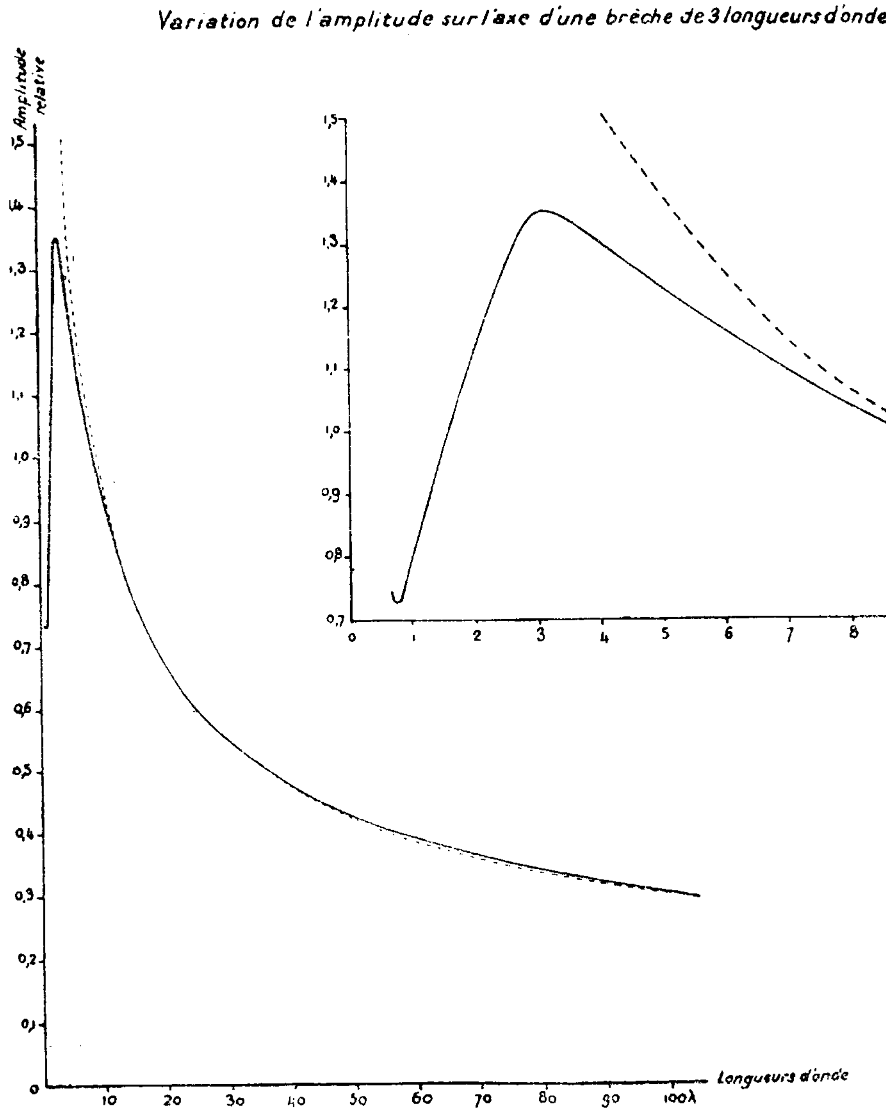


Planche VIII

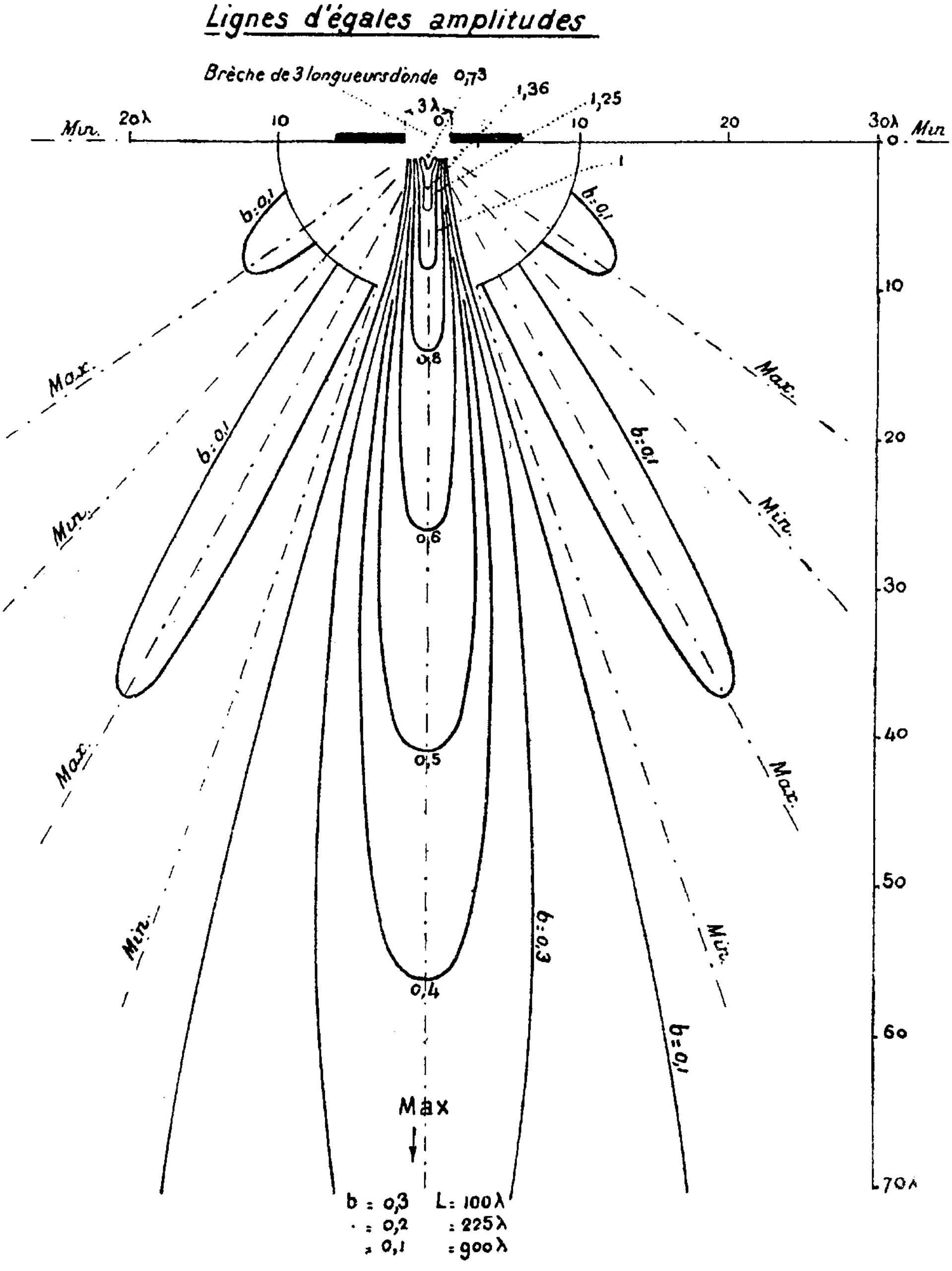

Lignes d'égales amplitudes
Brèche de 3 longueurs d'onde
0,73
1,36
1,25
3λ
0,1
1
Min.
20λ
10
10
20
30λ
Min
0
b:0,1
b:0,1
Max.
Max.
b:0,1
b:0,1
Min.
Min.
Max.
Max.
Min.
Min.
0,8
0,6
0,5
0,4
b:0,3
b:0,1
Max
10
20
30
40
50
60
70λ
b = 0,3 L: 100λ
" = 0,2 : 225λ
" 0,1 : 900λ

Planche IX.

ANALYSIS AND CONCLUSION

The above assumptions have all been made, and especially in the immediate vicinity of the jetties, under conditions of constant depth, i.e. involving no refraction.

Mr. Lacombe lists the solutions as follows :

(a) *Correct solution :* the hydrodynamic solution deduced from Sommerfeld and mentioned in the American note of Putnam and Arthur.

(b) *Simplified solution :* the solution derived from it by making the approximations indicated in the above-mentioned note.

(c) *Optical solution :* obtained by applying the gross results of optical analogy.

(d) *First solution now developed :* the first possible solution offered by Mr. Lacombe in his article, assuming no directional property of the elementary sources.

(e) *Second solution now developed :* the second possible solution briefly set forth by him, assuming a directional property of the elementary sources with regard to $\cos \theta$.

The two solutions a and b are valid :

1. In case of oblique as well as in case of perpendicular incidence ;
2. Only, however, for a jetty lying along half a straight line of indefinite length.

Solutions c, d, and e are applicable :

1. Only in perpendicular incidence ;
2. Also, however, for any combination of openings and passages perpendicularly affected by swell.

Comparing the amplitude values given by the different solutions, *for a jetty lying along half a straight line of indeterminate length,* and for various areas relating to the jetty, Mr. Lacombe demonstrates that *on an average, solution d is nearer solution a than all other solutions* (the discrepancy being no greater than 3 per cent of original amplitude) and that *the five solutions lead to almost similar expressions near the geometrical shadow.* Discrepancies appear only in the case of very oblique rays.

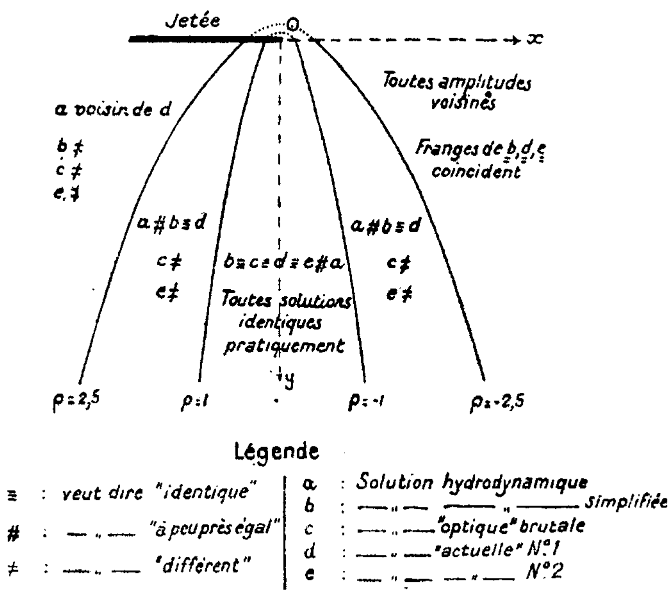


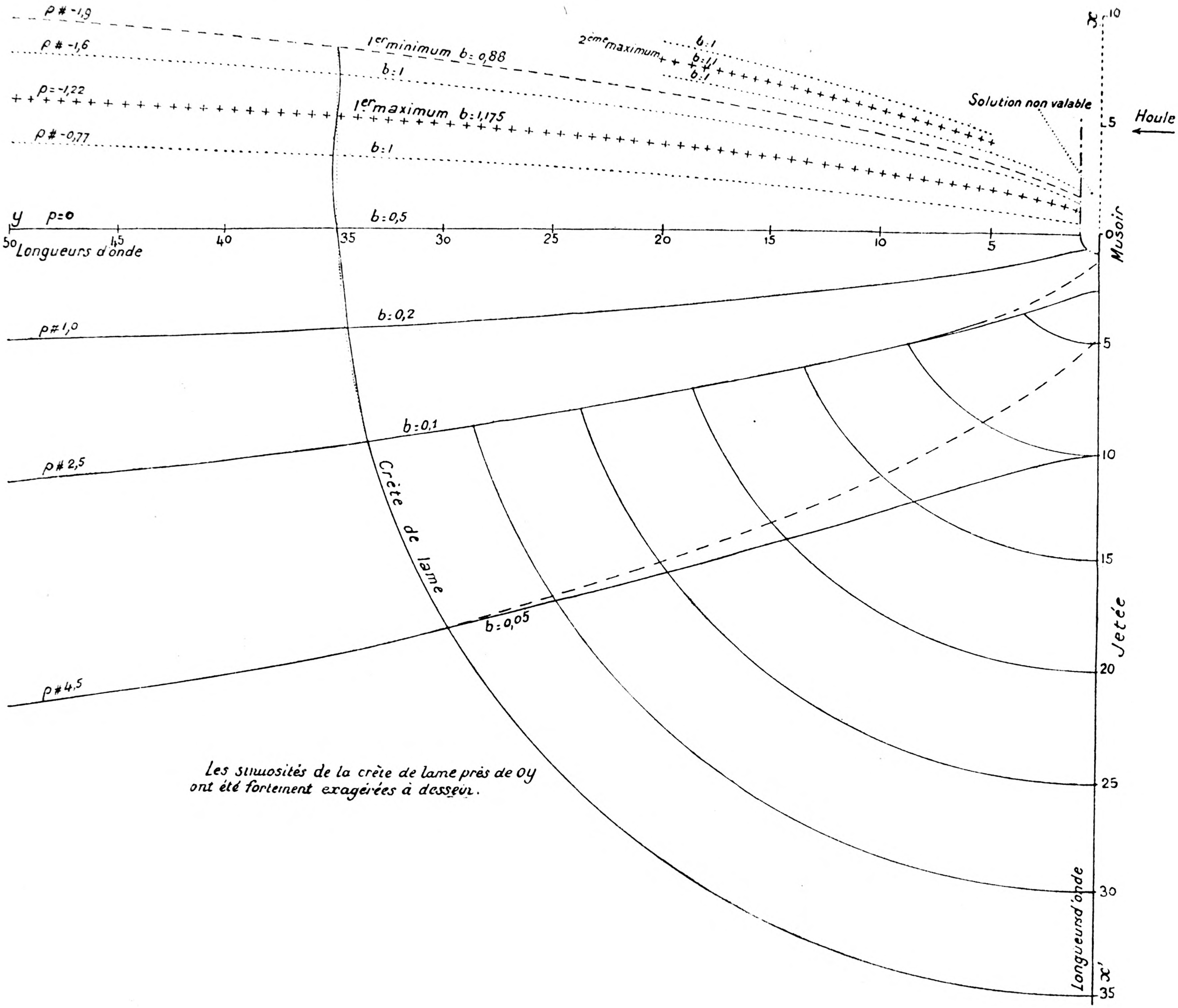
Fig. 11.

Figure 11 gives a résumé of the development and plate XI illustrates the discrepancies for $Y = 4$.

Mr. Lacombe concludes that the interest of his solution d resides in the following points :

Diffraction par une jetée indéfinie dans un sens

Lignes d'égales amplitudes, de maxima et minima d'amplitude et forme des crêtes



Pl. VI, p. 30

— Contrarily to solutions a and b, it is very simple to apply, the spirals being plotted once for all ;

— It is valid in perpendicular incidence only but is applicable to any system of openings and jetties ;

For a single passage, it offers an expression with respect to relative amplitude which *fully resolved is of extremely simple application.*

In concluding, Mr. Lacombe points out that, besides its practical interest, the fact that the application of Huyghens' principle produces results nearly analogous to those of solution a, shows that the principle interprets to some extent the laws of the mechanics of fluids.