THE ANALYSIS OF TIDAL PHENOMENA IN NARROW EMBAYMENTS.

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This work is an attempt to treat the tidal behavior of coastal embayments in such a way that the observed changes in elevation and motion of the water along the path of the wave are used to determine the distribution of phase of the primary and reflected waves along the channel and to measure the damping.

It may be summarized as follows.

1° On the assumption that the tides of embayments are the resultants of a primary progressive wave and its reflected counterpart, both undergoing damping, the following equations are developed relating the tidal range, time of high water, time of slack water, and phase difference of the primary and reflected waves along the channel, for given coefficients of damping :

In a uniform channel the elevation of the primary wave is given by :

$$\eta_1 = A \cos(\sigma t - Kx)\, e^{-\mu x}$$

and that of the reflected wave by :

$$\eta_2 = A \cos(\sigma t + Kx)\, e^{\mu x}$$

where : A is the amplitude of each of the waves at the barrier ; σ the change in phase per unit of time ; t the time measured from the time of high water at the barrier when $t = 0$; K the change in phase per unit of distance ; and x the distance measured from the barrier where $x = 0$; μ the damping coefficient.

The elevation of the water at any time and place is given by $\eta = \eta_1 + \eta_2$.

Thus the time of high water is deduced by equalling to zero $\frac{\delta \eta}{\delta t}$. Which gives :

$$\sigma t_H = \tan^{-1}(-\tan Kx \tanh \mu x) \qquad (1)$$

In the same way the ratio $\frac{\eta}{\eta_0}$ of the elevation at high water will be :

$$\eta/\eta_0 = \sqrt{\frac{1}{2}\left(\cosh 2\mu x + \cos 2Kx\right)} \qquad (2)$$

η_0 being the elevation of high water at the barrier. Besides, times of slack water at any point may be found by equalling to zero the addition $U_1 + U_2$, of velocities of current of the primary and reflected waves. Which gives :

$$\sigma t_s = \tan^{-1}\left(\frac{\tanh \mu x}{\tan Kx}\right) - \alpha \qquad (3)$$

where $\alpha = \tan^{-1} \frac{\mu}{K}$

The times of maximum current will precede or follow slack water by one quarter of period or 90°.

2° These equations may be applied to narrow embayments of simple form in which tidal currents are not rotatory, provided it is further assumed that the effect of irregularities in the cross section of the channel is merely to alter the velocity of propagation of the primary and reflected waves (i. e. to distort the geographical distribution of phase differences) ; that damping is proportional to the phase change in the primary and reflected waves (rather than to the distance traveled) ; and that the damping coefficient is constant along the length of the channel.

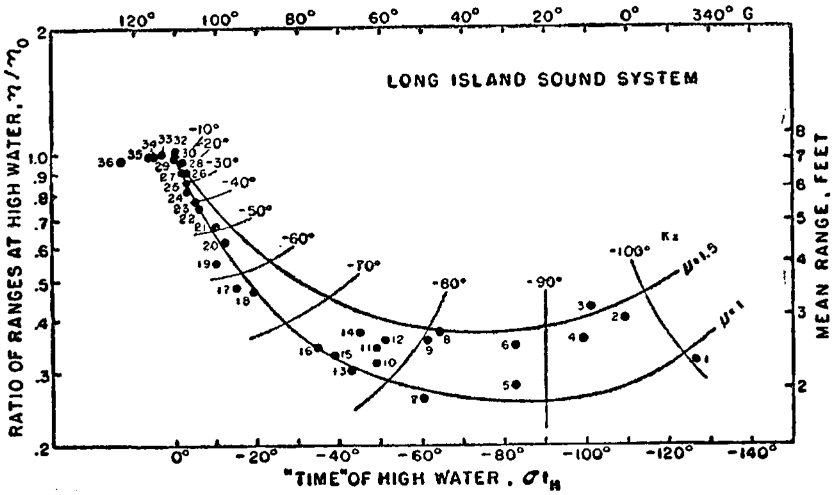
The procedure is the following.

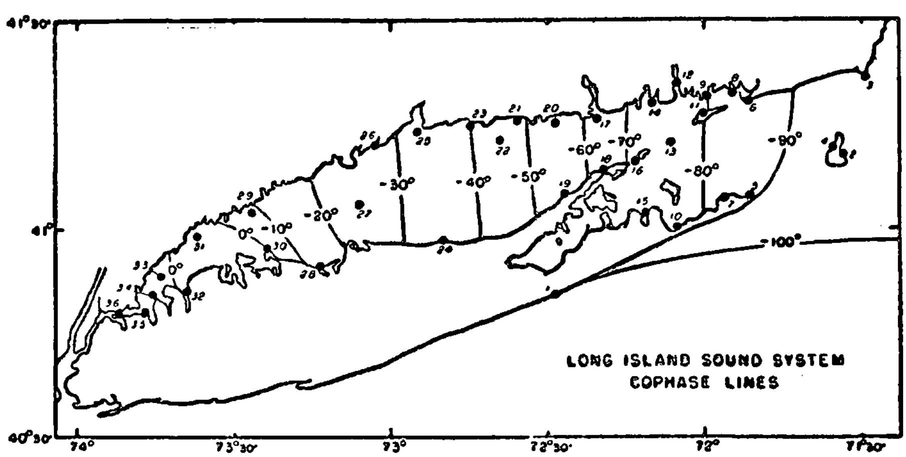
From equations (1), (2), (3), three nets of curves may be plotted showing respectively the time angle of high water, the ratio of high water, and the time angle of slack water with respect to phase difference K $\varkappa$, for different values of μ. By combining the curves defined by (1) and (2), a graph is constructed in which the observed variables (ratio of high water elevation, and the local time angle of high water) are represented by rectangular co-ordinates and the desired properties of the primary wave (phase relations and the coefficient of damping) are represented by a series of curves. This normogram being independent of the actual dimensions of the basin and period, it is consequently a simple matter to plot a series of tidal data giving elevation and times of high water on co-ordinate systems similar to that of the curves, and by superposing the curves to determine whether a satisfactory fit can be obtained.

In an entirely similar way the equations (1) and (3) are combined and a graph obtained in which the local time angle of high water is related to that of slack water.

This procedure is applied to the analysis of Long Island Sound, the Bay of Fundy and the Juan de Fuca-Georgia Straits system.

Abstract of the analysis of the Long Island Sound system appears hereunder.





The best fit of the data to the co-ordinate system is obtained by assuming that reflection occurs from a region extending from Eatons Point to Glen Cove (Stations 30 — 32) where the mean tidal range is maximum at 7.2 to 7.4 feet. Observations inside Long Island Sound proper, i. e. from —70° cophase line fall closely along the co-ordinate for the damping coefficient $\mu = 1.0$ Anomalous increase in range shown in stations 3, 6, 8, 9, 11, 12 — 2, 4, 14, may be explained by the rotation of the earth, and reflection from the coast between Block Island and No Man's Land. The conditions west of Glen Cove are also anomalous. They are evidently due to escape of water from the Sound through the East River.

The damping coefficient of the system is about 1.0, the velocity of the primary wave is about 27 knots. The properties of the wave may be traced seaward through a phase difference of 101°.

3° The three analyses dealt with indicate that the assumption underlying the equations are sufficiently valid to permit the detailed description of these systems in terms of the distribution of phase differences for the primary and reflected waves along the channel, the velocity of their propagation, and the coefficient of damping.

4° The damping coefficient of navigable embayments is large, representing an attenuation of two-thirds or more per cycle.

5° The departures of tidal behavior in embayments from the behavior of ideal standing waves receive rational and quantitative interpretation.

6° The limits placed by damping on the augmentation of the tidal wave by resonance are quantitatively defined.

7° It is suggested that exceptional tidal ranges observed in such embayments as the Bay of Fundy are due to two or more stages in amplification by the combined effects of reflection and resonance in systems successively tributary to one another and to the ocean.