COMPARATIVE PRECISION OF PRISMATIC ASTROLABES

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1. Introduction.

The precision of an astronomical position obtained from observations with the various types of prismatic astrolabes is always a subject open to discussion, but we think there is one method of drawing some useful conclusions regarding the matter. It is obvious that we must compare the various types of instruments by considering the magnifying power of the telescope in each and the number of times the same star can be observed.

H. Faye, in his work *Cours d'Astronomie*, gives a formula, from which the probable error in time seconds can be ascertained in the observation of a star which crosses the thread of the telescope of a transit instrument. To compute weights for the position lines, Claude and Driencourt, inventors of the prismatic astrolabe, considered it similar to the transit instrument and from the Faye formula derived another formula expressing the weight of a position line. Though this formula would be useful in making a comparative study of the various prismatic astrolabes, we have chosen another method eading to more convincing conclusions.

2. Theoretical comparison.

As we know, a position line is determined by the azimuth of the observed star, and by the difference between the zenithal distance z, computed in terms of the observed time and the true zenithal distance z_0. If we call d this difference, we have :

$$d = z - z_0 \tag{2a}$$

The mean error m_d of a single observation for d, will be obtained from

$$m_d^2 = m_z^2 + m_{z_0}^2 \tag{2b}$$

where m_z and m_{z_0} are, respectively, the mean errors of a single observation of z and z_0. Now, as the normal visual acuity is about 60″, we can assume that 60″ will be the largest error to be committed in the measurement of an angle, without use of the telescope, as the observer's eye will notice any shift larger than 60″. If the same angle is observed with a telescope of magnifying power G, it is obvious that the largest error committed in the observation will be 60″/G, and in a series of observations, all the errors will lie between zero and ± 60″/G. As in a prismatic astrolabe we see two images moving in opposite directions, with a relative velocity twice the zenithal velocity of the star, it follows that we have the sensation of looking through a telescope with a magnifying power $2G$. Then we conclude that the errors committed in observations with prismatic astrolabes, will be comprised between zero and ± 30″/G. Now, if all observations have the same weight, the probability of committing an error comprised between zero and 15″/G (neglecting the signs), will be 50%. Then, ± 15″/G is the probable error of a single observation. To obtain the mean error of a single observation, it suffices to divide this value by 0.6745, so that

$$m_{z_0} = \pm \frac{22{,}24}{G} \tag{2c}$$

Moreover, the mean error m_z is the mean error in z which results from the one committed in the time observation. If we use an electrical chronograph as timekeeper, we can assume a mean error of ± 0s.1. The mean error m_z can be expressed by

$$m_z = 15 \frac{\delta z}{\delta t} m_t$$

where (m_t) is the mean error of the time observation, given in time seconds, and $\delta z / \delta t$ will be obtained from the partial derivative of

$$\cos z = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos t$$

where φ, δ and t are, respectively, the latitude, the star declination, and the hour angle of the star. So we have

$$\frac{\delta z}{\delta t} = \sin A \cos \varphi$$

and it follows that, for $m_t = \pm$ 0s. 1.

$$m_z = \pm 1.5 \sin A \cos \varphi \tag{2d}$$

Now, we have from (2b), (2c) and (2d)

$$m_d^2 = \frac{495}{G^2} + 2.25 \sin^2 A \cos^2 \varphi$$

which gives the square of the mean error of a single observation for a position line. If we have n observations of the same star, the weight of the result will be

$$p_d = \frac{n}{m^2} = \frac{1}{\frac{495}{nG^2} + \frac{2.25 \sin^2 A \cos^2 \varphi}{n}} \tag{2e}$$

We can compute with this formula the weight for any position line obtained from observations with any kind of prismatic astrolabe. For the French 60° S.O.M. astrolabe we have G = 80x and n = 1, and for the British Cook 45° astrolabes we have : for the large model, G = 36x and n = 8, and for the small model, G = 25x and n = 6. Therefore, we have respectively :

French S.O.M. 60° astrolabe :

$$p_d = \frac{1}{0.077 + 2.25 \sin^2 A \cos \varphi} \tag{2f}$$

British Cook 45° — large model :

$$p_d = \frac{1}{0.048 + 0.281 \sin^2 A \cos \varphi} \tag{2g}$$

British Cook 45° — small model :

$$p_d = \frac{1}{0.132 + 0.357 \sin^2 A \cos \varphi} \tag{2h}$$

Each one the above formulae can be represented by a curve in which the ordinates represent the weight and the abscissae the values of sin A cos φ. By inspection of these curves (Fig.2A), we can reach the following conclusions :

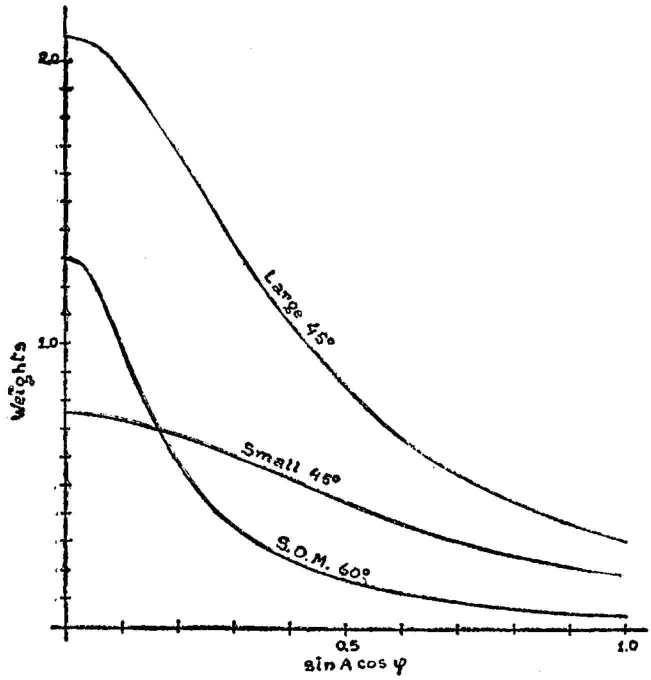


Fig. 2A

a) For the same azimuth, observations with the large model of the 45° prismatic astrolabe are always more accurate than those made with the S.O.M. ;

b) The relationship between the maximum and minimum weights is 30.2 for the S.O.M., 6.9 for the large model of the Cook astrolabe, and 3.7 for the small model ; and so we see that observations with the latter are the most homogeneous ;

c) Position lines obtained from observations with the small 45° prismatic astrolabes are less accurate that those obtained from observations with the S.O.M. 60° astrolabes, for a very restricted band of azimuths.

It should be pointed out that it is not advisable to observe, with the Cook 45° astrolabe, stars with azimuths NE, NW, SE or SW not exceeding 25°, otherwise a large second order correction — if not one of higher order — would be necessary. Actually it is a limitation that makes the comparison of the instruments difficult. To obviate this trouble, we think it would be sufficient to compare the astrolabes in such a way that all of them could give good results for determining a position by the Saint-Hilaire method. Now, as we know, a good position can be determined by groups of position lines at right angles, and these groups can be formed from observations of stars distributed at azimuths 45°, 135°, 225° 315°. As may be seen by inspection of formulae (2f) to (2h), weights vary with the latitude and we are therefore forced to particularize the comparison for a definite value of φ. Then, as the second term in the denominator of (2h) varies with φ, more than its correspondents in the two other formulae, we have chosen an approximate value of 34°S for the southern latitude of Brazil. For these values of A and φ we have

For the S.O.M. 60°	P*d* = 1.176
For the large 45°	P*d* = 6.197
For the small 45°	P*d* = 3.922

These figures show that for $\varphi = 34^o$ — if we observe stars near the chosen azimuths — the British astrolabes are better than the French. Then if we observe 48 stars with the S.O.M. astrolabe, the relationship between the weight for this instrument and the weights for the other two will show that an equally accurate result will be obtained by observing only about 8 stars with the Cook large model astrolabe and about 16 with the small model of the same maker.

3. Second order correction.

The second order correction to the observations with the British prismatic astrolabes, computed with the aid of a graph printed by the British Hydrographic Department, is given, we think, with some lack of accuracy. Interpolating by inspection is hard to do in this graph. The correction is given, as we know, by the product

$$- C \times K \qquad (3a)$$

where C is a function of instrumental constants and the number of times the same star is observed, and K can be computed by the formula

$$K = \text{cotg}^2 A - \tan \varphi \text{ cotg } A \text{ cosec } A$$

represented by the Cartesian graph issued by the Hydrographic Department. Now this formula can be changed to

$$\text{cotg}^2 A = \tan \varphi \text{ cotg } A \text{ cosec } A + K$$

which is of the form

$$f(x) = F(y)\, \Psi(x) + \Phi(z)$$

suitable for nomographic representation. We have therefore constructed a nomograph which is easier to use and which gives more accurate results than the Cartesian graph. Fig.3A shows this nomograph, to be used at the discretion of the reader.

4. Refraction.

Ever since it was invented by Captain Baker, the 45° prismatic astrolabe has suffered severe criticism because the amount of refraction variation with atmospheric conditions is larger for a 45° altitude than for 60°. It is therefore advisable to correct observations with 45° prismatic astrolabes for the effect of temperature and barometric pressure variations. In field astronomy, the actual refraction r, can be expressed in terms of the mean refraction r_m by

$$r = r_m . k$$

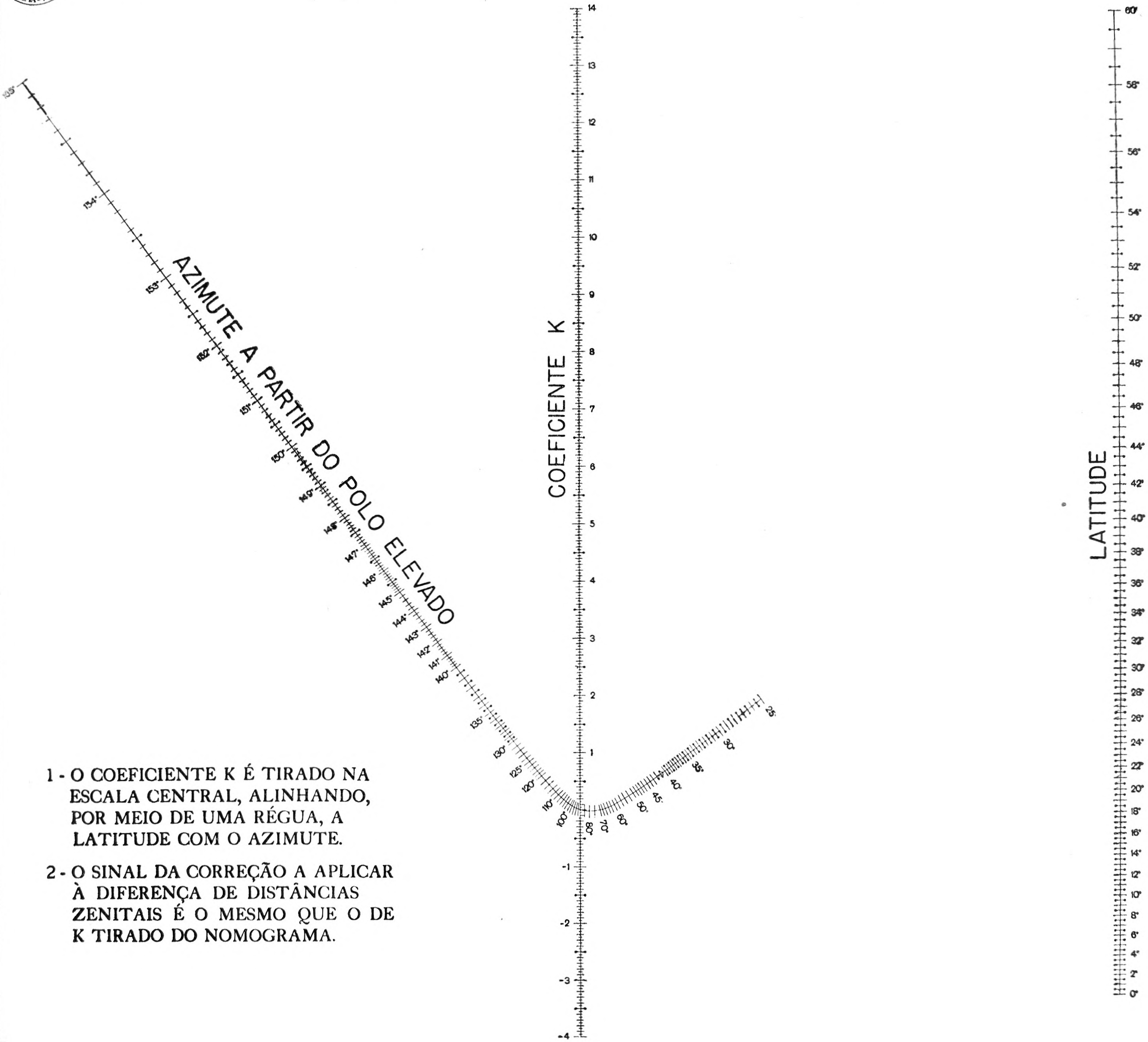
where k is a function of temperature and barometric pressure. It is obvious that if we neglect the correction — as we do with the French astrolabe — an error will be introduced that is expressed by

$$\Delta r = r_m (k - 1)$$

If we take from the *Anuário do Observatôrio Nacional* ((Brazilian Astronomical Ephemeris) the values of r_m for 60° and 45° altitudes we have, respectively, 32″.2 and 55″.7. Then, we conclude that if temperature and barometric pressure correction is neglected, the error of an observation with the 45° astrolabe will be 55″.7/32″.2, twice the error of an observation, under the same circumstances, with the 60° astrolabe. To reduce this error, the *Admiralty Manual of Hydrographic Surveying* recommends the observation of temperature and barometric pressure, necessary for the computation of the correction.

At the Brazilian Hydrographic Service we have a simple nomograph to obtain, without any computation, Δr in terms of temperature and barometric pressure.

CORREÇÃO DE 2ª ORDEM PARA O ASTROLABIO DE 45°



1 - O COEFICIENTE K É TIRADO NA ESCALA CENTRAL, ALINHANDO, POR MEIO DE UMA RÉGUA, A LATITUDE COM O AZIMUTE.

2 - O SINAL DA CORREÇÃO A APLICAR À DIFERENÇA DE DISTÂNCIAS ZENITAIS É O MESMO QUE O DE K TIRADO DO NOMOGRAMA.

FIGURE 3A

Second order correction for 45° astrolable
Azimuth from elevated pole. K coefficient. Latitude.

1. Coefficient K is read on the central scale by joining latitude to azimuth with a straight line.
2. Sign of correction for zenith distance difference is the same as for K as deducted from nomogramme.

5. Conclusion.

Though the theoretical comparison makes us very confident as to the results that can be attained with the various kinds of instruments, we believe that they should be confirmed by practical comparison on the part of experienced observers. Before this is done, it hardly seems advisable to give a definite opinion regarding the instruments referred to herein.