THE NEED FOR COORDINATION OF GEOGRAPHICAL GRIDS OF THE WORLD, IN RELATION TO POSITION-FIXING BY RADIONAVIGATION SYSTEMS

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(October 1951)

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§ I. — INTRODUCTION

At the Vth International Hydrographic Conference (1947) the following resolution was adopted :

Coordination of geographical grids of the world

« The conference recommends that the Directing Committee of the International Hydrographic Bureau get in touch with the International Union of Geodesy and Geophysics, to specify the needs of the Hydrographic Offices, and to offer such cooperation as may be possible in finding the best possible means of making and reducing observations for obtaining the absolute geographic coordinates of points on the globe, with the highest possible standard of accuracy ».

As a result of this resolution the President of the Directing Committee, Vice-Admiral J. D. Nares, D.S.O., R.N., has put forward this question at several international meetings; there was for instance a quite elaborate discussion on this subject at the General Assembly of the I.U.G.G. at Oslo in 1948 and the matter has also been discussed at this Union's General Assembly at Brussels in 1951.

To the best of my knowledge, however, a very important question has not yet been answered. This question is :

What order of accuracy in the coordination of geographical grids do the Hydrographers need ?

The aim of this paper is :

I To give an answer to this question.

II To review the methods to obtain the coordination to the desired degree of accuracy.

Any point on earth as well as any point of any track between two points on earth is fixed by 3 coordinates :

φ = geographical latitude.

λ = geographical longitude.

h = height above or below mean sea level.

If we denote the coordinates, determined by a ship or aircraft by (φ), (λ) and (h), it follows that the craft will be able to keep on any predetermined track, when all the way along this track :

$$\varphi = (\varphi)$$
$$\lambda = (\lambda)$$
$$h = (h)$$

The job of the chart-maker is for determining and to plot on a chart the coordinates φ, λ, h.

The job of the navigator is for determining the coordinates (φ), (λ) and (h), to plot them on his chart, and to draw conclusions as to the deviation from his predetermined track as plotted on the chart.

There are many methods to determine φ, λ and h and (φ), (λ), and (h) and it is obvious that the requirement in chart-making is to determine the coordinates φ, λ and h with an accuracy at least as high as that attainable by the navigator in determining (φ), (λ) and (h) *under the most favourable circumstances and with the best commercially available navigation techniques.* To avoid accumulation of errors and as a safeguard against further development of navigational methods, the accuracy in φ, λ and h should preferably be somewhat in excess of the accuracy in (φ), (λ) and (h).

Until the introduction of modern radionavigation systems, it was comparatively easy for the chart-maker to fulfil this requirement. At the present time, however, these modern navigation systems allow a much increased accuracy in determining (φ) and (λ) and as a result many existing nautical charts no longer meet the requirements of accuracy, because in many cases they have to be compilations of surveys whose triangulations have not been geodetically connected.

The first step in this investigation should be to make an estimate of the accuracy of determination of position by means of modern radio-navigation systems ; there are many such systems. However, hyperbolic systems are the systems most suitable for long range, as well as medium range and short range navigation. Although the accuracy of different systems need not be the same, the mathematical problems in investigating their accuracy are basically the same for systems employing pulse (e. g. Loran and Gee) or continuous wave = C. W. transmission (e. g. Decca and Radux). As pulse systems — contrarily to C. W. systems — occupy a very broad band in the already overcrowded radiospectrum, future development appears to be on C. W. systems. The system to be discussed in this paper is Decca, being a C. W. phase comparison system.

§ II. — HYPERBOLIC POSITION LINES

2.1. General

A radio receiver is designed to measure and to indicate the difference in length of the transmission paths of radio signals transmitted by two or more transmitters.

Any constant difference in distance results in a line of position. On a flat earth this position-line would be a hyperbola. On the chart representation of the ellipsoid it is a line of complicated curvature, which — although mathematically not correct — is usually also referred to as a hyperbola.

The readings on the dials of the navigator's instrument consequently do give hyperbolic coordinates. As the navigator, however, needs geographical coordinates, these hyperbolic coordinates have to be converted into geographical ones. For this purpose there are two methods :

a) appropriate tables ;

b) latticed charts.

The method b) is most commonly used.

Tables as well as lattices have to be computed beforehand ; the same method of computation applies to both.

2.2. Formulæ for the computation of Decca-lattices

Decca hyperbolae for a 3-slave chain are computed by formulae (1) or (3).

$$\left.\begin{aligned} L_R &= \frac{F_R}{V}\left(b_R + l_M - l_R\right) \\ L_G &= \frac{F_G}{V}\left(b_G + l_M - l_G\right) \\ L_P &= \frac{F_P}{V}\left(b_P + l_M - l_P\right) \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots (1)$$

L_R, L_G, L_P = « lane-numbers » Red, Green and Purple

= lattice lines (hyperbolae) with a fixed difference in distance.

F_R, F_G, F_P = comparison frequencies Red, Green and Purple.

V = phase velocity.

b_R = | distance, travelled by the radio-wave from Master to Red slave

b_G = | distance, travelled by the radio-wave from Master to Green slave

b_P = | distance, travelled by the radio-wave from Master to Purple slave

l_M, l_R, l_G, l_P = distance, travelled by the radio-wave from {Master, Red slave, Green slave, Purple slave} to observer.

F should be expressed in cycles/second.

V (per second), b and l should be expressed in the same unit ; the unit usually chosen is the metre.

As the number of lanes in a Decca pattern is

$$n = \frac{2bF}{V} \qquad \ldots\ldots\ldots\ldots (2)$$

formulae (1) may be written as follows :

$$\left.\begin{aligned} L_R &= \frac{n_R}{2} + \frac{F_R}{V}\left(l_M - l_R \right) \\ L_G &= \frac{n_G}{2} + \frac{F_G}{V}\left(l_M - l_G \right) \\ L_P &= \frac{n_P}{2} + \frac{F_P}{V}\left(l_M - l_P \right) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (3)$$

The formulae (1), (2) *and* (3) *are exact.*

In order to lattice a chart, the procedure is as follows : L_R , L_G and L_P are computed for a great number of intersections of parallels and meridians all over the coverage area of the chain. « Whole numbers » (= the hyperbolae with zero phase difference) along a parallel or a meridian are obtained by inverse interpolation and are plotted on the chart to be latticed. Finally the hyperbolae are drawn with the aid of splines. Errors in plotting and drawing are dependent on the scale of the chart and can be kept within the normal drawing error of 0.2 millimetre.

As formulae (1) or (3) are exact, the lattices thus computed and plotted on the chart, will be identical with the patterns actually radiated, when F, V, b (or N) and l are correctly known. In actual practice however these data will be known only within certain limits of accuracy and as a result, there will be a discrepancy between the radiated pattern (being the pattern in which the receiver measures a difference in distance) and the chart lattice. Moreover the observations themselves will be subject to instrumental errors.

§ III. — EFFECT OF ERRORS

3.1. General

In most cases formula (3) is to be preferred when a new Decca Chain is set up, because n is a quantity that can be obtained from observations, while b, (in formula (1)) possibly will be known only with an accuracy less than the accuracy attainable in n. As an example the effect of errors will be investigated for the *Red* pattern.

$$L_R = \frac{n_R}{2} + \frac{F_R}{V}\left(l_M - l_R \right) \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

L_R = hyperbola-number in radiated pattern.

$$L^1_R = L_R + dL_R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

L^1_R = hyperbola-number in computed pattern.

dL_R = combined influence of errors in the asumed values of n_R, F_R l_M and L_R

$$L''_R + dL''_R = L_R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

L''_R = hyperbola-number as abserved

dL''_R = influence of instrumental errors in the receiver.

As we are interested in the discrepancy between a computed hyperbola (as printed on the chart) and the observed hyperbola, (6) should be subtracted from (5) :

$$D = L^1_R - L''_R = dL_R + dL''_R$$

After differentiating formula (4) we find :

$$D = \frac{1}{2} dn_R + \left(\frac{l_M - l_R}{V}\right) dF_R - \left[\frac{F_R (l_M - l_R)}{V^2}\right] dV + \left(\frac{F_R}{V}\right) d\left(l_M - l_R\right) + dL''_R \quad (8)$$

The errors d may be :

1) of a systematic character, to be denoted by Δ
2) of a random character, to be denoted by m.

3.2. Evaluation of n_R

n_R is the number of lanes, actually existing in the radiated Red pattern : n_R may be obtained from observations at both base-line extensions. As this observed value of n_R is used as a constant in the computation of L^1_R it follows that an error in n will have a systematic effect on L^1_R.

Provided the transmitters are favourably sited (propagation over water), it may be assumed that :

$$dn = \Delta n = \pm 0.02 \text{ lanes}$$

3.3. Evaluation of dF_R

The systematic error in the comparison frequency F_R will not exceed $\Delta F_R = \pm 2$ cycles/second. Besides there may be — for some reason or other — a slight random variation to an amount of $mF_R = \pm 2$ c/sec.

3.4. Evaluation of dV

The best known value of the phase velocity of Decca-waves (frequencies around 100 kc/sec.) for propagation over sea water is V = 299680 km/sec. It is unlikely, that this value will be in error more than ± 50 km/sec. ; hence the systematic error in using V = 299680 will be

$$\Delta V = \pm 50 \text{ Km/sec.} = \pm 5 \times 10^4 \text{ m/sec.}$$

In addition there may be a random variation, the amount of which is assumed by me to be of the order of

$$mV = \pm 20 \text{ Km/sec.} = \pm 2 \times 10^4 \text{ m/sec.}$$

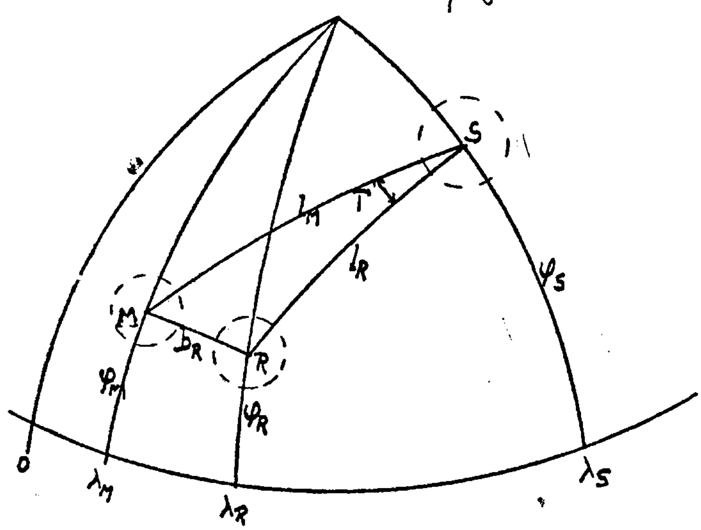
3.5. Evaluation de $d(l_M - l_R)$

l_M and l_R are the distances, travelled by the radio-waves from Master and Red Slave to observer. There is no simple way of knowing along which way these waves actually travel. It is however always assumed that they travel along the shortest way and there are no indications that the difference in length between the actual path along which they travel and the assumed shortest path, is of any practical importance.

For practical applications at sea-level the elevation of transmitting and receiving aerials may be neglected, and consequently the computation of l_M and l_R is reduced to a geodetic problem on the ellipsoid.

The fundamental data for computing l_M and l_R are the geographical latitude and longitude of M and R and of a number of points S on the chart to be latticed. It is obvious that these coordinates should be known in one and the same geographical system (1). In actual practice however it may — and often will — happen, that the geographical coordinates of M, R and S are based on 3 different surveys, having no mutual geodetic connection. This situation is illustrated in fig. 1a ; the (unknown) errors in the geographical coordinates of M, R and S respective to a common geographical grid — i. e. the grid adopted for a chart to be latticed (2) — are represented by the dotted circles with radii ε_M , ε_R and ε_S; (ε_M, ε_R and ε_S need not be nearly equal as indicated in fig. 1a).

fig. 1a



Errors ε_M, ε_R and ε_S do have a systematic effect on the computed values of l_M and l_R. This effect may be derived from fig. 1b ; in this plane triangle the sides are equal in length to the sides of the ellipsoidal triangle MRS.

From triangle ABC :

$$l_M^2 = (X_C - X_A)^2 + (Y_C - Y_A)^2$$

$$l_R^2 = (X_C - X_B)^2 + (Y_C - Y_B)^2$$

$$dl_M = \frac{XC-XA}{l_M} dX_C - \frac{XC-XA}{l_M} dX_A + \frac{YC-YA}{l_M} dY_C - \frac{YC-YA}{l_M} dY_A$$

(1) The effect of different ellipsoids will be mentioned in 3. 7.

(2) As the ellipsoid is very nearly a sphere, the influence of errors in the absolute geographical coordinates — contrarily to their differences — is very small and may be neglected in this investigation.

$$dl_R = \frac{XC-XB}{l_R} dX_C - \frac{XC-XB}{l_R} dX_B + \frac{YC-YB}{l_R} dY_C - \frac{YC-YB}{l_R} dY$$

$$d(l_M - l_R) = (\sin\alpha - \sin\beta)\, dX_C + (\cos\alpha - \cos\beta)\, dY_C - \sin\alpha\, dX_A$$
$$- \cos\alpha\, dY_A + \sin\beta\, dX_B + \cos\beta\, dY_B$$

The standard error in $(l_M - l_R)$ consequently will be :

$$\sigma^2 (l_M - l_R) = (\sin\alpha - \sin\beta)^2 \sigma^2 X_C + (\cos\alpha - \cos\beta)^2 \sigma^2 Y_C + \sin^2\alpha\sigma^2 X_A$$
$$+ \cos^2\alpha\sigma^2 Y_A + \sin^2\beta\sigma^2 X_B + \cos^2\beta\sigma^2 Y_B$$

As the errors ε_M, ε_R and ε_S are unknown, we may assume that they do not exceed a certain value ; their azimuth, however, may be in any direction and therefore we may put :

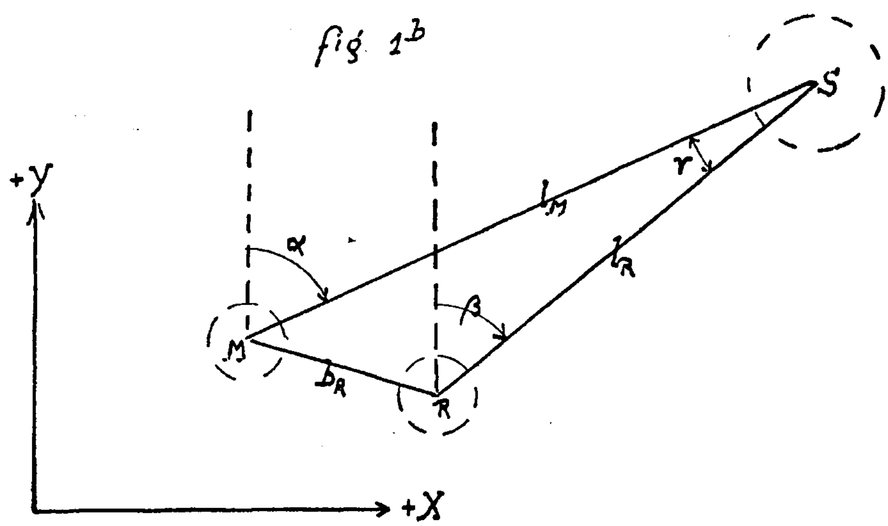
$$\sigma^2 X_A = \sigma^2 Y_A = \varepsilon_A^2 = \varepsilon_M^2$$
$$\sigma^2 X_B = \sigma^2 Y_B = \varepsilon_B^2 = \varepsilon_R^2$$
$$\sigma^2 X_C = \sigma^2 Y_C = \varepsilon_C^2 = \varepsilon_S^2$$

and it follows, that :

$$\sigma^2 (l_M - l_R) = \varepsilon_M^2 + \varepsilon_R^2 + 2 (1 - \cos\gamma) \varepsilon_S^2$$

For the purpose of estimating the order of magnitude of errors, the angle γ (fig. 1^b) may be considered equal to angle Γ (fig. 1^a); (from the examples in par. 4 it will be seen that the effect of this approximation can be neglected).

fig. 1^b



3.6. Evaluation of dL''_R

The systematic error of a well-calibrated Decca receiver will be of the order of

$$\Delta L''_R = \pm 0.01 \text{ lanes}$$

In addition there may be a random error of $mL''_R = \pm 0.01$ lane.

3.7. Effect of different ellipsoids

This matter is rather complicated ; complete formulae are given in textbooks on geodesy. The order of magnitude of possible errors may however be illustrated by the following example.

Most geographical coordinates are computed from geodetic distances obtained by triangulation ; the computation of the geographical coordinates always starts from a geographical datum (point of origin) somewhere in the survey area.

Dependent on azimuth and latitude, a geodetic distance of *exactly* 200 kms. may result in an arc distance of approximately :

6478″.52 on the International ellipsoid.
6479″.45 on the Bessel ellipsoid.
6479″.28 on the Airy ellipsoid.
6478″.78 on the Clarke 1866 ellipsoid.

As the arc distance is very nearly proportional to the geodetic distance it follows that the discrepancy between the Bessel and International arc distances may be of the order of 5″ (150 m) for a geodetic distance of 1000 kms; consequently the discrepancies in latitude or longitude may be also of the order of 5″ at a distance of 1000 kms. It will be shown later in this paper, that total errors as small as 1″ can hardly be accepted and consequently errors from other sources should not be aggravated by the use of different ellipsoids in computing the triangulation.

The effect of errors resulting from the use of different ellipsoids will not be taken into account in this paper, because this effect may be computed and be applied as (known) corrections.

3.8. Recapitulation

For our purpose of investigation, the errors, discussed in the foregoing subparagraphs (except 3. 7.), will be divided in two groups :

I. Errors in geographical coordinates.

II. Other errors to be expected.

Nautical charts should meet the requirement, that the errors in group I do not exceed the errors in group II to be expected under favourable (1) *conditions.*

The standard error in group I is represented by the formula :

$$\varepsilon_{I} = \frac{FR}{V} \sqrt{\varepsilon_{M}^{2} + \varepsilon_{R}^{2} + 2\,(1 - \cos\gamma)\,\varepsilon_{S}^{2}} \qquad \ldots\ldots\ldots (9)$$

(1) It is for this reason, that multipath radiowave propagation (sky-wave) is not considered in this paper.

The errors dn_R, dF_R, dV and dL''_R — their systematic as well as their random parts — are uncorrelated and consequently the standard error in group II is represented by the formula :

$$= \sigma_{II}\sqrt{\left(\frac{\Delta n_R}{2}\right)^2 + \left(\frac{l_M - l_R}{V}\right)^2\left(\Delta F_R\right)^2 + \left[\frac{F_R\,(l_M - l_R)}{V^2}\right]^2 (\Delta V)^2 + (\Delta L''_R)^2} + \sqrt{\left(\frac{l_M - l_R}{V}\right)^2 m^2 F_R + \left[\frac{F_R\,(l_M - l_R)}{V^2}\right]^2 m^2 V + m^2 L''_R} \qquad (10)$$

Similar formulae apply to the Green and Purple patterns.

§ 4. — EXAMPLES

4.1. General remarks

Three different situations should be considered, as indicated in fig. 2. It is supposed, that the charts, I, II and III are based on different surveys, not geodetically connected to each other.

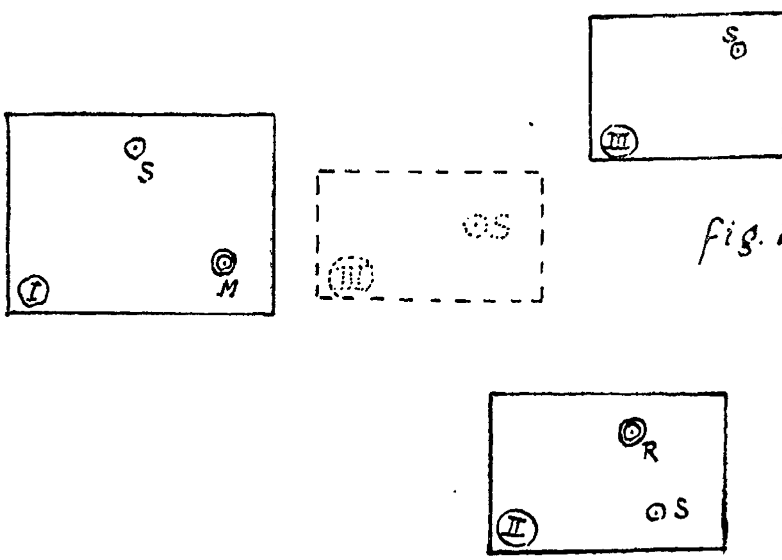


Fig. 2

When computing the Red lattice for chart I, ε_S and ε_M — being the uncertainty in the geographical positions of S and M respectively to the geographical chart datum of chart I — will be small, while ε_R will be larger. On chart II ε_S and ε_R will be small as compared with ε_M. In computing the Red lattice for chart III, we will have to take into account a small error in ε_S and larger errors in ε_M and ε_R . The examples to be given are relative to Decca Navigation chains of the type now existing.

4.2. Example 1

$b_R = 150$ km $= 15 \times 10^4$ m.

$n_R = 339{,}259$ lanes

$F_R = 34 \times 10^4$ c./sec.

$V = 3 \times 10^8$ m/sec.

$\frac{F_R}{V} = 113 \times 10^{-5}$

$\Delta n_R = \pm 2 \times 10^{-2}$ lane.

$\Delta F_R = \pm 2$ c./sec.

$mF_R = \pm 2$ c./sec.

$\Delta V = \pm 5 \times 10^4$ m./sec.

$mV = \pm 2 \times 10^4$ m./sec.

$\Delta L''_R = \pm 1 \times 10^{-2}$ lane

$mL''_R = \pm 1 \times 10^{-2}$ lane

Chart I

$\varepsilon_M = \pm 5$ m.
$\varepsilon_R = \pm 30$ m.
$\varepsilon_S = \pm 5$ m.

Chart II

$\varepsilon_M = \pm 30$ m.
$\varepsilon_R = \pm 5$ m.
$\varepsilon_S = \pm 5$ m.

Chart III

$\varepsilon_M = \pm 30$ m.
$\varepsilon_R = \pm 30$ m.
$\varepsilon_S = \pm 5$ m.

(for Charts I, II and III:)
$\gamma = 20°$
$\gamma = 90°$
$\gamma = 180°$

Chart I and chart II

1) $\gamma = 20°$

$$\sigma_1 = 113 \times 10^{-5} \times \sqrt{25 + 900 + 2 \times 0.06 \times 25} = 0.034 \text{ lane}$$

2) $\gamma = 90°$

$$\sigma_1 = 113 \times 10^{-5} \times \sqrt{25 + 900 + 2 \times 1 \times 25} = 0.035 \text{ lane}$$

3) $\gamma = 180°$

$$\sigma_1 = 113 \times 10^{-5} \times \sqrt{25 + 900 + 2 \times 2 \times 25} = 0.036 \text{ lane}$$

Chart III

1) $\gamma = 20°$

$$\sigma_1 = 113 \times 10^{-5} \times \sqrt{900 + 900 + 2 \times 0.06 \times 25} = 0.049 \text{ lane}$$

2) $\gamma = 90°$

$$\sigma_I = 113 \times 10^{-5} \times \sqrt{900 + 900 + 2 \times 1 \times 25} = 0.049 \text{ lane}$$

3) $\gamma = 180°$

$$\sigma_I = 113 \times 10^{-5} \times \sqrt{900 + 900 + 2 \times 2 \times 25} = 0.050 \text{ lane}$$

In formula (10) the maximum value of the coefficient $(l_M - l_R)$ is equal to b_R (i. e. along the base-line extension). As the pattern will seldom be used along these extensions, in these examples a value of $(l_M - l_R)$ equal to $0.8\ b_R$ will be used in formula (10).

$$\sigma_{II} = \sqrt{\left(\frac{2 \times 10^{-2}}{2}\right)^2 + \left(\frac{12 \times 10^4}{3 \times 10^8}\right)^2 \times 4 + \left(\frac{113 \times 10^{-5} \times 12 \times 10^4}{3 \times 10^8}\right)^2 \times 25 \times 10^8 + 1 \times 10^{-4}}$$

$$+ \sqrt{\left(\frac{12 \times 10^4}{3 \times 10^8}\right)^2 \times 4 + \left(\frac{113 \times 10^{-5} \times 12 \times 10^4}{3 \times 10^8}\right)^2 \times 4 \times 10^8 + 1 \times 10^{-4}}$$

$$\sigma_{II} = \sqrt{(10.000 + 64 + 51075 + 10.000) \times 10^{-8}}$$

$$+ \sqrt{(64 + 8172 + 10,000) \times 10^{-8}}$$

$$\sigma_{II} = 0.0267 + 0.0135 = 0.040 \text{ lane}$$

4.3. Example 2

All data as in example I, except :

Chart I.	$\varepsilon_M = \pm 15$ m. $\varepsilon_R = \pm 100$ m. $\varepsilon_S = \pm 15$ m.	
Chart II	$\varepsilon_M = \pm 100$ m. $\varepsilon_R = \pm 15$ m. $\varepsilon_S = \pm 15$ m.	$\gamma = 180°$
Chart III	$\varepsilon_M = \pm 100$ m. $\varepsilon_R = \pm 100$ m. $\varepsilon_S = \pm 15$ m.	

Chart I and chart II

$$\sigma_I = 113 \times 10^{-5} \times \sqrt{225 + 10,000 + 2 \times 2 \times 225} = 0.0119 \text{ lane}$$

σ_{II} = (from example 1) = 0.040 lane

Chart III

$$\sigma_I = 113 \times 10^{-5} \times \sqrt{10{,}000 + 10{,}000 + 2 \times 2 \times 225} = 0.164 \text{ lane}$$

$$\sigma_{II} = \text{(from example 1)} \qquad = 0.040 \text{ lane}$$

4.4 Conclusions from examples I and II

Example I shows that the effect of discrepancies between the geographical grids, even as small as 30 metres (I second of arc), is — or anyhow somewhere on a chart to be latticed may be — in excess of the effect of other errors to be expected under favourable conditions of observation.

The largest scale of a nautical chart will be about I : 25.000. At the base-line 0.01 lane is equivalent to 5 metres (terrestrial) and more than 5 metres elsewhere in the pattern; 5 metres is equivalent to 0,2 millimetre at a scale of I : 25.000, being the theoretical error of plotting (printing) of the computed lattice on the chart.

It follows, that roughly 30 metres (I second of arc) is the acceptable discrepancy between geographic grids.

Frequently discrepancies as supposed in example 2 — and even considerably larger discrepancies (1) — will exist in nautical charts, because very often they are compilations of different surveys. From example 2 it may be seen that such discrepancies — although fully acceptable when using classic navigation methods — cannot be accepted, or at any rate preferably should be avoided, for position fixing by modern radio navigation systems.

§ V. — REVIEW OF THE METHODS TO OBTAIN COORDINATION OF GEOGRAPHIC GRIDS

5.1. General

To be able to carry out geodetic computations we need :

1) an initial point *from* which to compute ;
coordinates : geogr. *lat.* φ and *long.* λ.

2) a direction *in* which to compute :
azimuth : A.

3) a surface *along* which to compute :
the surface satisfying all practical needs is a two-axial *ellipsoid* of revolution with parameters *a* = *equatorial radius* and *f* = *flattening.*

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(1) The following discrepancies were quoted by Admiral Nares at the Oslo Conference of the I.U.G.G. (reduced to metres).

Denmark-Sweden	= + 12 m.	= — 91 m.
Denmark-Norway	= — 195 m.	= + 146 m.
Denmark-Norway	= — 170 m.	= — 16 m.
Gr. Britain-France	= + 168 m.	= — 80 m.
U.S.A.-Bahamas (1)	= + 2877 m	= + 10420 m.
(2)	= — 124 m.	= + 1368 m.
(3)	= — 371 m.	= — 466 m.

These discrepancies were determined after the World War and *now* may be taken into account in the compilation of nautical charts.

These five quantities φ, λ, A, a and f constitute a *geodetic system*. If size and shape of the used ellipsoid agree within narrow limits with the real earth, if one and the same ellipsoid is used throughout the world and if we are able to make all our geodetic computations starting from the same initial quantities φ, λ, A, we will have a *world geodetic system* in which all geographic coordinates φ and λ are coordinated (= absolute geographic coordinates as termed in the Resolution of the Vth International Hydrographic Conference).

5.2. Triangulation method

A terrestrial triangulation consists of measuring baselines, angles and azimuths. All these observations are carried out on the *physical surface* of the earth and are in a direct or indirect way related to the local direction of gravity (plumbline, spiritlevel, horizon). The curvature of the physical earth — apart from being unknown — is too complicated to enable the developing of mathematical formulae to make triangulational computations on it. As a first step the observations have to be reduced to a sort of « standard » level surface : the *geoid* (mean sea level) ; the reduction from physical surface to geoid is not very complicated, because the main thing we need to know is the elevation above mean sea-level. The surface of the geoid is however still of too complicated curvature for the development of mathematical formulae, and therefore the second step must be to reduce the observational data from geoid to ellipsoid. In this second step we need to know :

1) The ondulations N of the geoid about the ellipsoid.

2) The components ξ and η (in the direction of the meridian and perpendicular to it) of the deflection of the vertical respectively to the ellipsoid.

As geodetic observations alone cannot give N nor ξ and η, it is impossible :

a) To reduce the length of a base line from the geoid to the ellipsoid. A base may well be measured with an accuracy of 1 part in 10°. As N may be of the order of 100 metres, the error in neglecting the reduction from spheroid (sea-level) to ellipsoid may be of the order of

$$\frac{100}{6{,}378{,}000} \times 1 \text{ part in } 64{,}000.$$

b) To reduce angles and azimuths to their corresponding values on the ellipsoid.

Further drawbacks of the terrestrial triangulation method are that errors accumulate thus limiting the accuracy over long distances, and that this method can span only comparatively small stretches of water, due to the curvature of the earth and the limited elevation at which observations can be made.

5.3. Astronomical observations

Astronomical determination of latitude φ', longitude λ' and azimuth A', is always relative to the local direction of gravity and thus refers to the geoid. The relation between φ, λ, A (ellipsoid) and φ, λ', A′ (as reduced from physical surface to geoid) is given by the following equations

$$= \varphi' - \varphi$$

$$\eta = (\lambda' - \lambda) \cos \varphi$$

$$\eta = (A' - A) \operatorname{cotan}\varphi$$

The last two give the LAPLACE equation $A' - A = (\lambda' - \lambda) \sin \varphi$. By a combination of terrestrial observations and astronomical observations it is possible :

a) To determine ξ and η relative to an ellipsoid adopted for the area under consideration.

b) To determine N relative to this adopted ellipsoid.

As a terrestrial triangulation cannot span the whole globe (1/3 = land ; 2/3 = oceans) this combination does not enable to determine ξ, η and N relative to one single ellipsoid to be adopted for the whole earth. In other words : the relative orientation of two ellipsoids (even when they are of the same size and shape) remains unknown, when two areas are not connected by triangulation.

5.4. Flare triangulation and Shoran

These two methods enable the spanning of greater distances than hitherto had been possible by terrestrial triangulation. The methods do not give any direct information as to ξ , η and N. Flare triangulation is restricted in its applications to distances of at the most 200 kms. Appreciably longer distances can be measured by Shoran ; for the time being the accuracy cannot compete with first order triangulation and it is very unlikely that the method ever will enable bridging the oceans.

5.5. Survey Decca

This system is more suitable for hydrographic surveys than Shoran. This system enables a very high accuracy of hydrographic position fixing to be obtained in surveying up to distances of a few hundred kilometres offshore.

5.6. Solar eclipse and lunar occultation method

In the solar eclipse method the moments of 1st, 2nd, 3rd, and 4th contact are observed at distant stations along the central line of the shadow path over the earth. From these observations the arc distance between the stations can be computed.

The lunar occultation method is a modification of the solar method, employing the disappearance and reappearance of a star behind the moon's limb.

The solar method is very limited in its applications, eclipses being rare and the central line seldom passes over or near the desired stations. Lunar occultations are much less limited in their application.

The greatest difficulty in these methods is to get sufficiently accurate values for the moon's declination and right ascension. In lunar occultations there are additional difficulties due to incorrectly known irregularities in the profile of the moon's limb, as a high mountain causes the star to disappear earlier or reappear later.

The observations and calculations are elaborate. Both methods can give the arc distance only in one direction, i. e. the direction of the shadow path. The accuracy so far obtained is of the order of 150 metres; this accuracy is of course independent of the distance between the stations.

5.7. The gravimetric method

This method enables computation of ξ, η and N relative to one single ellipsoid or in other words enables to compute their *absolute* values at the initial datum points of the surveys in different countries, *without establishing a connection by triangulation between* those countries.

It thus opens the possibility to convert the already existing geodetic systems to the *world geodetic system.*

The *mathematical* basis for this system was published over 100 years ago by the famous British scientist STOKES. Until 25 years ago his work had mainly academic interest, due to lack of suitable gravity measuring instruments.

The *geophysical* basis for this method is the fact, that the gravity anomalies Δg, the ondulations N of the geoid and the vertical components ξ and η have the same cause, namely the disturbing mass-layers of the earth. Of these quantities Δg, as referred to the adopted gravity formula, now can be observed; in the formula of Stokes ξ, η and N are functions of Δg.

The *gravity formula* now *adopted* is related to the *international ellipsoid.*

To determine N, ξ and η for the initial datum stations of the various national or continental surveys we need:

1) One and the same reference ellipsoid throughout the world; the international ellipsoid appears to be the best available.

2) Gravity maps of the whole world, giving iso-anomaly curves and the average gravity anomaly value of squares of $5° \times 5°$ or preferably $3° \times 3°$ (roughly 100,000 sq. kms).

3) Detailed gravity measurements at some 50 to 100 stations in an area with a radius of roughly 100 kms. surrounding the datum station.

4) A very detailed gravity survey in an area with a radius of roughly 10 kms. surrounding the datum station.

When the data 1 to 3 are available, N may be computed with an accuracy of a few metres and ξ and η with an accuracy of $0''.5$ to $1''.0$.

When the data under 4) are available also, a considerably higher accuracy (f. i. for the determination of the geographic coordinates of a national or continental datum) can be obtained.

5.8. Summary

Terrestrial triangulation methods cannot give any information as to N, ξ and η, making it impossible to reduce the observational data to the ellipsoidal surface of computation. From a combination of terrestrial triangulation and astronomical observations N, ξ and η may be computed. As a terrestrial triangulation cannot span the oceans, the values of N, ξ and η thus computed are relative to a chosen ellipsoid, of which the orientation remains unknown. Flare triangulation and Shoran are just extensions of the classic terrestrial triangulation method.

The only possibilities for arriving at a *world geodetic system* are :

I) a) Terrestrial triangulation of the continents, combined with astronomical observations of latitude, longitude and azimuth.

b) Geodetic connection of the continents by the solar eclipse or lunar occultation methods.

II) Astronomical observations of φ' and λ' to be corrected by the values of ξ and η obtained by the gravimetrical method described in 5. 7.

The method to be preferred appears to be no. II. It should be emphasized that this method does not make terrestrial triangulation, Shoran and survey Decca obsolete, as the latter are indispensable in practical survey work and enable high or very high relative accuracy over limited areas for the construction of large scale maps.

The gravimetric method enables to *connect* different surveys and to establish a world geodetic system ; in unexplored areas, where no terrestrial triangulation exists or is difficult to establish, it may be used for the determination of geographical control points with an accuracy of $0''.5$ to $1''.0$, without setting up any kind of triangulation.

5.9. Data available and supplementary data needed

5.9.1. Data available

The terrestrial triangulation of the U.S.A. has been adjusted on the Clarke 1866 ellipsoid. This large area of triangulation is being extended to the North and to the South. Large areas of triangulation in Africa and in India as well as in Russia and Siberia have been adjusted.

As a result of the initiative of Colonel Hough, of the U.S.A., the combined triangulations of Western Europe and parts of North Africa have been adjusted on the international ellipsoid by the Army Map Service and the Coast & Geodetic Survey. Very unfortunately the triangulation of Great-Britain has not yet been connected to this vast block ; plans seem to be under discussion to establish this connection by Shoran by the following junctions : Northern Scotland-Norway, S.E. England-Holland and S.W. England-France. As to the data necessary to apply the gravimetric method to connect the above mentioned and other triangulation blocks to each other, many millions of these gravity data are available on land and some 3000 at sea.

From these data it has been possible to compute the *absolute* values of ξ and η for Meades Ranch (included in the U.S.A. adjustment) and Potsdam (included in the European adjustment) with an accuracy of less than $0''.5$.

5.9.2. Data needed

The existing gravity measurements have been carried out for geophysical, geological or exploratory purposes, or just by chance. In many cases they are densely concentrated, but vast regions are entirely without any gravity observations. To be able to construct the general gravity map of the world, it will therefore be necessary to fill up the gaps.

Following a suggestion of Prof. Harding, director of the Mapping and Charting Research Laboratory of Ohio State University, Prof. Heiskanen planned a detailed gravity analysis survey of the whole world. The ultimate goal — the world geodetic system — cannot be reached in a short time ; however, Prof. Heiskanen believes that very important results will have been obtained within 3 years.

§ VI. — CONCLUSIONS

6.1. It has always been and always will be a good and sound hydrographic policy to construct hydrographic charts with a geometric accuracy at least as good and preferably better than the accuracy attainable by the navigator in fixing his position by the equipment at his disposal.

6.2. As a result of the steadily increasing use of modern radio navigation systems, the navigator is now able to fix his position relative to far-away shore-based radiotransmitters, with an accuracy many times greater than hitherto had been possible and could be foreseen at the time of making the charts.

6.3. Nautical charts very often have to be compilations of hydrographic surveys carried out in different countries and based on triangulations, not mutually connected.

6.4. In order to be able to make a hydrographic compilation, fulfilling the requirement of a geometric accuracy on large scale charts, equal to that attainable by modern radio position fixing under favourable circumstances, this paper concludes that — apart from the internal accuracy of the triangulations on which the surveys are based — the geographic grids of the charts used in the compilation, should be coordinated with an accuracy of the order of 30 metres. (i. e. $1''$ in latitude and $1''$ sec φ in longitude).

It is extremely unlikely that there will ever be a need for higher accuracy for navigational purposes.

6.5. Few existing hydrographic charts (based on compilation) meet this requirement. It is therefore necessary that *new* hydrographic surveys and compilations aim at an accuracy as concluded in 6.4.

6.6. It does not necessarily follow that existing nautical charts, not meeting this requirement, should be considered obsolete or unusable for modern radio navigation.

In the first place less accuracy is required on small scale charts and even on large scale charts there will not always be a need for ultimate accuracy ; in the second place the present-day accuracy of many systems is smaller than supposed in the examples given in this paper. Moreover the discrepancies between chart patterns and transmitted patterns (1) may be determined from operational trials and be applied as corrections to the observations (2).

It is however obvious that :

a) there may be future improvements in the radio-navigation systems (accuracies better than 30 m. are unlikely to be achieved and moreover would have no practical meaning in navigation);

b) applying corrections to the system instead of to the charts can never be the right solution and is acceptable only as a temporary measure.

6.7. The coordination of national or continental geographic grids to an accuracy of the order of 30 metres in therefore of great importance in the compilation of hydrographic charts.

The methods for achieving this coordination in the shortest possible time appears to be the *gravimetric method.*

The Navies of various nations have already contributed very much by making gravimetric observations in submarines. Until shortly these observations were merely of scientific value, but today additional observations at sea are indispensable for purely practical applications like the compilation of up-to-date nautical charts.

6.8. As the European adjustment now has been completed, nautical charts of that area should be based on this adjustment.

6.9. Today there are two Decca chains in Great Britain and a third chain will be put into operation in that country in the near future. In addition there are the Danish and German chains and others under construction or consideration.

The connection of the British triangulation to the European network is therefore considered to be of the utmost importance for construction of nautical charts.

's-Gravenhage, 30 October 1951.

(1) These discrepancies need not and often will not only be due to errors in geographic coordinates.

(2) This method is for instance used for the Decca patterns of the Chains in Great-Britain and in Denmark.