MODIFICATION OF MEAN PROJECTION PLANE OF THE MULTIPLEX APPARATUS OF THE « DIRETORIA DE HIDROGRAFIA E NAVEGAÇAO » AND ITS EFFECTS ON PHOTOGRAMMETRIC RESTITUTION OPERATIONS APPLIED TO CARTOGRAPHY

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Early in the year 1948 the *Diretoria de Hidrografia e Navegaçâo* modified the technical orientation followed up to that time in its aerophotogrammetric surveys, with a view to employing for restitution work the ZEISS wide-angle type of aero-projector. This new technique permits greater advantage to be derived from the aerial triangulation and has shown itself to be most trustworthy for this kind of work. In the case of plans of ports, the practical results were in fact highly satisfactory. With regard to coastal surveys, however, the fact that the apparatus magnified the scale of the photograms rendered its application questionable from the economic point of view, since all the restitution work has to undergo a reduction of ten times in order to obtain the scale of the sounding sheets and in these conditions the advantages remain restricted to a small portion of the needs of chart construction.

To solve this problem, Lieutenant-Commander Alberto dos Santos Franco devised and developed a system of reduction of the mean projection plane distance of the Multiplex apparatus, to 26 cm., the intention being to obtain the restitution on a scale approaching that of the photogram. The official report presented at the time contained the following statements :

At present the wide-angle projectors have the projection plane 360 mm. from the nodal point of the lens. For restitution in this plane to scale E, it will be necessary to fly to the height $H = \dfrac{0.360}{E}$, which means that if we diminish the projection distance, it will be possible, for the same altitude flights, to make the restitution to a smaller scale. It was precisely for this reason that the firm of ZEISS, Jena, constructed its latest model of wide-angle projectors (See : *Fotogrametria Terrestre y Aerea* : K. Schwidefsky : Page 154) for which the projection plane is situated 260 mm. from the nodal point of the lens. The possibility of modifying the present wide-angle projectors possessed by the *Diretoria de Hidrografia e*

Navegação so as to render them equal to the modern wide-angle instruments, will be readily understood, the alteration being a very slight one.

Let a be the principal distance of the interior orientation of the projector; b the true projection distance; c the projection distance required; f the focal distance from the projector lens.

We then have in the present conditions :

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

For a new projection distance e we have an increased value Δa and the above equation becomes :

$$\frac{1}{a + \Delta a} + \frac{1}{c} = \frac{1}{f}$$

Equalizing these two formulæ we have :

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{a + \Delta a} + \frac{1}{c}$$

$$\frac{1}{a + \Delta a} = \frac{bc + ac - ab}{abc}$$

$$\Delta a = \frac{abc}{bc\text{-}a(b\text{-}c)} - a = \frac{a^2(b\text{-}c)}{bc\text{-}a(b\text{-}c)}$$

The constant a is equal to 22 mm., therefore :

$$a = \frac{484 \times 100}{360 \times 260 - 2200} = \frac{48400}{92400} = 0.52 \text{ mm.}$$

Consequently, if we increase by 0.52 mm. the principal projector distance we shall have attained our object. This can be very easily realized by fixing a metal band of 0.52 mm. thickness, with rigorously plane and parallel surfaces, between the lens and its stand. The modification of the principal distance by the value indicated produces an altimetric error of 1 %, which can be checked by the relation between 22.22 which was the principal distance taken as basis for the computation of the reductor of the ZEISS RMKP-20 cm² (30 x 30 cm.) camera and the actual distance of 22.52.

Six projectors were transformed and in two of them a pair of reticular parts (diapositives) were introduced in order to check the very unlikely appearance of any distortion whatever of the optical system. These diapositives having been levelled and set, readings were obtained in nine conveniently distributed points, without any difference whatever greater than the admissible reading error (0.2 mm.) being noted. These trials finished, there was then selected one of the strips of the aerophotogrammetric survey made in Jacuacanga, Angra-dos-Reis, Brazil, to scale of 1 : 5000 in order that an aerial triangulation with the six transformed projectors might be made.

Owing to the great depth of field it was possible to accomplish this aerial triangulation to scale of 1 : 10000, which means that the same area can be restituted in half the time formerly necessary ; also on a scale equal to half of that used before the transformation. Now, in hydrographic surveys, the scales of which are normally small, particularly for the coastal line, the transformation corresponds to 100 % increase of efficiency compared with the primitive conditions. In addition to this unquestionable increase in output of work, the following results were derived from the transformation :

a) *LUMINOSITY* : It is known that the value of the luminosity is expressed by :

$$e = \frac{i}{d^2} \cos \alpha$$

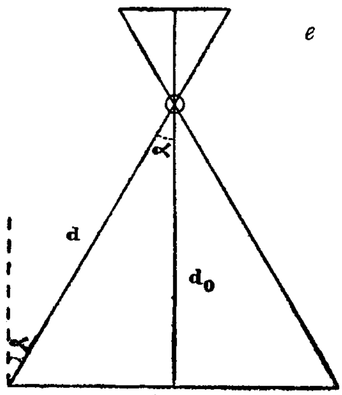

Fig. A

where i is the intensity of the light source, α the incidence angle of the light ray on the lighted surface and d the focal distance to the point of this surface for which the incidence angle is α (Fig. A).

Taking d as the focal distance to the projection plane, we have :

$$e = \frac{i}{d^2} \sec \alpha \qquad \text{(a)}$$

Now, the value of d_0 before the transformation was 36 cm.; afterwards 26 cm.; consequently, according to formula (a), where i remains the same, we have a new value of e indicated by e' corresponding to the ratio :

$$\frac{e'}{e} = \frac{d_0^2}{d_0'^2} = \frac{(36)^2}{(26)^2} = 1.92$$

which means that the luminosity is practically doubled since its increase comes out as 92 %.

As it is acknowledged that one of the difficulties of operation with the wide-angle Zeiss projectors is precisely the deficiency of luminosity, the increase of efficiency which results from the transformation can be imagined.

b) *CLEARNESS OF OUTLINE :* It is obvious that clearness of outline is also increased by the modification since it varies inversely with magnification, which is considerably reduced.

(c) *CONVENIENCE IN OPERATING* : Although the convenience of the operator may be somewhat projudiced by diminution of the distance between the lens and the plane, this inconvenience is largely compensated by the results enumerated in paragraphs (a) and (b).

(d) *ALTIMETRIC ERROR :* On the construction of the reductor for the RMKP-20 (30 x 30 cm.) instrument, the value of 22.22 mm. was adopted as the Multiplex constant. Afterwards a diapositive was introduced, mounted in the reductor, the projector being levelled and at the mean distance, and it was ascertained that the constant was very close to 22.00 mm.; it was concluded that there existed an error of -1 % in the altimetric restitution scale. With a wedge of 0.52 mm. the actual constant is 22.52 so that the restitution error will be :

$$1 - \frac{22.22}{22.52} = + 1\%$$

The error in question, in addition to being small, can readily be corrected.

(e) *INCREASE IN NUMBER OF PROJECTORS* : The aviation of the *Diretoria de Hidrografia e Navegaçâo* can fly without difficulty at heights of 4500 metres and the photographs can be taken at this height with the Zeiss RMKP-20 (30 x 30 cm.) apparatus ; the side of each photograph represents a ground distance of 6750 metres. Consequently, for a longitudinal covering of 60 %, the aerial base is 0.40 x 6750 m. = 2700 m. In order that the strip may be restituted to scale of 1 : 20000, the distance between the projector lens and the projection plane should be 4500 ÷ 20000 = 0.225 m. -- this distance differing from that of the plane of maximum clearness (0.260) by scarcely 0.035 m., a value which is perfectly within the limits of the depth of field which is about ± 7 cm. In its turn the aerial restitution base will be 2700 ÷ 20000 = 0.135 m. Now, the bar of the larger Multiplex in the possession of the *Diretoria de Hidrografia e Navegaçâo* having a useful length of 2.80 m., the number of projectors which this bar can support will be 2.80 ÷ 0.135 = 21. With regard to the smaller Multiplex the bar of which has a useful length of 1.40 m., it is logical that with it 10 projectors can be used ; 31 projectors would be necessary in order that full benefit might be reaped from the transformation.

(f) *USE OF STATOSCOPE :* With the argument (e) we prove that strips of 56 kilometres can be restituted with the 3-metre bar Multiplex. It may happen, however, that it is possible to obtain the necessary accuracy for strips of that extent, with a minimum of control points, by using the statoscope with recording camera synchronized with the aerial camera. This is an accessory which should be acquired in order to obtain all the advantages of the transformation carried out.

The number of control points used for the strip selected for the trials was never great enough to permit a definite conclusion to be reached ; the *Diretoria de Hidrografia e Navegaçâo* decided, therefore, to make a second experiment. For this the Multiplex apparatus, with the projection plane distance of 26 cm. was used in the execution of the plan of Magé, which was to be the first restitution operation of the current year. For this special purpose, a large number of reference points were fixed by the topographic Service, the planimetric positions of which were obtained by means of polygons and the altimetric positions by geometrical levelling. The co-ordinates of these points having been computed and adjusted, the latter were plotted by means of their rectangular co-ordinates. The formation of the stereoscopic model was accomplished by rigorously following the technique previously used for the 36 cm. distance, the scale and absolute orientation being based on only five of the reference points.

The trial consists in comparing the planimetric and altimetric positions of the number of points plotted by the topographic Service with the positions of their images obtained in the optical model.

The following results were obtained :

Mean planimetric error	0.2 mm.
Mean altimetric error	1/4 interval of contour lines
Planimetric error	0.5 mm.
Altimetric error greater than 1/2 interval of contour lines	5 %
Altimetric error equal to interval of contour lines -..............	0

For work of this nature, the International Conference of Caracas (1946) the decisions of which were officially (among others by the Brazilian government) accepted, drew up the following text in Chapter II, paragraphs 11 and 12 of its results :

11. In maps published on a scale of 1 : 20000 or less, the admissible error in planimetric horizontal accuracy shall not be greater than 0.5 mm. ; 90 % of the height figures of the control points must lie within the specified limits of accuracy.

12. With regard to vertical tolerance, 90 % of the points must be fixed with an accuracy not exceeding half of the equidistance and none must lie beyond the total equidistance.

Comparing the results it will be seen that they fall within the fixed limits, which leads us to accept the accuracy obtained as fully satisfactory. The *Diretoria de Hidrografia e Navegação* concluded therefore that the modification of the mean projection plane of the Zeiss aero-Multiplex wide-angle model from 36 cm. to 26 cm., as developed by Lieutenant-Commander Alberto dos Santos Franco, neither prejudices the accuracy nor affects the fundamental operating conditions of the instrument.

Having obtained these preliminary results, a new examination of the benefits that the transformation in question could bring to restitution work applied to cartography, was undertaken. First, the principal aim, i. e. to make the restitution scale approximate to that of the 30 x 30 photograph without affecting either the accuracy or the fundamental mechanism of the instrument, was fully attained. In practice this offers the following advantages :

1) A restitution scale more nearly approaching that used on the sounding sheets ;

2) 50 % economy in drawing for restitution work ;

3) Extension of the benefits of aerial triangulation to coastal surveys without any major modification in the instrument already at the *Diretoria de Hidrografia e Navegação* ;

4) 50 % economy in topographical work for fixing control points ;

5) Doubles the capacity of each instrument, permitting restitution of large coastal areas for each strip.

To these advantages must be added those derived from increase of luminosity and which may be resumed as follows :

1) Improvement in relative and absolute orientation procedure ;

2) Increased certainty in the plotting of the points ;

3) Greater stereoscopic sensitivity.

As inconvenience, the fact may scarcely be mentioned that the operator has to work with his head very near the upper part of the projectors and therefore subject to a certain amount of heating. This difficulty is, however, perfectly surmountable, and is insignificant compared with the proved advantages. From the final weighing of the results it may be concluded that the reduction of the distance between the mean projection plane and the lens, as developed by Lieutenant-Commander Alberto dos Santos Franco should be immediately adopted in the cartography of the *Diretoria de Hidrographia e Navegação.*

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