VARIATIONS IN THE LENGTH OF THE DAY

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In opening a Geophysical Discussion on « Variations in the Length of the Day » held at the Royal Astronomical Society on November 24, 1950, Sir Harold Spencer Jones, Astronomer Royal, pointed out that the variations observed can be divided into three types : the slow secular lengthening, irregular fluctuations and a periodic variation with a period of a year. The first two types can be demonstrated by astronomical observation alone, while the yearly variation was discovered only when accurate standards of time controlled by quartz crystals became available.

Halley was the first to point out that the motion of the moon relative to the stars appeared to be accelerating. Laplace showed that the effect could be accounted for by a variation in the eccentricity of the earth's orbit as a result of planetary perturbations. John Couch Adams revised Laplace's calculations in 1854, however, and found that only half the observed acceleration could be accounted for in this way Emmanuel Kant suggested that tidal friction might be slowing down the earth's rotation, but it was not until Sir Geoffrey Taylor pointed out the importance of narrow seas as a source of dissipation of the earth's rotational energy that it was believed that this source alone could account for the 2×10^9 H. P. necessary. Calculations by Jeffreys for all the seas of the world showed that the slowing down is explicable in this way, the Bering Straits in particular being responsible for about seventy per cent of the total dissipation. Cavell later showed that the longitude of the sun also has a secular acceleration, and recent work has revealed the same effect for Mercury and Venus, thus confirming that the cause of the acceleration lies in the earth.

Newcomb was the first to note that, after allowing for the secular acceleration, the moon's motion showed irregularities greater than the estimated experimental errors. By studying records of eclipses between 1660 and 1900, he showed that the major portion of the discordances between the observed and tabular positions could be represented by a periodic term of about 300 years in period and $\pm$ 15 seconds of arc amplitude, afterwards referred to as the « great empirical term » ; it is included in E. W. Brown's tables of the moon. It was not until the completion of Brown's theory of the motion of the moon that it was possible to assert with confidence that no gravitational term had been overlooked, and that the fluctuations in the moon's motion were probably due to irregularities in the earth's rotation. To establish this it was necessary to prove that there were corresponding fluctuations in the motions of other bodies. First of all, the secular acceleration of the moon was determined as accurately as possible from a study of ancient eclipses, and this term and the great empirical term were subtracted from Brown's tables. If the residuals were attributable to a fluctuation in the rate of rotation of the earth, and effect on the observed longitudes of the sun, Mercury and Venus would be in proportion to the ratios of their mean motions to those of the moon. Allowance had also to be made for the effect of the secular retardation of the earth's rotation on the motions of these bodies. The secular accelerations of the sun, Mercury and Venus must be in the ratio of their mean motions ; but the ratio of the secular accelerations of the sun and moon could not be determined theoretically, because of the tidal interaction of the earth and the moon. With these assumptions, the motions of the sun, Mercury, Venus and the moon could be accounted for, within the limits of observational error.

Sir Harold Spencer Jones said he did not think it possible to account for the irregular changes in the earth's rotation by movements of material over the surface of the earth. He also pointed out that we do not know how smoothly the changes occur : observations cannot at present decide whether a change occurs suddenly or is spread over several weeks or months, or even over a couple of years.

The periodic annual change in the rate of rotation has been found by analysis of the errors of a number of quartz clocks compared with time-signals controlled from astronomical observations. It was assumed that each clock would have a linear drift of rate so that its error would contain a quadratic term in the time. The third differences of the tabulated errors should therefore be the same for all clocks, apart from accidental errors. This was, in fact, found to be so, and, by averaging the values from all the clocks and integrating ,it was possible to discover the form of the original periodic term in the error. It turned out that the earth is slow in the spring and fast in the autumn, when compared with a uniform time. The variation in the length of the day during the year amounts to ± 60 millisec. departure from the mean over the year. This annual variation is probably due to seasonal movements of superficial material over the earth.

Dr. R.C. Sutcliffe discussed the contribution which atmospheric movements can make to the annual variation of rate. Changes may occur either in the moment of inertia of the atmospheric shell or in its mean speed of rotation. In either case the change in angular momentum will be communicated to the earth. The former effect would show up as a seasonal change in the distribution with latitude of the mean atmospheric pressure, and the latter as a change in the mean west-east wind-speed. The variations of rate referred to by Sir Harold amount to about $1 \times 10-{}^{8}$, and, since the ratio of the moments of inertia of the atmosphere and of the earth is 1.8×10^{-6}, then a variation in angular momentum of the atmosphere of about 7×16^{-3} is in question. If this were due to a change in the distribution of air mass between poles and equator, then a seasonal variation of order 7 mb. should be sought in the pressure data. If it were due to variations in angular velocity of the atmosphere, since the earth's surface moves at about 1,000 m.p.h. at the equator, variations of about 7 m.p.h. would be expected in the east-west wind-speed. Of course, when considering the data in conjunction with these estimates, it is necessary to bear in mind the factor $\cos^3\varphi$ representing the effect of latitude.

The variation with latitude of the atmospheric pressures, averaged over a month and all longitudes, for the months of February and August, shows an increase of about 7 mb. in February just north of the equator, almost exactly balanced by a decrease at about the same latitude south. Fluctuations in higher latitudes are smaller. An exact formal integration shows that the total effect is about a hundredth of what is required.

The variation with latitude of west-east wind-speeds up to 300 mb. above sea-level, averaged over June-August and over December-February, appears more hopeful. An increase, west-east, of about 10 m.p.h., occurs in December-February at about lat. 30° N., while there is no such variation in the southern hemisphere. This is due to the greater area of land in the northern hemisphere. Integration over all latitudes gives a total effect of only about one-fifth of that required. It is possible that further work may show that this estimate should be increased, but it is unlikely that this cause alone will turn out to be the whole explanation.

Dr. J.N. Carruthers introduced his colleagues, Commander A.L. Lawford and Instructor-Lieut. V.F.C. Veley, who later described the review made at the Admiralty Hydrographic Department of seasonal changes of mean sea-level. It appears that winds urge the water into the northern hemisphere in August more strongly than into the southern hemisphere in March, thereby bringing about an annual variation in the moment of inertia of the oceans. It is also possible that the annual variations in extent of the ice-caps could contribute to this. The change in glaciated area at the North Pole is from 8.2×10^6 sq. km. in September to 16.8×10^6 sq. km. in March, and at the South Pole the change, although in antiphase, is five times as great.

Commander Lawford displayed a chart exhibiting the anomaly in mean sea-level for spring over the whole earth, derived from data at seventy widely distributed stations. Most of the continental coasts are covered, except Antarctica and the east coast of Brazil, but there is a dearth of reliable records from oceanic islands. Except for Norway, there are no stations north of lat. 60° N., and none south of 50° S. Data for a period of ten years have been examined, and charts prepared for both autumn and spring. The spring chart is seen to show anomalies of -2 to -4 cm. over the Gulf Stream in the North Atlantic, but positive values as great as + 4 cm. are found over most of the South Pacific and South Atlantic. Commander Lawford stressed that the data are very incomplete; but, in areas where there is a reliable knowledge both of currents and of sea-level anomaly, a marked correlation between the two is observed, and this principle is used as a guide in areas where the data about the currents are more reliable than those about the sea-level. From these charts, graphs of mass anomaly per degree of latitude can be constructed for each season.

Lieut. Veley said that the calculated change in moment of inertia of the oceans from spring to autumn on the basis of these figures is -2.06×10^{36} gm. cm.^{2}. Munk's[1] figure for the change from January to July is $+ 0.8 \times 10^{36}$ gm. cm.^{2}. If these results results are plotted on one sine curve, it turns out that this effect may not be in phase with the main effect, which Munk considers to be due to the change in the angular momentum of the atmosphere. Munk's figure for the increase of this quantity above the mean, expressed as a fraction of the angular momentum of the earth, is $+ 0.8 \times 10^{-8}$ for January, whereas the corresponding quantity for the oceans derived from the present work is -0.5×10^{-8}. Both effects together give a resultant only about one-tenth of the observed fluctuation of rate.

Dr. Bondi described the work he and Dr. R.A. Lyttleton are carrying out on the motions in the earth's core caused by the known motions of the earth's mantle. These motions can be divided into those due to tidal friction, the lunar and the free nutation, which are small in the sense that all the displacements and velocities are small, and those due to precession which, though of small velocity, involve a large change in the direction of the earth's axis. The former type show no motions of very great interest ; tidal friction, for example, sets up a slow, steady internal circulation. The effect of precession has not been fully worked out ; but already it appears that its peculiar feature is the concentration of motions with appreciable shear along the surfaces of two intersecting cones of semi-angle 30° the apices of which are at the North and South Poles. This would lead to the formation of eddies which might well possess time-constants of order 10^2-10^3 years, and it is therefore possible that the complete calculation could give an explanation of Newcomb's term.

At this point in the meeting there followed a short discussion in which various speakers made suggestions as to the physical causes which might account for the difference between the observed variations in the earth's rotation and the variation attributable to the causes so far considered. It appears that variations in land glaciation can scarcely amount to much, since it has been calculated that, if the high central Asian plateau including the whole of the Himalayas were reduced to sea-level, the day would change its length by only 1 millisec. It has also been calculated that variations in oceanic circulation can contribute only one-eighth of the required effect ; this is probably because ocean currents circulate mainly in confined areas and so would give no total east-west transport. A suggestion that variation in the thickness of the ice-caps would produce an effect was examined, but it was considered that this can scarcely amount to more than a few cm. and so would change the earth's moment of inertia by a very small amount.

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(1) Munk W. H. and Miller, R. L. Tellus 2, 93 (1950).