

DETERMINATION OF PLANE OF REDUCTION OF SOUNDINGS AT ANY PLACE

by Marciano A. BALAY, Technical Advisor of the Dirección General
de Navegación e Hidrografía, Buenos Aires

FOREWORD

Among the different planes of reference that can be deduced from tidal observations and systematic recording, the following are considered of special importance :

Plane of reduction of sounding, or Tidal height datum ;

Mean Sea Level, or Geodetic datum;

Level of Shoreline or that marking limit between public and private property.

The first is a function of Low Water and of the type of tide at the place where it occurs.

The second is the result of the integration of the tidal curve.

The third is a function of High Water and of the type of tide at the place where it occurs.

The extreme levels, i.e. the plane of reduction of soundings and that which determines the shoreline, mark between them the coastal area normally and periodically covered and uncovered by the tide.

It appears natural that the same principle should apply in the determination of both planes.

This should be the principle which permits establishment of the higher and lower limits of the covering and uncovering coastal area at the usual Higher High Water level.

In this article, the subject treated is the lower limit considered as the plane of reduction of soundings or tidal datum, with a view to establishing a *general principle* for its determination.

It should be determined with the greatest possible accuracy, not only for purposes of chart construction, but also for use as a datum in harbour works and other public undertakings.

The *principle* set forth herein does not involve the establishment of any *single mathematical formula* since its application is a function of the type of tide of the area, i.e., dependent upon the nature of the *Low Waters considered* at each place.

The determination of the plane of reduction of soundings is not a function of tidal amplitude, but of the Low Waters reckoned from an arbitrary zero, the values of which show a certain degree of discrepancy in relation to its arithmetical mean, a discrepancy which will provide a criterion in considering the Low Waters entering into the computation.

It is not intended in this article to modify existing or earlier concepts but merely to define the principle to be followed in determining the plane of reduction of soundings, with the object of collaborating with those organizations who have not yet completed their chart construction or desire to solve this problem.

M.A.B.

DETERMINATION OF PLANE OF REDUCTION OF SOUNDINGS AT ANY PLACE

by Marciano A. BALAY (*)

The determination and definition of the plane of reduction of soundings have seriously preoccupied hydrographers all over the world and have repeatedly formed the subject of discussions at International Assemblies, especially at the Hydrographic Conferences held under the auspices of the International Hydrographic Bureau (1) and of the P. I. G. H. (2)

As is known, the motivation for these discussions lay in the danger to navigation represented by the existence of distinctly different planes of reduction of soundings in the various countries.

Actually, too low a plane of reduction makes the dangers of any area appear in exaggerated form, thus constituting an artificial obstacle for navigation, since the depths given on the chart will always be less than the true depths.

If the plane of reduction is too high, it will notably change the aspect of the area, resulting in a topography which will only infrequently be observed at Low Water, and it will often be necessary to apply large negative corrections to depths indicated on the charts, with consequent danger for the navigator with no good tidal prediction at his disposal.

The problem thus posed, a series of suggestions were made at successive Conferences, among which we shall cite the following :

a) That the idea of establishing an international plane of reduction based on a definite formula be abandoned.

b) That the planes of reduction be determined in accordance with the type of tide at each place.

(*) Technical Advisor of the Dirección General de Navegación e Hidrografía (Ministerio de Marina). Argentina.

(1) I.H.B. Special Publications, Nos. 5 and 10.

(2) Meetings at Buenos-Aires, 1948, and S.-de-Chile, 1950.

c) That its definition be simple and of concrete significance.

d) That it be easy to deduce, both from harmonic constants and from direct tidal observations.

At the I.H.B. Hydrographic Conferences held in the years 1926 and 1937 the question was intensively treated, and the resolutions given below (1, 2), were adopted, partly including the above-mentioned considerations :

1) « Tidal datum should be the same as chart datum, and should be a plane so low that the tide will but seldom fall below it. »

2) « A reference to Mean Sea Level of the datum of reduction for soundings should be shown clearly on charts and in Tide Tables. »

However, the expression « that the tide will but seldom fall below it » does not specify clearly the number of times or by what magnitude the tide may fall below datum for a designated period of time and place.

Hence there arises a series of definitions for the datum of reduction of soundings that are for the most part vague, and ambiguous, and of no actual significance.

These give rise, chiefly in areas where banks and channels with very marked inequalities of Low Water exist, to irregular cartographic representations which are never as seen by the navigator, nor do they define the state of the tide corresponding to the time represented by the chart.

The conflict between the plane of reduction and the tidal characteristics of the area considerably affects the safety of the navigator owing to the difficulty involved in making the topography of the chart agree with the corresponding state of the tide, which should be capable of being clearly deduced from the definition of the adopted plane.

From a rapid examination of the definitions for planes of reduction of soundings given by various charts and Tide Tables, the following may be extracted :

1. International Low Water.
2. Mean Low Water.
3. Lower Low Water.
4. Mean Low Water Springs.
5. Low Water Mean Springs.
6. Low Water Indian Springs.
7. Approximate Mean Low Water Springs.
8. Mean Low Water Diurnal Springs.
9. Lower Low Water Diurnal Springs.
10. Approximate Mean Low Water Springs.
11. One or two feet below Mean Low Water Springs.
12. One or two feet below Low Water Mean Springs.
13. Solstitial Low Water Springs.

14. Equinoctial Low Water Springs.
 15. Equinoctial Low Water Springs at perigee.
 16. Mean Lower Low Water at six-month intervals.
 17. Mean Sea Level (or approximately).
 18. Mean Lower Low Water Monthly Springs.
 19. Mean Lower Low Water Semi-Diurnal Springs.
 20. Mean Daily Lower Low Water.
- Etc...

From this lengthy series of definitions emerges the necessity of establishing a concept for determining the plane of reduction of soundings according to the type of tide in the area, in order to avoid the present state of confusion.

The plane of reduction of soundings is dependent upon Low Water at the place involved, but account must be taken of the type of tide of the area which actually characterizes the manifestation and sequence of its Low Waters.

This datum should be determined in such a manner that it may not be necessary to apply positive corrections other than those peculiar to the tide, and negative corrections only in extraordinary circumstances chiefly due to meteorological disturbances of an unusual kind.

The same rule should be closely applied in the case of Low Water, by representing the lower limit normally attained by the tide.

By " Ordinary Low Water " is meant all such latter peculiarities of the tide as recur periodically according to the sequence of the type of tide under consideration, with more or less inequality, the effects of strong meteorological or seismic influences being rejected.

For each separate type of tide, " Ordinary Low Water " will show varying diurnal inequality, producing a different distribution about its mean value, and consequent various degrees of dispersion.

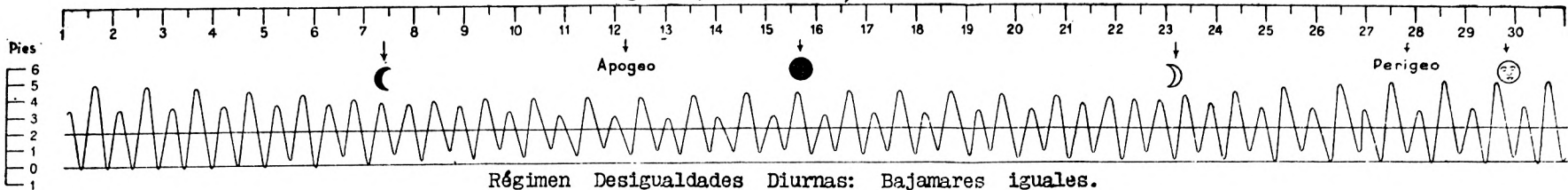
From an analysis of these, the actual significance of the arithmetical mean of observed Low Waters may be established. For example, in the case of the tide at Salina Cruz (Mexico) (fig. 1), where the tide can be defined as belonging to the " Diurnal inequality, Low Water equality " type, the arithmetical mean of its values will have an actual significance, since its dispersion in relation to the arithmetical mean will be extremely slight.

This will not apply in the case of Puerto Foster (Argentina), where the tide can be considered as being of the " Diurnal inequality, High Water equality " type, but where there exists great Low Water inequality. Its arithmetical mean will therefore not be significant, owing to the considerable dispersion of values in relation to it.

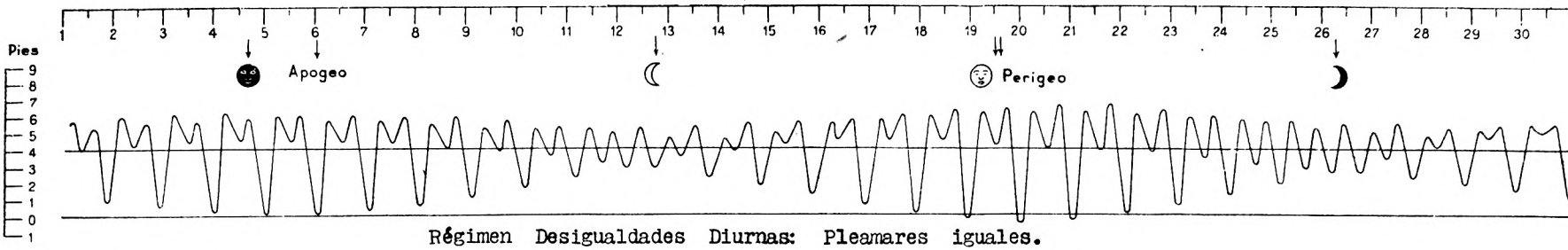
It may easily be seen that the plane of reduction obtained by the simple arithmetical mean of Low Water gives rise to an arbitrary cartographic representation of the area under consideration.

The same is true for other types of tide.

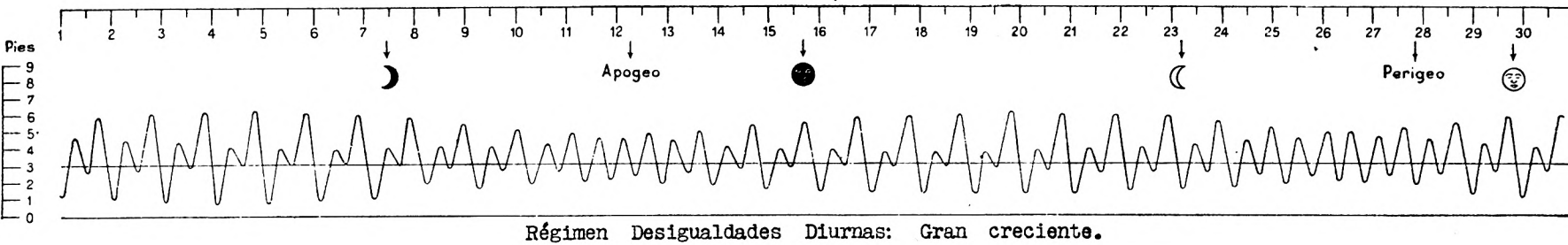
SALINA CRUZ, MEJICO



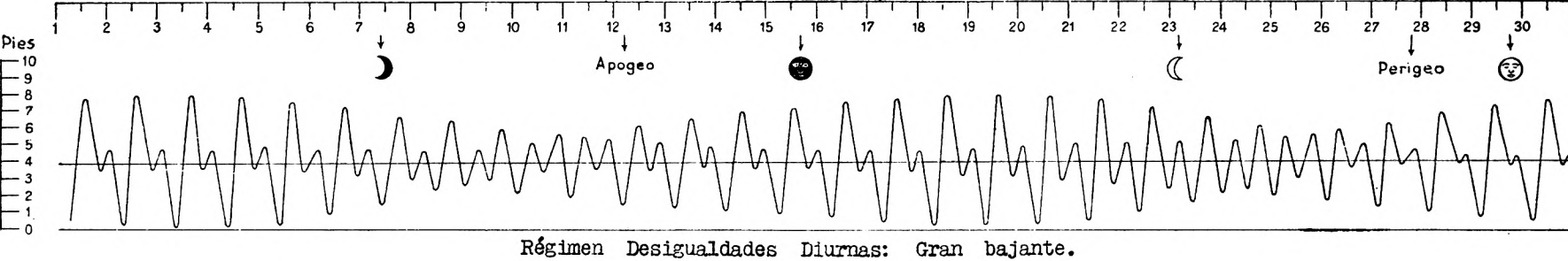
FOSTER, ARGENTINA



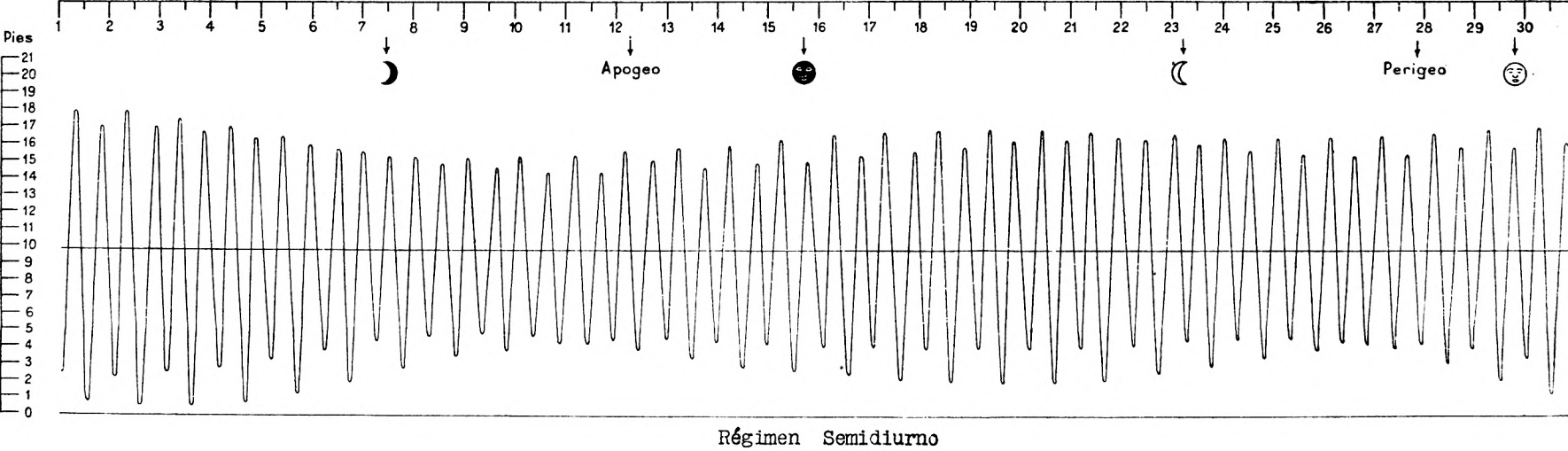
MAR DEL PLATA, ARGENTINA



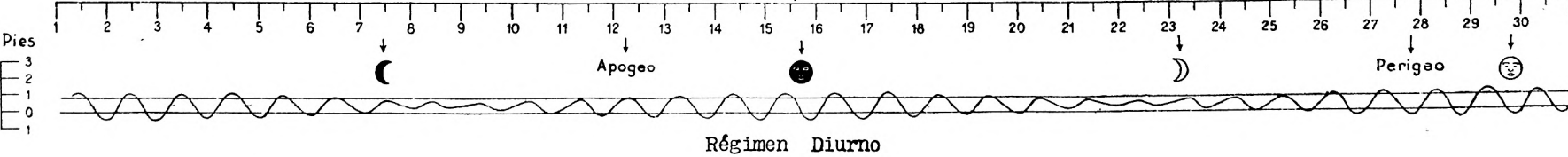
PUNTA ARENAS, CHILE



MADRYN, ARGENTINA



TAMPICO, MÉJICO



CRISTÓBAL, PANAMÁ

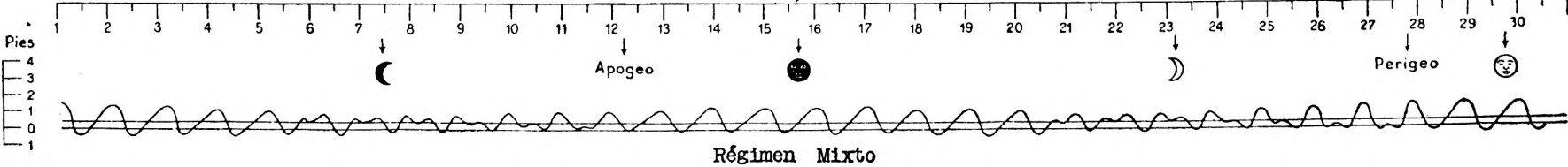


Fig. 1

Various tidal regimes

The arithmetical mean of a series of values will have real significance if these represent measurements of one and the same phenomenon originated by identical action or corresponding to equivalent conditions and times as in the case of tides at syzygies, quadratures, tropics, etc...

The arithmetical mean of all observed Low Waters, except in the case corresponding to the " Equal Low Waters type ", will result in a higher value having no actual meaning and frequently exceeded by the tide.

Hence the necessity of establishing for each type of tide the Low Waters which have to be considered so that the arithmetical mean corresponds to a definite condition of the tide at the place and is reached only a number of times as defined by the statistical analysis of the series.

These Low Waters should be " the Lowest Low Water " of the ordinary tides which are recorded periodically, presenting only certain differences due to the logical result of the variations of the characteristic positions of the actuating heavenly bodies in their successive cycles, which slightly modify the generating force in relatively equivalent situations ; and to varying meteorological action whose effects cannot be checked.

From the foregoing, the natural definition of the plane of reduction of soundings would appear to be : « The lowest limit of ordinary Lowest Low Water ».

The same concept of normality applies in establishing the higher limit of ordinary Highest High Water which a few authors of treatises have adopted for the determination of the shoreline (3).

It appears evident that in both extreme planes of reference, the concept of normality should be accepted as being that which is common or frequent.

This definition is satisfactory when it is a question of mutually exclusive phenomena and when it is possible to establish clearly the normal as distinguished from the abnormal, and to set their limits accurately.

In the case of tides, however, it is difficult to discern when the Lowest Low Waters at any place are normal or ordinary and when they are not so, leaving aside those influenced by important atmospherical disturbances.

The problem becomes more complicated when we consider the different types of tide because each of them offers particular characteristics in the sequence of its Low Waters (fig. 1).

With the help of statistical analysis we can nevertheless establish the concept of normality in the manifestation of Lowest Low Water, which will permit determination of the lowest limit representing the normal and materially defined plane of reduction which we are trying to find.

(3) Ing. H. MEOLI, R. del C.E. de I. No. 434-35 and 36.

For this purpose it will first of all be necessary to establish the tide cycles which represent all the manifestations of the phenomenon, with its maxima and minima, for the classification of Low Waters in groups according to appropriate intervals.

By means of this classification the frequency with which Low Waters occur may be established and its limits of variation deduced by analysis.

The graphical representation of these groups of Low Waters by histogram enables study of the frequency-polygon and later of the frequency curve, if we increase indefinitely the number of observations.

If we examine all the Low Waters of a series of observations, the form of the resulting frequency diagram will show various asymmetrical characteristics, with one or two maxima, according to the type of tide analysed.

However, as has already been shown, it will be sufficient to determine for each type of tide which Low Waters should be considered as the ordinary Lowest LW.

According to M. A. Courtier (4) the tide can be classified in four perfectly defined types according to the value of the coefficient $\frac{K_1 + O_1}{M_2 + S_2}$, which repre-

sents the ratio of the semi-range of the diurnal and semi-diurnal waves at the respective times of syzygies.

These four most commonly accepted tidal types have well-defined characteristics, which enable the establishment in each case of the classification relating to *ordinary Lowest Low Waters*.

They are :

(1) *Semi-diurnal Type.*

$$\frac{K_1 + O_1}{M_2 + S_2} < 0.25$$

Two High Waters and two Low Waters daily of appreciably equal heights are noted. The amplitude varies during synodical months.

Ordinary Lowest Low Water : Low Water Springs.

(2) *Type with daily inequalities.*

Generally two HWs and two LWs daily are noted, but of unequal heights owing to the action of the diurnal wave. This inequality is maximum at the time

(4) M. A. COURTIER, B.H.T., « Int. Hydr. Review. Vol. XVI, No. 1, 1939.

of tropic tides. The tidal sequence consists of four forms according to the value of the expression :

$$2K = K_{M_2} - (K_{K_1} + K_{O_1}) - n 720^\circ$$

Let :

- a) $2K = 0$: Two equal Low Waters
- b) $2K = 180$: Two equal High Waters
- c) $\left\{ \begin{array}{l} 0 < 2K < 180^\circ \\ 180 < 2K < 360^\circ \end{array} \right.$: Great rise (Unequal High and Low Waters)
: Low fall (Unequal High and Low Waters)

Ordinary Lowest LW :

For a) All Low Waters.

For b) All Low Waters at springs.

For c) All Low Waters at tropic springs.

(3) *Mixed Type.*

$$1.5 < \frac{K_1 + O_1}{M_2 + S_2} < 3$$

Sometimes a single HW and a single LW are noted (moon transit at tropics), and sometimes two HWs daily (moon transit at equator). Full tides occur at the time of diurnal springs or at the time of Tropic Tides.

Ordinary Lowest LW : Low Water at diurnal and semi-diurnal springs.

(4) *Diurnal Type.*

$$3 < \frac{K_1 + O_1}{M_2 + S_2}$$

Generally one single HW and one single LW daily are noted. When the moon is near the equator the diurnal wave tends to disappear and the semidiurnal wave prevails, giving rise to two HWs and two LWs of small range. The largest tides occur at the time of tropic tides.

Ordinary Lowest LW : Low Waters at tropic springs. (*See fig. 1*).

This classification of ordinary Lowest Low Waters for each type shows the Low Waters which will follow a law of normal frequency distribution, whose graphical representation will take a more or less symmetrical bell-shape.

The frequency diagrams drawn with the Low Water values thus classified will be similar to the curve of normal probability or Gauss curve.

The abscissae represent the value of the Low Waters and the ordinates the number of times that each of them has been recorded.

The most usual measurement for dispersion in the statistical series is the mean square deviation.

Statistics prove that, in frequency curves which obey the simple law of probability, the value of the mean square deviation determines on both sides of the mean ordinate the points of inflection that, by indicating a change in the frequency rate, are accepted as the normal upper and lower limits of the arithmetical mean of the series represented (5).

Before applying these concepts to the frequency diagram let us put down the equation of the normal probability curve and some of its properties :

$$y = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x}{\sigma} \right)^2}$$

where

y=ordinate

σ =mean quadratic deviation

x=abscissa

e=2.7182

The ordinate of the point of origin $x=0$ gives :

$$y = \frac{1}{\sigma \sqrt{2\pi}}$$

The curve is symmetrical and extends regularly on both sides of the maximum ordinate.

It is customary to compute the ordinates making $\sigma=1$ and expressing x in deviations σ related to the central value.

The equation takes the following form :

$$y = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

The area which contains the normal probability curve represents the total number of frequencies so that the normal probability curve contained between the ordinates $x_1 = -1\sigma$ and $x_2 = +1\sigma$ (points of inflection) will be represented by :

(5) Ingeniero A. ARMANI. D.G. de Navegación y Puertos M. O. P.

$$S = \frac{1}{\sqrt{2\pi}} \int_{x_1=-1}^{x_2=+1} e^{-\frac{x^2}{2}} \cdot dx = 0.68$$

which being greater than 0.50 satisfies the concept of normality.

It can therefore be stated that the observations contained in the central group of the diagram thus determined (68 %) represent the usual or most frequent values and of those remaining, consisting of 16 % on either side of the higher and lower limits, indicate the unusual or less frequent values.

The abscissa for the point of inflection corresponding to the lower limit determines, as will be seen further on, the plane of reduction that we are trying to find.

CURVA NORMAL DE LA PROBABILIDAD

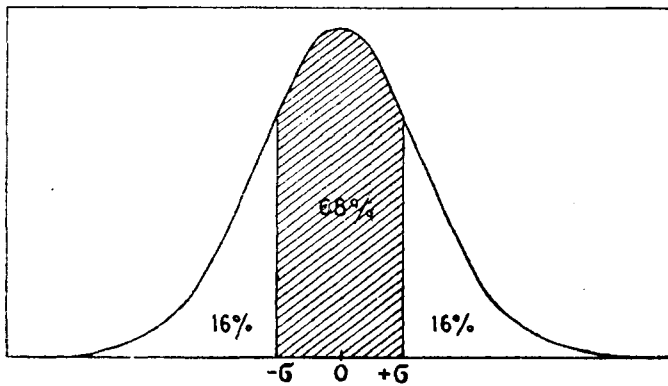


Fig. 2

Normal Probability Curve

Diagram of Observed Mean Frequencies

For the construction of the frequency diagram it is first necessary to arrange the observations in a certain statistical order, and secondly to classify them at intervals depending upon the number of observations available.

For the selection of the time interval the following considerations, deduced from experience, will be taken into account :

a) For an equal number of observations, the diagram tends to become regular as the interval increases, its distribution varying, however, as related to the arithmetical mean.

b) The diagram tends to become regular when the time interval is amplified more rapidly than when the number of observations is increased.

c) As the time interval becomes smaller and the number of observations increases indefinitely, the diagram tends towards the normal frequency curve.

Let the series of observations corresponding to Low Water Springs at Puerto Madryn be referred to the zero of the tide-gauge, which here reaches 356 cm. below Mean Sea Level.

1944	1945	1946	1947	1948	1949	1950
cm.	cm.	cm.	cm.	cm.	cm.	cm.
—	—	—	—	—	—	—
	86	128	68	147	138	89
	83	83	147	55	81	89
	113	86	88	122	120	64
	41	122	102	19	58	97
	106	83	76	100	108	76
	64	91	82	106	66	111
	111	73	151	123	150	78
	39	55	72	87	76	159
	59	60	101	111	108	77
	54	139	103	123	77	129
	161	84	107	107	105	83
	76	110	97	68	85	128
	81	83	204	104	51	120
69	—11	72	58	161	102	172
86	102	59	82	—7	82	89
43	85	82	69	139	95	97
48	37	58	120	54	76	
105	87	73	41	97	80	
53	36	107	101	72	58	
117	96	55	92	126	109	
36	59	113	113	93	94	
113	138	76	94	131	92	
71	65	138	92	118	67	
103	142	126	125	119	119	
	75	138	113		103	

Computation of Arithmetical Mean and of Mean Square Deviation (*)

Classification interval : a=20 cm.	Middle Point m.	Fre- quency y ₀	δ''	(δ'') ²	δ'' y ₀	(δ'') ² y ₀	x y ₀
— 90/ — 71	— 80	1	— 7	49	— 7	49	— 80
— 70/ — 51	— 60	0	— 6	36	0	0	0
— 50/ — 31	— 40	0	— 5	25	0	0	0
	— 20	1	— 4	16	— 4	16	— 20
— 10/ 9	0		— 3	9	— 3	9	0
10/ 29	20	1	— 2	4	— 2	4	20
	40	8	— 1	1	— 8	8	320
50/ 69	60	24		0		0	1.440
70/ 89	80	38	1	1	38	38	3.040
90/ 109	100	34	2	4	68	136	3.400
110/ 129	120	26	3	9	78	234	3.120
130/ 149	140	10	4	16	40	160	1.400
150/ 169	160	5	5	25	25	125	800
170/ 189	180	1	6	36	6	36	180
190/ 209	200		7	49	7	49	200

$\Sigma y_0 = 151$

$\Sigma \delta'' y_0 = 238$

$\Sigma xy_0 = 13820$

$\Sigma (\delta'')^2 y_0 = 864$

Arithmetical mean

$$M_x = \frac{\Sigma xy_0}{\Sigma y_0} = \frac{13820}{151} = 91,52$$

$$\Delta = \frac{\Sigma \delta'' y_0}{\Sigma y_0} = \frac{238}{151} = 1,576$$

Mean Square Deviation

$$\sigma = a \cdot \sqrt{\frac{\Sigma (\delta'')^2 y_0}{\Sigma y_0}} - \Delta^2 = 35,8$$

$M_x = 92 \text{ cm.}$

$\sigma = 36 \text{ cm.}$

With the frequencies (y₀) as ordinates, let us draw the diagram of observed mean frequencies (fig. 3) which follows the law of the normal curve.

(*) See text re Mathematical Statistics.

DIAGRAMA DE FRECUENCIAS MEDIAS OBSERVADAS

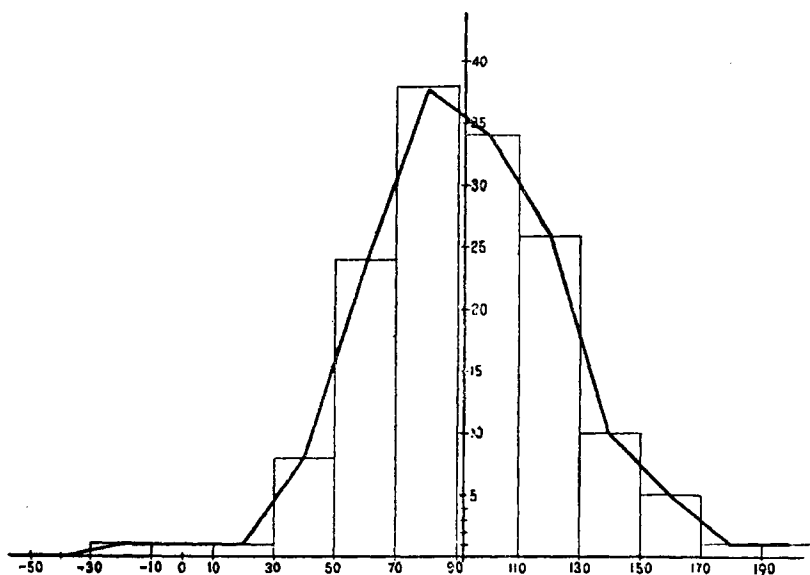


Fig. 3

Diagram of Observed Mean Frequencies

Theoretical Frequency Diagram

The process of adapting the frequency diagram to the normal curve can be simplified with the help of tables of ordinates given as fractional parts of the maximum ordinate.

$$y_0 = \frac{N}{\sigma \sqrt{2\pi}} = 33,65$$

in terms of $\frac{x}{\sigma}$ which represents the deviations with respect to the mean, expressed

in units of the mean square deviation (6).

For example, the ordinate corresponding to the mean square deviation is equal to 0.60653 y_0 .

To adapt a simple probability curve to the diagram of observed mean frequencies, the deviations should first be expressed in fractions of σ ; secondly, the value of the maximum ordinate should be determined; and, thirdly, by means of the table given below, each of the ordinates should be calculated.

(6) If greater precision in the determination of the theoretical frequencies is desired, areas instead of ordinates should be measured using the Tables of the areas of the curve in terms of its of abscissae.

Table

Ordinates of the Normal Curve expressed in Fractions of the Maximum Ordinate (7)

x/σ	y/y_0	x/σ	y/y_0
0,0	1,00000	2,5	0,04394
0,1	0,99501	2,6	0,03405
0,2	0,98020	2,7	0,02612
0,3	0,95600	2,8	0,01984
0,4	0,92312	2,9	0,01492
0,5	0,88250	3,0	0,01111
0,6	0,83527	3,1	0,00819
0,7	0,78270	3,2	0,00598
0,8	0,72615	3,3	0,00432
0,9	0,66689	3,4	0,00309
1,0	0,60653	3,5	0,00219
1,1	0,54607	3,6	0,00153
1,2	0,48675	3,7	0,00106
1,3	0,42956	3,8	0,00073
1,4	0,37531	3,9	0,00050
1,5	0,32465	4,0	0,00034
1,6	0,27804	4,1	0,00022
1,7	0,23575	4,2	0,00015
1,8	0,19790	4,3	0,00010
	0,16448	4,4	0,00006
2,0	0,13534	4,5	0,00004
2,1	0,11025	4,6	0,00003
2,2	0,08892	4,7	0,00002
2,3	0,07100	4,8	0,00001
2,4	0,05614	4,9	0,00001
		5,0	0,00000

(7) Cecil Mills Statistical Methods.

Calculation of Normal Theoretical Frequency Curve,
adapted to Distribution of Observed Frequencies

Middle point m	Deviations with respect to M (interval as unit) x	x — σ	y — y ₀	Theoretical Frequency (Ordinates) y
— 80	— 8.576	— 4.79	0.00001	0.00
— 60	— 7.576	— 4.23	0.00013	0.00
— 40	— 6.576	— 3.67	0.00120	0.04
— 20	— 5.576	— 3.12	0.00775	0.26
0	— 4.576	— 2.56	0.03801	1.28
20	— 3.576	— 2.00	0.13534	4.55
40	— 2.576	— 1.44	0.35505	11.95
60	— 1.576	— 0.88	0.67874	22.84
80	— 0.576	— 0.32	0.94942	31.95
100	0.424	0.24	0.97052	32.66
120	1.424	0.80	0.72615	24.43
140	2.424	1.35	0.40243	13.54
160	3.424	1.91	0.16157	5.44
180	4.424	2.47	0.04760	1.60
200	5.424	3.03	0.01023	0.34

With these ordinates let us draw the normal frequency curve (Fig. 4).

Mean Square Error of Theoretical Frequencies used for Adaptation

Figure 4 shows the figure of observed frequencies jointly with the normal curve adapted according to the preceding computations.

From observation emerges the existence of a law of distribution of the Low Waters considered (Springs) agreeing with the Gauss normal law of error or probability curve.

The small differences are attributable to the restricted number of observations.

However, we can determine the range of the greatest differences between the observed and the theoretical differences.

As is known, the mean square error of a distribution of frequencies is supplied by the following formula :

$$\sigma = \sqrt{\frac{f(N-f)}{N}} \quad \text{where}$$

f = theoretical frequency at any point.

N = total number of observations.

**CURVA NORMAL DE FRECUENCIAS
ADAPTADA AL DIAGRAMA DE FRECUENCIAS OBSERVADAS**

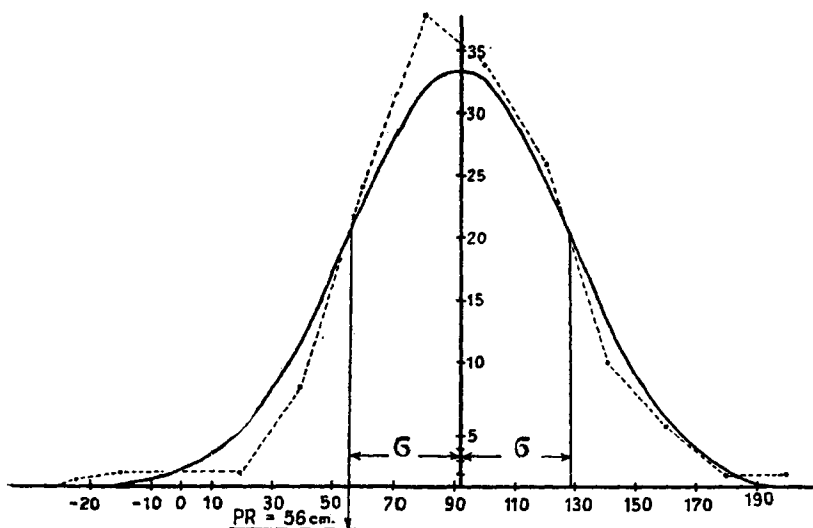


Fig.4

$M_x = 92 \text{ cm.}$

$G = 36 \text{ cm.}$

Plano Reduc. = 56 cm. sobre el cero del mareografo

Nivel medio = 356 cm. " " "

Plano Reduc. = 300 cm. debajo del nivel medio del mar

Normal Frequencies Curve adapted to the Observed Frequencies Diagram.

Plane of Reduction above Tidal Datum.

Mean Level.

Plane of Reduction below Mean Sea Level.

An examination of the cases showing large discrepancies gives :

m	y	y°	y - y°	σ
40	12	8	4	3
80	32	38	6	5
120	24	26	2	4

We see that the differences are within the mean error of each of the theoretical frequencies, so that it may be asserted that the frequency distribution of LW springs at Puerto Madryn effectively follow the Gaussian law or the normal probability curve. The points of inflection of the normal curve determined by the ordinates corresponding to the abscissae - σ and +σ reckoned from the mean value will respectively fix the lower and higher normal limits of the arithmetical mean of the series represented. The abscissa corresponding to the point of inflection - σ from the origin determines the figure of the plane of reduction of soundings as being the *lower limit of Ordinary Lowest Low Water*.

Consequently, the figure of the plane of reduction of soundings at any place remains determined by the arithmetical mean of Ordinary Lowest Low Water with reference to its type of tide less the mean square deviation.

DEFINITIONS OF THE PLANE OF REDUCTION

From the foregoing study the following can be accepted as a general concept in determining the plane of reduction of soundings: « *Lower limit of Mean Ordinary Lowest Low Water* ».

This general concept is developed according to four types following the behaviour of the tide at the place under consideration:

Type :

Semidiurnal and

Diurnal Inequalities (Equal HW type).

1) « Lower limit of mean LW springs ».

Type :

Diurnal Inequalities (Equal Low Water type).

2) « Lower limit of mean of all Low Waters ».

Type :

Diurnal and

Diurnal Inequalities (Great rise and low fall type).

3) « Lower limit of mean of tropical Low Water springs ».

Type :

Mixed.

4) « Lower limit of Mean Diurnal and Semidiurnal Low Water springs ».

In the first case it is deduced from the definition that the type of tide at the place is « Semidiurnal » with diurnal inequalities, chiefly at Low Water.

The cartographic representation of the area will correspond to the aspect most frequently occasioned by semidiurnal LW Springs, and the navigator will be enabled to make a useful comparison with the help of Tide Tables.

The second case means that the type of tide at the place is semidiurnal and is characterized by equal, or appreciably equal Low Waters. Here, the charted aspect of the area at all Low Waters, with the slight deviations indicated in the Tide Tables, will be available to the navigator.

The third case shows that the type of tide at the place is chiefly « diurnal » and also « semidiurnal » but with great daily inequalities in its Low Waters and High Waters.

At places where the tide shows strong daily variations, the navigator can bring the cartography into useful comparison only at times of tropic tides.

At other times he will find, at certain Low Waters, great differences between the aspect of the chart and the area in question.

From case No. 4 emerges the presence of a mixed type, with one single High Water (tropic tides), or two diurnal High Waters and Low Waters (moon at the equator), with strong inequalities during a lunation.

At such places the range of the tide is generally small and very irregular, and consequently the navigator can make an approximation of the area as charted only at certain times of semidiurnal springs coincident with tropic tides.

In the case of these definitions, expressions and their meanings can more or less be adapted in terms of harmonic constants, as follows :

Semidiurnal :

« Lower limit of Mean Low Water Springs ».

Expression : $ML - (M_2 + S_2 + N_2 + K_2)$.

Meaning : Perigean Equinoctial Springs.

Diurnal Inequalities :

a) *Type Equal Low Waters :*

« Lower limit of mean of all Low Waters ».

Expression : $ML - (M_2 + S_2 + N_2)$.

Meaning : Perigean Springs.

b) *Type Equal High Waters :*

« Lower limit of Mean Low Water Springs ».

Expression : $ML - (M_2 + S_2 + N_2 + K_1 + O_1)$.

Meaning : Perigean Tropical Springs.

c) *Type Great Rise or Low Fall :*

« Lower limit of mean of Low Waters of Tropical Springs ».

Expression : $ML - (M_2 + S_2 + K_1 + O_1 + P_1)$.

Meaning : Solstitial Tropical Springs.

Mixed :

« Lower limit of Mean Diurnal and Semidiurnal Low Water Springs ».

Expression : $ML - (M_2 + S_2 + K_1 + O_1)$.

Meaning : Diurnal and Semidiurnal Springs.

Diurnal :

« Lower limit of mean Tropical Low Water Springs ».

Expression : $ML - (M_2 + S_2 + K_1 + O_1 + P_1)$.

Meaning : Solstitial Tropical Springs.

However, it ought to remain an established rule that the plane of reduction of soundings at any port of reference (or Standard Port) must be obtained directly from Low Waters observed over a long period of time.

A series of observations covering a minimum period of from four to five years is considered satisfactory (Semi-period of revolution of lunar perigee), but more trustworthy results will be obtained from a complete cycle of 18.6 years (period of revolution of ascending node of the moon).

When a long series of values is not available, the plane of reduction should be deduced from simultaneous observations obtained at a standard tide-gauge station or a place where it has been accurately determined.

This will always be more satisfactory than the calculation by harmonic constants obtained from a short period of observations, since it will not be possible to take into account the « shallow water » and long-period waves which at some places considerably affect the tide, principally in areas that are most dangerous to navigation.

This concept for determining the figure of the plane of reduction of soundings (or tidal height datum) has been applied to observations obtained at different places along the coast with satisfactory results.

In all of them the frequency distribution of Lowest Low Water with reference to the type of tide at the place follows the law of the normal probability curve, as in the case of Puerto Madryn illustrated in this article.
