NEW GEODETIC ASTRONOMY METHOD: THE METHOD OF AZIMUTH LINES

(Lecture delivered by Ingénieur Hydrographe en Chef André GOUGENHEIM, during the VIth International Hydrographic Conference, Monaco, April-May 1952).

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1. *Astronomical Determinations during Hydrographic Surveys.* — Hydrographers seldom have to effect very accurate astronomical determinations while surveying. Whenever they operate on the coast of an important land area, their geodetic work, which is for the purpose of fixing their signal positions, consists of additional triangulations extending the main net covering the country along the coast ; the geographical positions of the vertices are derived from a basic astronomical position that has been determined previously. If a new country is involved for which no geodetic description as yet exists, or if one has not been made of the area being surveyed, the hydrographic survey is based on an astronomical position adopted temporarily, pending the connection of the triangulation carried out by the hydrographic survey to the main net. Only in the case of isolated areas and remote islands or archipelagoes do astronomical determinations by hydrographic surveyors retain any final value, owing to the fact that the present state of geodetic technique does not permit the geometric connection of such areas to continental nets.

It will moreover be realized that in either the case of new countries or insular areas, where astronomical operations must be carried out by the hydrographic surveyors themselves, extreme accuracy is theoretically not essential. The principal requirement consists in the obtaining of a sufficiently close approximation so that no appreciable error will result in the construction of charts on any scale. Hydrographic surveyors are therefore justified in making their astronomical observations in accordance with relatively expeditious methods. This has considerable advantages, since it is a well-known fact that the search for greater accuracy involves along with it a vast increase in the costliness of and space taken up by instruments, the volume and length of observations and computations, as well as in the amount of personnel and resources that must be made available.

2. *Purpose and Conduct of Astronomical Operations.* — The sole purpose of the astronomical operations to be carried out is the determination of the geographical co-ordinates of a triangulation vertex taken as a basic position and the azimuth of a side issuing from this vertex. Actually, although latitude is supplied directly by these, the same is not true in the case of longitude and azimuth ; they merely enable the setting of a timepiece to local time and the orientation with respect to the meridian of the horizontal limb of an instrument. It is by means of other determinations made with the latter instrument and the timepiece that the longitude of the place and azimuth of a land bearing are obtained.

To sum up, the three unknown quantities in geodetic astronomy are latitude (φ), the clock correction (Cp) to be made with respect to the indications of a chronometer in order to obtain local sidereal time, and finally the azimuth (V) of the zero of a horizontal limb. Examination of the triangle of position in relation to a star (pole, zenith, star) shows that it is impossible in theory to determine each of the unknowns separately and that they can only be obtained by combining several observations.

Until the early part of this century, owing to the lack of a method of calculation enabling the solution of systems of observational equations supplied by spherical trigonometry, special cases were taken in which calculation or the elimination of unknowns became a simple matter, or else (which amounts to the same thing) it was considered that determination of each of the three unknown quantities involved a separate problem ; and the essential purpose of observation and computation methods was to supply each of the unknown quantities independently of the other two, which were generally assumed as being either unknown or known approximately.

The portable meridian circle was essentially resorted to for the obtaining of accurate results, but use of this instrument required careful installation, a lengthy and arduous process in the field. Use of the theodolite was far more expeditious, but the degree of accuracy obtained left a great deal to be desired ; methods of observation attempted especially to mitigate difficulties of a material nature, due to the fact that before radiotelegraphy the keeping of time by means of chronometers was a delicate operation ; and as there was no electric source of supply for the illumination of reticules and graduated arcs, field observations of stars were a troublesome performance.

Shortly after 1900, a great advance was made with the coming into use of equal altitudes instruments, the most widespread of these being the prismatic astrolabe ; good simultaneous determination of latitude and local time is rapidly arrived at with the latter, but the method has the disadvantage of not supplying azimuth, which involves a separate determination by theodolite.

3. *Possibility of Recourse to the Theodolite.* — This is the situation existing at present. Among the stock of instruments on board surveying vessels, there is always an instrument for determining equal altitudes which, for reasons we mentioned earlier, is rarely used ; and this, quite apart from the aspect of economy, naturally has a most unfavourable effect on the quality of results, owing to the lack of training of the personnel. How many hydrographic surveyors have never effected the determination of a geographical position during the course of their careers, or have done so only once ! With respect to these intruments, the hydrographer is usually a novice ; whenever he has to use them, he may spend a great deal of time practising beforehand and yet fail to achieve perfect mastery during effective operation or to make the utmost use of all the accuracy of which the instrument is capable.

He would certainly show greater skill with a theodolite, with which he is much more familiar ; his experience with the theodolite would compensate to a certain extent for the slightly inferior accuracy of the instrument as compared with the equal altitudes instrument, and it is likely that the degree of approximation finally attained would be of the same order. During the past thirty years, as a matter of fact, improvements have taken place in the construction of precision theodolites that have greatly increased the accuracy and convenience of operation of these instruments, with particular reference to astronomical observations.

But what methods of observation should appropriately be used ? The existence of radio time signals, of instrument illumination devices, recording chronographs and rational calculation methods, is ample justification for not returning to earlier systems of operation, but rather for directing one's attention towards new methods. Up to now, efforts have especially been made to adapt the theodolite to the accurate determination of geographical positions through application of the method of equal altitudes. The method of azimuth lines, which we recommend and are about to describe, is more complete as it enables the three unknowns — latitude of station, local time and orientation of the arc of the instrument — to be obtained simultaneously.

4. *General Method of Azimuth Lines.* — The method of azimuth lines — justification for this designation will be made later — is merely an extension to position astronomy of the method currently used in geodesy known as « determination of position by station ». When sights are taken around the horizon, that is when azimuthal sights are taken by theodolite from an unknown position on known positions, it is possible to derive the position of the station and orientation of the instrument.

This method has an identical application in astronomy, by taking stars as the known positions ; the position derived is that of the zenith of the station, whose co-ordinates are the latitude and local time. The only extra precaution that should be taken is a note of the instant of each pointing in order to account for diurnal motion and to reduce all observations to the same instant.

Each sighting of a star (right ascension α , declination δ) therefore involves a reading (L) of the horizontal limb and the recording by means of a chronometer of the corresponding instant (θ). These four given quantities are combined with the three unknown quantites (φ Cp V) by the cotangent formula applied to the position triangle.

$$\sin\varphi \cos(\theta - \alpha + Cp) - \cot(V + L) \sin(\theta - \alpha + Cp) - \cos\varphi \tan\delta = 0$$

As the unknown quantities are three in number, it is necessary to make three observations and solve the system constituted by the three corresponding equations. The use of an approximate solution (φ_0 Cp_0 V_0) enables all difficulties of computation to be avoided, as the observation equations may then be rendered linear. If one takes as the new unknown quantities

$$\Delta\varphi = \varphi - \varphi_0 \quad \Delta p = (Cp - Cp_0)\cos\varphi_0 \qquad \Delta q = (Cp - Cp_0)\sin\varphi_0 - (V - V_0) \;;$$

if the approximate zenithal distance of the star is called ζ, its *computed* approximate azimuth Z', both quantities being determined with the help of the approximate solution ; finally, if the quantity $V_0 + L$ representing the approximate azimuth of the star *deduced from the observation* is termed Z, the observation equation is written

$$\sin Z\, \Delta\varphi - \cos Z\, \Delta p + \tan\zeta\, \Delta q = \tan\zeta\, (Z - Z')$$

It is entirely analogous to the one used in geodesy,

$$\sin U\, \Delta Y - \cos U\, \Delta X - D\Delta V = D(U' - U)$$

where X Y V designate the unknowns, co-ordinates of the station and bearing of the zero of limb ; X_0 Y_0 V_0 an approximate solution ; $\Delta X = X - X_0$, $\Delta Y = Y - Y_0$, $\Delta V = V - V_0$, the new unknown quantities ; D and U' the approximate distance and bearing of the sighted point obtained with the help of the approximate solution, and $U = V_0 + L$ the approximate bearing derived from the observation.

A comparison of the two equations shows that distance D from the sighted point corresponds in the astronomical method to the tangent of the zenithal distance of the celestial body.

In effecting calculations, only Z' need be computed with a high degree of accuracy, of the order of that supplied by the observation instrument ; while for ζ an approximation of a few minutes will be satisfactory. If the solution $\varphi_0 C p_0 V_0$ is sufficiently approximate, it is enough to know the coefficients of the observational equations within three places of decimals.

In general, in order to rid results insofar as possible of accidental errors that may for various reasons occur, more than three observations are made ; the observational equation system then becomes super-abundant and the most likely solution is obtained by application of the method of least squares. One might also operate as in geodesy, by combining the equations, in order to determine $\Delta\varphi$ and Δp separately by a graphical method and then Δq.

5. *Special Case. Method of Azimuth lines for Constant Altitude.* — We shall not, however, enlarge upon the general case, as it is possible in astronomy to make use of a special case of the method which greatly facilitates its application.

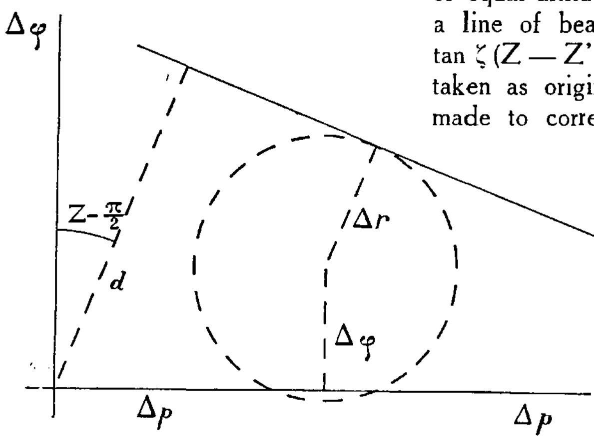
While known positions in geodesy are few in number and are located at well-defined distances, stars in very large numbers may be used during astronomical observations, whose diurnal motion moreover causes constant variation in the zenithal distance. It is therefore possible in this case to impose upon observations the additional condition of their being taken at an appreciably constant altitude above the horizon, while in geodesy it is obviously impossible to choose known positions that would all be at the same distance from the station.

We saw earlier that it generally sufficed to know coefficient tan ζ within three decimals ; this is the approximation, then, that defines the constancy of the height of observation. The telescope of the theodolite should therefore be set at the selected height, but the passage of the celestial body need not be observed exactly at the crossing of the hairs of the reticule, which can be accomplished accurately only with difficulty. As in the general case, the passage of the body under the vertical hair should be sighted, but it can easily so be arranged that the passage occurs in the vicinity of the horizontal hair, as a discrepancy of one or two minutes of a degree is one that can perfectly well be tolerated.

Under these conditions tan ζ retains practically the same value for all observations, and the auxiliary unknown quantity $\Delta r = \tan \zeta \, \Delta q$ can be substituted in the observational equation for the unknown Δq. The equation is then written :

$$\sin Z \, \Delta\varphi - \cos Z \, \Delta p + \Delta r = \tan \zeta (Z - Z')$$

It can be seen immediately that the system of observational equations is capable of a graphical resolution similar to that which is used for observations of equal altitudes. On a large-scale diagram, a line of bearing Z passing at a distance tan ζ (Z — Z') from the approximate position, taken as origin of the co-ordinates, can be made to correspond with each observation. The straight lines should as closely as possible envelop a circumference whose centre has as co-ordinates $\Delta\varphi$ and Δp and whose radius has a value of Δr. From these quantities recorded on the diagram it is an easy matter to proceed to the three unknown quantities φ Cp and V, which are thus determined.



The circumference-envelope is all the better defined as the stars are well-distributed in azimuth ; moreover, this distribution offers the advantage of the greatest possible elimination of the influence of limb division errors on the value of instrumental azimuth, since all parts of the limb contribute readings for its determination. The straight lines represent the various observations resulting from azimuth sights, and it is therefore legitimate to term them « azimuth lines », in analogy with the term « position lines », adopted for the designation of lines corresponding to measurements of star altitudes (1).

6. *Practical Considerations.* — (a) A compromise must be made in the *common altitude of observation* between two contradictory conditions ; for it would be appropriate to make observations fairly close to the zenith in order to obtain a good geographical position and relatively near the horizon to determine orientation with accuracy. The analogy in geodesy is cause for similar comment ; as we have already stated, the method under consideration is equivalent to the determination of a geodetic station by sighting known points which are all located at approximately the same distance from the station : a purely theoretical case, in view of the low density of geodetic positions ; the position of the station is all the more accurate as the common distance of the points sighted is short ; for the orientation of the station, however, a greater distance would be preferable.

For astronomical observations of azimuth at a constant altitude, the most advantageous height appears to be 45° ; as tan ζ is equal to 1, the three unknowns are obtained with the same order of approximation ; at lower altitudes, determination of azimuth is better than that of position, while the reverse is true at higher altitudes.

(b) The *preparation of observations* is identical to that of observations of equal altitudes : the approximate time of passage of the celestial bodies at the altitude adopted and the corresponding azimuth must be determined. For an altitude

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(1) In French, the analogy is more evident, as 'position lines' are termed 'altitude lines' (*droites de hauteur*). *Ed.*

of 45°, one may therefore use the tables computed for Baker's pentagonal prismatic astrolabe. But the most common tables are those for the equilateral prismatic astrolabe, which has an instrumental height of 60°, and this is naturally the height that one will be most tempted to adopt for azimuth observations, although it leads to a somewhat lesser degree of accuracy with respect to orientation of the station.

(c) The *practice of observations* is quite simple, and involves the recording of the time of passage of the body under the vertical hair of the reticule and the corresponding reading on the horizontal limb.

It is generally possible, if one is dissatisfied with the sight taken on a star, to perform the operation over again by turning the azimuth adjusting screw so that the vertical hair of the reticule is rapidly brought slightly ahead of the star ; this can be done in a few seconds, and in the majority of cases the altitude of observation will have remained within the limit allowed.

As observations of transits are involved, exactly as in time determinations with the meridian transit or in determination of latitude by transit instrument observations on the prime vertical, the use of numerous cross hairs might be considered in order to decrease accidental errors in sighting, or even the use of a micrometer equipped with a mobile wire to eliminate the personal equation of the operator. These improvements, however, would singularly complicate observational practice, which is in contradiction with the desired goal, and do not appear to be essential in view of the degree of accuracy that the method may lead one to expect.

7. *Discussion.* — A certain number of objections in connection with just this final accuracy now remain to be answered.

(a) A first assumption in the method is that the main axis of the theodolite is *absolutely vertical.* If this be not the case and it is slightly off-centre, the axis of the instrument passes through the celestial sphere at a point in the vicinity of the zenith and the result of observations supplies the co-ordinates of this point and not of the zenith.

Precision theodolites, however, are always equipped with a sensitive level perpendicular to the plane of sight ; one can always read the level for each pointing that is made and correct the observations for the residual slant measured in this way by simply adding it in the appropriate direction to the second member of the observational equation.

This argument likewise pre-supposes *stability of the main axis,* or at least that of its average direction throughout the length of observations. Modern instruments actually appear to be constructed with sufficient accuracy to ensure such stability. This same requirement as to instrumental stability likewise involves observations of the meridian, although these are considered as being extremely accurate, as well as at geodetic stations, where the fact is of lesser significance, as the sighted positions are generally not much above the horizon.

(b) But, one may ask, do not these residual deviations to a certain extent affect *constancy of orientation* of the instrument ? This same problem arises just as acutely in the setting up of geodetic stations and does not seem to create a major obstacle. The stability of the theodolite in azimuth is regularly controlled by the closure of azimuth sights on a definite distant point. One may also, in the case of astronomical sights, verify the stability in azimuth by periodically sighting on a distant luminous signal.

(c) Where the *connection of observations with the vertical* is concerned, we have already mentioned that each observation could be corrected by means of a reading of the level. However, we recommend that no too great importance be attached to the levelling of the main axis of the theodolite, and that elimination of its defect in verticality be effected by taking a second series of observations, without tampering with the levelling, with the telescope pointed under the horizon, 180° from the previous position, the stars then being observed by reflection in a mercury bath. This series may be considered as a normal series obtained by means of an instrument that is symmetrical with the previous one with respect to the mercury surface. Dealt with in exactly the same way as the previous series, it supplies data with reference to the point on the celestial sphere that is symmetrical to the point of the direct series with reference to the zenith, defined by the normal to the mercury bath. If both series of observations have been carried out in such a way as to furnish results of the same order of accuracy, the average of the data obtained in the two series will be free from the error in verticality of the main axis, the only condition being that the error shall have remained constant for the duration of both series.

This method of observation by reflection is easy to put into practice, and is likewise applicable to the equal altitudes method when use is made of a theodolite.
