GRAVIMETRIC TIDES

(Lecture delivered by Ingénieur hydrographe en Chef André GOUGENHEIM at Sixth International Hydrographic Conference, Monaco, April-May 1952).

1. *Definition.* — A gravimetric tide is the oscillatory phenomenon displayed by gravity when it is continuously observed by means of a highly sensitive gravimeter. This designation, patterned after that of « barometric tide », may perhaps bear criticism, but it has the advantage of being both concise and expressive, since the ascertained phenomenon is essentially caused by the luni-solar force generating tides.

2. *Tide-Generating Force.* — It would be useless to supply hydrographers with details concerning this force, of which they are well aware, and which in spite of its extreme smallness causes them great concern owing to the magnitude of its effects upon the oceans.

The vertical component of the generating force enters into combination with gravity, whose intensity it causes to vary. When the moon is at its zenith or nadir, gravity amounting to 981.000 milligals decreases by 11/100ths of a milligal ; when the body is at the horizon, it decreases by half as much again, so that the total difference reaches 17/100ths of a milligal. The action of the sun, 2.17 times weaker, causes a maximum variation of a 8/100ths of a milligal under similar conditions. It follows that in the most favourable case where the two celestial bodies pass at the zenith and horizon at the same time, gravity undergoes a variation of 25/100ths of a milligal, that is, very appreciably of 1/4.000.000th of its intensity.

The horizontal component of the tide-generating force, on the other hand, slightly alters the direction of the plumb-line. Deviation is maximum when the attracting body is 45° from the horizon, when lunar action causes it to reach 0''017, and solar action 0''008. In the most favourable case there is consequently a difference of 1/20th of a second between the two extreme directions of the vertical, resulting from the superposition of both actions.

If one penetrates within the Earth, the disturbing forces, in ratio to the distance of the considered point from the centre of the globe, therefore decrease in a linear manner, but the relative variation of gravity and relative deviation of the vertical would retain the same value as on the surface if the globe were homogeneous ; the increasing densities towards the centre appreciably diminish the magnitude of these effects.

3. *Effect of Earth's Elasticity. Earth Tides.* — But the Earth is not a rigid undeformable body ; it is an elastic solid, unceasingly modified by the forces that attract it, and especially by the variable action of the Sun and Moon. Kelvin, who was the first to hit upon this concept, succeeded in proving that the Earth must have an elasticity coming between that of glass and steel. These elastic deformations of the globe are called earth tides. It may easily be seen that

contrarily to what happens in the hydrosphere, this phenomenon comes under static theory and that the globe constantly takes on an equilibrium-form whose free surface is normal at every point to the resultant of gravity and the generating forces. In the hydrosphere the liquid particles, having no rigid connection with their neighbours, may assume an ample horizontal motion and by inertia go beyond the position for which there is maximum attraction, so that a state of balance can only occur in the case of long-period (semi-monthly, monthly and semi-annual) tides ; recourse must be had to dynamic theory to explain short-period (diurnal and semi-diurnal) tides. On the other hand, since no current can be originated within the solid core, the particles that make it up need only move a few decimetres, with plenty of time for a state of equilibrium to be reached, even in the case of short-period tides. One can also follow another line of reasoning, and, considering a steel ball having the dimensions of the earth, compute the period of the basic vibration that alternately transforms it into a flattened ellipsoid and an elongated ellispoid having the same axis. The result is between 1 and 2 hours ; with respect to this vibration, luni-solar action, even though it be semi-diurnal, can be considered as being long-period action and can be treated by the theory of equilibrium.

Under the influence of this disturbing action, then, the Earth alters its shape ; a correlative result is a change in the field of gravity. Ordinarily, the geometric distortion is made dependent upon a parameter h, if it is referred to a level surface ; and upon a parameter l, if referred to the direction of the Earth's axis ; the variation in the potential is dependent upon a parameter k. These three parameters are pure numbers and were introduced by Love. In the case of a homogeneous and incompressible Earth, they may be evaluated in terms of the modulus of rigidity ; but account should properly be taken of the internal distribution of densities, concerning which several formulas have been proposed. However, even though the globe be assumed as being incompressible, the theoretical calculations are extremely complex and have only been carried out in particular cases.

Certain phenomena reveal the existence of earth-tides in purely qualitative fashion, such as subterranean tides occurring in wells or flooded mines, or the relative shiftings of the sides of faults as observed in Japan.

Other phenomena, on the other hand, are capable of measurement, whereby the elastic deformation of the globe can be investigated quantitatively. These consist of oceanic tides, which are but relative tides ; deviations of the vertical in relation to the ground or with respect to the global axis ; and finally, of variations in gravity.

Results obtained may be checked to a certain extent by evidences of elasticity other than earth-tides ; the period of shift of the terrestrial pole around the mean pole leads to the most interesting results, as it involves the globe as a whole ; while the measuring of the velocity of propagation of transversal seismic waves only applies to actions that are localized.

4. *The Gravimetric Tide.* — If the Earth were absolutely rigid, a gravimeter placed upon its surface would register exactly the same periodic variations as gravity does under the influence of luni-solar attraction. But as the Earth becomes distorted owing to this action, measuring apparatus undergoes a slight vertical displacement that modifies the variation in gravity ; as this deformation moreover slightly alters the field of gravity, the variation involved is still further modified. It can finally be established that the relationship of observed variations with

respect to theoretical variations is equal to $1 + h - 3/2\ k$, in terms of Love's numbers. It may immediately be stated that experimentation supplies an average of 1.2 for this relationship, so that the maximum range attainable by the variations is about 30/100 of a milligal ; this is an extremely small amount, and it is easy to understand why it was necessary to wait for the recent improvements in gravimeters before it could adequately be made evident.

5. *First Attempts at Measurement.* — The German scientists Thomascheck and Schaffernicht twenty years ago constructed an extremely sensitive special gravimeter whose basic feature was a weight suspended between a coil spring and a bifilar system. The apparatus was first set up at Marburg in a deep cellar, then at Berchtesgaden 130 metres underground, and led to disconcerting results : instead of ratios larger than unity between observation and theory, much lower figures were obtained — 0.55 and 0.57 ; phase lags of several hours' duration were moreover noted.

6. *High-Sensitivity Gravimeters.* — A few years later, a great advance from the instrumental point of view took place with the coming into use of highly sensitive gravimeters for geophysical prospecting purposes. It is well known that extensive use of various methods of geophysical prospection is made in the search for oil and ore throughout the world, and that here gravimetry plays a leading part. Inequalities between mineral masses at shallow depths show up as slight variations in gravity, after corrections have of course been made for station elevations. These geological anomalies of gravity only amount to a few milligals, and in certain cases there is reason for attaching importance to anomalies of a few tenths of a milligal, and therefore scarcely greater than variations due to the gravimetric tide.

Prospecting organizations set up closely integrated gravimetric networks by means of highly sensitive portable equipment, of which the most modern instruments are nearly all of American manufacture. They belong to widely different types, but generally all include a weighted horizontal beam maintained in a state of stable equilibrium that is very close to indifference ; under these conditions, a very slight variation in gravity alters the weight of the ballast and quite appreciably changes the position in balance of the beam, whose shifts can be observed by means of a reading microscope. A double thermostatic casing ensures to within 1/100th of a degree the strict thermal stability that is essential for the correct operation of the instrument. It is of course not possible to effect absolute determinations of gravity with this instrument, but it is sensitive to variations amounting to approximately one-hundred-millionth of its intensity, or about 1/100th of a milligal. The graduation of the reading dial is calibrated by means of observations taken at two stations where the absolute value of gravity has been obtained by pendulum measurements.

7. *Practical Importance of Gravimetric Tide.* — The idea quickly sprang up of applying these gravimeters to the study of periodic variations in gravity ; the theoretical interest of the question did not of course escape prospectors, but an evident practical interest was likewise offered by such an investigation. These instruments have one considerable drawback, consisting of what is called instrumental drift, or the progressive displacement of the origin of reading graduations ; for lack of a better procedure, it is assumed that this variation is linear during relatively short periods. From this point of view the utilization of gravimeters recalls former methods of determining longitudes by transport of chronometers,

which invariably involved closure on a known base and interpolations for intermediate stations. In gravimetry, closure is likewise affected by earth tides, and the very high degree of accuracy at present desired induces the taking into account during adjustments of periodic variations in gravity. Thanks to the detailed gravimetric tide predictions supplied by the French Navy Hydrographic Office since the early part of 1949, the Geological and Geophysical Research Bureau has been able in conducting prospecting operations to perfect and apply new methods, thus enabling the saving of an appreciable amount of time as compared with previous procedures, whether in the execution of basic networks or in surveying extensive areas for the purpose of obtaining a first approximation of the distribution of gravity.

8. *Observations taken.* — Systematic observations of daily gravity variations were undertaken in the United States just prior to the Second World War. Recently, in 1949, a large Dutch petroleum company had gravimetric tidal observations taken over a two-week period at 26 stations distributed over practically the entire surface of the globe. Analysis of this rich harvest was only made in order to determine a mean value for the relation $1 + h - 3/2\ k$ for the whole world, and a value of 1.22 was found.

For the past four years France has also played an active part in the study of the phenomenon, and groups belonging to various public and private organizations have carried out series of observations lasting several days, both in France and its overseas territories, under the most favourable conditions for accuracy. The analysis of these series by the French Navy Hydrographic Office has shown that the approximation reached amounted to 2 hundredths of a milligal, and that within these accuracy limits the phenomenon had the character of a static tide, nor was it disturbed by the thermal deformation of the earth nor by its yielding under the weight of oceanic tides. A similar conclusion was drawn from the Dutch observations mentioned. This result is of the utmost importance in the study of the earth-tides, as the observation of deviations of the vertical, which is another method of showing the elastic deformation of the globe, is on the contrary considerably hampered by these two disturbance factors, whose effects conceal those of luni-solar action and can only be eliminated by averages spread over a relatively long period.

9. *Analysis of Gravimetric Tide Observations.* — As the gravimetric tide has the same origin as oceanic tides, it can be analyzed in the same manner, and the collaboration of hydrographic offices is occasionally sought by prospecting agencies taking series of observations for the above purpose.

Certain precautions should be taken, however, that so far do not always seem to have been used. I am not speaking of the dispersion of individual values (noted down every hour or half-hour), which causes the gravimetric tide curve to be less sharply drawn than the oceanic tide curve, and which for purposes of analysis makes it useless to look for other than the most important waves. Neither do I refer to instrumental drift, which is not sufficiently stable for making it possible to show long-period waves.

It is the principle itself of analysis which is involved. The phenomenon observed by means of a gravimeter is the resultant of two terms : one is the vertical component of the perturbing acceleration due to the Moon and Sun ; this is the theoretical tide to which the observation would be reduced if the Earth were absolutely rigid. It can be computed easily : the gravimeter is completely and immediately sensitive to it. The second term represents the variation in gravity resulting from elastic deformation of the Earth owing to the preceding luni-solar action. This latter term, therefore, which is the difference between the observed tide and the computed action, is the one that shows the elastic reaction of the globe at the point of observation ; generally, it has the appearance of a static tide ; the phenomenon is to an appreciable degree in phase and in constant ratio (1/5) with the luni-solar action that produces it.

(a) *Correction of Prospection Measurements.* — All that prospectors in correcting their observations need to know is the global phenomenon affecting the gravimeters, and in fulfilment of their requirements, analysis of this latter phenomenon should take place in exactly the same manner as for oceanic tides, except that a small number of waves should be dealt with.

(b) *Investigations regarding Global Elasticity.* — But if investigation of the Earth's elastic behaviour is desired, the second term, or the difference between the observed and theoretical tide, must be resorted to. As the latter is on the average five times larger than the former, mere analysis of the observed tide may not suffice in causing the special features of this term, which superposition of the theoretical tide almost completely conceals, to appear. The term may not fully show the character of a static tide ; it may, for instance, be somewhat dephased with respect to luni-solar action ; according to certain writers, the mechanical constitution of the globe should involve a different range ratio between the diurnal components and semi-diurnal components of elastic deformation and the corresponding components of the theoretical tide. If these points are to be brought out, the term depending upon the elasticity of the globe must be isolated and analyzed separately.

We will not of course be very well informed with regard to the phenomenon, since its range during the course of a single day barely exceeds 5/100 of a milligal, and since the best observations now made are within 1 or 2 hundredths of a milligal. It is exactly as if one had to analyze the tide in a harbour where the tide rises or falls 5 meters, but where surveys could only be made to within 1 meter.

This uncertainty justifies, for the time being, a very brief analysis of the phenomenon, limited, for instance, to the diurnal and semi-diurnal tides for each day of observation. By comparing the results with the corresponding elements of the theoretical tide, a ratio of amplitude and dephasing with reference to theoretical action is deduced for the diurnal component and semi-diurnal component of elastic deformation. It may be pointed out that for nearly all observations handled in this way practically inexistent dephasings were found, as well as nearly equal amplitude ratios for the diurnal and semi-diurnal waves, which confirms the hypothesis of a static tide. But it has not conclusively been shown that a different behaviour might not be encountered elsewhere. Analysis seems to indicate, moreover, that although there is an almost complete absence of dephasing, amplitude ratios, which average 2/10, show fairly different values according to areas generally somewhere between 1/10 and 3/10.

In any case, a great deal of caution should be exercised in the conduct of analyses owing to the smallness of the phenomenon, which is on the borderline of sensitivity of the measurement instruments.

10. *Further Evidence of Gravimetric Tide.* — There is another way of showing up luni-solar variations of the acceleration of gravity : this consists in the comparison of the rates of astronomical pendulum clocks, which are subject to

these variations, and those of quartz clocks, which are free from them. But pendulum clocks are subject to the disturbing effects we mentioned with reference to deviations of the vertical, and a result can only be reached by averaging long series of observations. Research carried out at the Paris Observatory during a four-year period on five astronomical clocks led to a value of 1.2 for the ratio between the observed tide and the theoretical tide. This value is nearly identical with that resulting from the Netherlands gravimetric observations carried out on a global basis.

11. *Conclusion.* — Geophysical prospecting gravimeters at the moment appear to be the instruments enabling investigations of the elastic behaviour of the earth to be carried out with the greatest speed, convenience and accuracy. Geophysics therefore derive benefit from instruments that in principle were not designed to further their development, and doubtless would never have seen the light of day if the object had been other than disinterested investigation. No scientific research laboratory would have had sufficient funds at is disposal to construct and develop these instruments ; they owe their existence to the part they are able to play in the conquest of oil, in which so many different interests are involved. Owing to this favourable circumstance, there is good reason to hope that the experimental study of earth tides is about to enter into a new phase, and that before long it will be possible to define and explain — as has already been done to a certain extent in the case of the oceans — the manner in which each fragment of the Earth's crust responds to the attraction of celestial bodies.