THE STABILITY OF DECCA PATTERNS UNDER VARIOUS CIRCUMSTANCES AND THE INFLUENCE OF TIME ON THAT STABILITY

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Note of the Directing Committee : *The following article has been received from the Hydrographer of the Royal Netherlands Navy an is published. in the hope that it will lead to further articles, both from interested persons and from others using this system of navigational aid, for publication in the* International Hydrographic Review.

Short Description of Decca

The Decca system is developed by engineers of Decca, the well-known British radio and gramophone firm. It consists of a number of transmitters ashore in accurately known positions and of a receiving apparatus somewhere within the coverage of the transmitters. The system is used as an aid to navigation for both ships and aircraft, as well as a survey-system. It is a phase-comparison system contrary to Loran which is a pulse-system.

Two stations M and R transmit continuous unmodulated waves that will be assumed to be of the same wavelength. In some way the result is obtained that there remains a constant phase-difference between the transmissions of M and R. It is easily seen that curves on earth, representing lines of equal difference of phase will be hyperbolae as long as the area used can be considered as plane. The curvature of the earth has some well-known consequences tending to disturb the form of these hyperbolae. However, though mathematically not quite correct, the Decca system is called a hyperbolic navigation system.

The space between two hyperbolae of zero phase-difference is called a « lane ». The position of all hyperbolae relative to the transmitters remains unchanged as long the phase-difference between M and R remains constant. The station M is called the Master station and R the Slave station, as the function of the latter is to receive the Master transmissions and to keep the phase of its own transmissions locked to those of the Master.

The number of lanes in any hyperbolic pattern is always equal to two times the length of the base-line (the line connecting Master and Slave) divided by the wave-length. It will also be clear that the lane-width along this base-line is equal to half the wave-length.

From the above it follows that the electro-magnetic fields produced by the two transmitters may be considered as radiating a pattern of hyperbolic coordinates. To fix one's position relative to the transmitters, a second set of hyperbolic coordinates intersecting the first pattern is necessary, and this can be achieved by operating a second pair of transmitters. This is more easily done by « attaching » a second Slave to the original Master. Such a group of transmitters is usually called a « 2-slave chain » or « 3-station chain ». A 3-slave chain consists of one Master and 3 Slave transmitters. For the purpose of discrimination between the different patterns, they are indicated by different colours, e. g. the

Red, the Green and the Purple patterns and consequently the 3 Slave transmitters that, together with the one Master, give birth to these patterns are called the Red, the Green and the Purple Slaves.

The phase-difference at a certain position can be measured with a special receiving apparatus, consisting of receiver, phase-comparator and decometer. The indications of phase-difference are given on the dial of the decometer. The Red decometer gives the difference of phase for the Red pattern (between Master and Red Slave) and so forth. The Decca receiver is able to measure phase-difference only, and it cannot distinguish between one lane and another. Consequently only sub-divisions of a lane can be indicated by the so-called « fractional pointer » of the decometer. These sub-divisions have divisions having a value of one-hundredth of a lane, i. e. 3.6 degrees, and one complete revolution of the fractional pointer comprises 100 of these sub-divisions or exactly one lane.

Though no distinction can be made between the different lanes, the number of revolutions of the fractional pointer can be counted by a coarse pointer which is attached to each decometer and geared to the fine pointer indicating the number of revolutions of the latter. All the lanes of all patterns have a number shown on a lattice-chart (a nautical chart with the different patterns in their respective colours printed on it) and corresponding to the numbers shown on the dials of the decometers. So the coarse pointer will always point to one of these numbers, but the difficulty is to know which is the right one. The fractional pointer which is shorter than the coarse pointer indicates the parts of a lane on the inner division of the dial. The lane numbers form the outer ring of this same dial. The problem of finding the right lane-number can be solved if a fix (preferably terrestrial) can be obtained and the right lane-number thus found from the lattice-chart. By resetting the coarse pointer accordingly the problem is solved, as this pointer will continuously count the number of revolutions of the fractional pointer and thereby stay on the right lane-number provided no irregularities occur.

As it is not always possible to obtain fixes and as anomalous propagation of the radio-waves way be the cause of so-called « lane-slip », a lane-identification system is provided which on a fourth dial gives the right lane-numbers for the three different colours every minute. This rather complicated device will not be discussed further as it is irrelevant to the present investigation.

It is clear that the Decca receiver must receive the transmissions of all 4 transmitters and in order to discriminate between these the 4 radiated frequencies differ. Therefore the receiver has 4 channels with crystals resonant to the 4 frequencies. Phase-comparison between incoming signals of different frequencies is technically impossible, but this difficulty was overcome by choosing the 4 frequencies in such a way that simple multipliers can be used to obtain the same comparison frequency for the pair of signals between which the phase-difference is to be found. One of these signals is always the Master signal, the other one is the Slave signal. This frequency-multiplication is done in the Decca receiving set. The actual transmitting frequencies are for the English chain: Master 85.0, Red 113, Green 127.5, and Purple 70.8 kcs. These frequencies get a multiplication factor after arriving in the receiving set as follows. In order to measure the phase-difference between Master- and Red Slave signals the Master frequency is multiplied by 4 and the Red frequency by 3, which gives a comparison-frequency of 340 kcs. In the same way the comparison frequency for Green is 255 kcs (master $\times 3$ and green $\times 2$) and for Purple 425 kcs (master $\times 5$ and purple $\times 6$).

Though perhaps not seen at once it will eventually be clear that the hyperbolae in the lattice-charts have to be calculated for the comparison-frequencies and not for the actually radiated frequencies.

All Decca-chains have the same multiplication-factors as described above; only the radiated frequencies differ slightly so as to give a minimum of interference with neighbouring chains. It should be kept in mind that all disturbances of the incoming signals are also multiplied by the same factors as the frequencies they influence. The main cause of anomalies is the sky-wave, the wave reflected by the ionized layers of the atmosphere at a height of approximately 60 km. The effective height of these layers depends on the frequency used. The sky-wave has still the same frequency as the direct- or « ground-wave », but it has a phase-difference relative to this ground-wave because of the longer path it has travelled and the phenomena occurring at reflection.

Near the transmitters the field-strength of the direct wave is so big as not to be disturbed by the weak sky-wave, but at medium and long range the ground-wave is weaker (about proportional to the inverse distance) as the sky-wave tends to grow stronger. Now if the sky-wave were constant in a certain area, the application of local sky-wave corrections would be profitable, but sky-wave differs with time and there also occur local disturbances in the reflective layers which tend to make the sky-wave irregular.

The wave reflected from the ionosphere, arriving at the earth's surface, is again reflected by the earth. Since for the frequencies used and shallow incidence the reflection coefficient for the sea is practically equal to unity, the resultant field-intensity at the earth's surface is equal to about twice the value of the primary sky-wave. However, for very large distances, i. e. for grazing angles of incidence, the phase of the wave reflected from the earth's surface changes very rapidly, and when the angle of incidence approaches 0° becomes equal to 180°. The reflection-coefficient is then equal to —1 so that the primary and the earth-reflected sky-wave fields cancel out.

It is an accepted approximation to assume that the ionosphere acts as a reflecting surface and that for shallow angles of incidence the reflection-coefficient of the ionosphere has a constant value and is therefore the cause of a constant phase-shift. Thus the difference in phase between sky-wave and ground-wave is also due to reflection. If the height of the reflective layer changes, the angle of incidence will change too and a change in phase-shift will occur. It is probable that the effective height of the ionosphere is a function of the sun's altitude and though not yet certain, it is thought that sunspots also influence this effective height.

There is still another factor that materially increases the influence of the sky-wave, especially at large distances. This is the convergence-factor which expresses numerically the effect of concentration of electro-magnetic energy by reflection from the ionosphere acting as a concave mirror. The opposite effect occurs upon reflection from the earth's surface acting as a convex mirror, but the so-called « divergence-factor » does not enter into considerations of « single-hop » propagation, since it only produces effects at some distance above the earth's surface.

The above shows that the sky-wave is a troublesome item. Not only the phase of the incoming sky-wave but also its intensity is of interest, as may readily be seen from a vector-diagram in which the groundwave G and the sky-wave S are the components. The sum or resultant vector R is the one that is materialized in the Decca-receiver. Its phase-difference from the ground-wave not only depends on the difference of phase of the sky-wave and ground-wave but also on the ratio G :S. All four incoming signals of the various transmitters are disturbed by the sky-wave, but as the frequencies are not the same as the differences in distance from the receiving point to the transmitters may be considerable, all these skywave vectors will be different in scalar and in phase.

Prima facie it is sound to assume the existence of a fairly high correlation between the various sky-wave vectors and it has been proved that this correlation increases with increasing distance from the transmitters. This phenomenon can be accounted for by realising that the larger the distances become the smaller will be the influence of local disturbances of the ionosphere. In the present study of the stability of Decca patterns, sky-wave is considered as the major cause of all instabilities found.

1. *Introduction*

1.1 *Problems of Stability.*

This paper contains some information on the stability of Decca-patterns over several periods and the influence of time on observations. Some results are recorded of an analysis of pattern variations at long and medium ranges of the English and German chains. The following questions are considered to be of importance and an attempt was made to find answers to at least some of them :

a. What shape has the histogram of a large number of observations ? Is the frequency-distribution normal ? If not, is there any other known frequency-distribution that could serve as a mathematical model for the observed histogram ?

b. If no such frequency function can be found is there an acceptable hypothesis for the observed distribution ?

c. What is the influence of the time-interval between successive readings on the shape of the histogram ?

d. What is the influence of the length of the set of observations on the shape of the histogram ?

e. Is there any difference between the relative frequency histogram of readings taken at long and at medium ranges ?

f. Can any relation be found between simultaneous observations of two different chains ?

1.2. *Summary of Conclusions.*

a. The relative frequency-distribution reduced to standard units (or half standard units as was done in this paper) of histograms of decometer-readings is not Gaussian. Usually they are not wider than normal distributions. No known frequency function could serve as mathematical model to observed histograms of this kind.

b. This kind of histogram can be considered as typical for time-series. An acceptable hypothesis for the observed distributions not yet been found, but depends on the kind of time-series able to serve as a mathematical model for a sample of decometer-readings. Correlogram analysis can probably provide the solution to this problem. It is not impossible that the time series able to represent samples of decometer-readings is of the trendless, oscillating stochastic type, with oscillations composed of sums of disturbed cyclical movements (see pars. 4.2.1 and 4.2.3.).

c. The shape of a histogram of decometer-readings is due mostly to serial correlation between successive readings. The histogram tends to normality when the serial correlation is nearing zero value, which will occur when the time interval

Table 1

SERIAL CORRELATION COEFFICIENT

for a 2-minute interval of RED decometer-readings, daylight, GERMAN chain.

	t	.76	.77	.78	.79	.80	.81	.82	.83	.84	.85	.86			
	x	- 6	- 5	- 4	- 3	- 2	- 1	0	1	2	3	4	sum	sum y	sum y^2
t + 2	y														
.86	4									1	1		2	8	32
.85	3							1		6	13		20	60	180
.84	2							2	6	11	3	1	23	46	92
.83	1							10	32	11			53	53	53
.82	0		1			1	10	26	12				50	0	0
.81	- 1					4	18	13	2				37	- 37	37
.80	- 2			2	7	16	10	2					37	- 74	148
.79	- 3			7	22	6	1						36	- 108	324
.78	- 4	1	4	19	9								33	- 132	528
.77	- 5		5	5	1								11	- 55	275
.76	- 6			1									1	- 6	36
sum		1	10	34	39	27	39	54	52	29	17	1	303	- 245	1705
sum x		-6	-50	-136	-117	-54	-39	0	52	58	51	4	- 237		
sum x^2		36	250	544	351	108	39	0	52	116	153	16	1665		

$m'_1(x) = -0.78218$

$m'_1(y) = -0.80858$

$m_2(x) = 4.88324$

$m_2(y) = 4.97324$

sum xy $=$ · 1576

cov.(xy) $= 4.56887$

$r_2 = +0.927$

SERIAL CORRELATION COEFFICIENT

for an 8-minute interval of RED decometer-readings, daylight, GERMAN chain.

	t	.76	.77	.78	.79	.80	.81	.82	.83	.84	.85	.86	sum	sum y	sum y^2
	x	- 6	- 5	- 4	- 3	- 2	- 1	0	1	2	3	4			
t + 8	y														
.86	4									1	1		2	8	32
.85	3								1	7	5		13	39	117
.84	2							1	5	5		1	12	24	48
.83	1						1	5	10	8	1		25	25	25
.82	0					1	6	9	11				27	0	0
.81	- 1				2	4	10	5	1				22	- 22	22
.80	- 2			1	6	5	5	2					19	- 38	76
.79	- 3			3	7	3	2						15	- 45	135
.78	- 4		3	9	8	1	1						22	- 88	352
.77	- 5		1		4								5	- 25	125
.76	- 6				2								2	- 12	72
sum			4	13	29	14	25	22	28	21	7	1	164	- 134	1004
sum x			-20	-52	-87	-28	-25	0	28	42	21	4	- 117		
sum x^2			100	208	261	56	25	0	28	84	189	64	1015		

$m'_1(x) = -0.71341$

$m'_1(y) = -0.81707$

$m_2(x) = 5.68007$

$m_2(y) = 5.45435$

$\text{sum } xy = 829$

$\text{cov.}(xy) = +4.47197$

$r_8 = +0.803$

SERIAL CORRELATION COEFFICIENT

for a 30-minute interval of RED decometer-readings, daylight, GERMAN chain.

	t	.76	.77	.78	.79	.80	.81	.82	.83	.84	.85	.86			
	x	- 6	- 5	- 4	- 3	- 2	- 1	0	1	2	3	4	sum	sum y	sum y^2
t + 30	y														
.86	4									1			1	4	16
.85	3								1	2			3	9	27
.84	2							1	2	3	3		9	18	36
.83	1							4	6	3	1		14	14	14
.82	0					1	2	9	5	1	1		19	0	0
.81	- 1					1	2	3	3				9	- 9	9
.80	- 2				1	1	2	3	1				8	- 16	32
.79	- 3	1	1	4	3	3	2	2	2				16	- 48	144
.78	- 4		1	4	5	2	1						13	- 52	208
.77	- 5			2									2	- 10	50
sum		1	2	10	9	8	9	22	18	10	5		94	- 90	536
sum x		-6	-10	-40	-27	-16	-9	0	18	20	15		- 55		
sum x^2		36	50	160	81	32	9	0	18	40	45		471		

$m'_1(x) = -0.58511$

$m'_1(y) = -0.95745$

$m_2(x) = 4.66829$

$m_2(y) = 4.78542$

sum xy $= 421$

cov.(xy) $= +3.91851$

$r_{30} = +0.829$

TABLE 4

SERIAL CORRELATION COEFFICIENT

for a 2-minute interval of PURPLE decometer-readings, daylight, GERMAN chain.

	t	.11	.12	.13	.14	.15	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30	.31	.32	.33	.34	sum
	x	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
t + 2	y																									
.35	15																								1	1
.34	14																					1			1	2
.33	13																									0
.32	12																									0
.31	11																				1	2				3
.30	10																		1		1	1				3
.29	9																									0
.28	8																		3	1						4
.27	7																4	2		1	1					8
.26	6														1	3	6	2		2						14
.25	5											1			6	9	1	1								18
.24	4												1	8	14	10										33
.23	3												4	11	5											20
.22	2									1		7	11	3	2											24
.21	1										7	15	5	1												28
.20	0								1	3	11	5	3													24
.19	-1								8	11	3	2														24
.18	-2							3	13	8	2															26
.17	-3					2	3	8	3																	16
.16	-4					9	12	3																		24
.15	-5				5	8	7	3																		23
.14	-6				7	4																				11
.13	-7		3	4	1	1																				9
.12	-8	1	1	1																						3
.11	-9			1																						1
sum		1	4	6	13	24	22	17	25	23	23	30	24	23	28	22	12	5	4	4	3	4	0	0	2	319

sum x = 153	sum y = 139	$m'_1(y)$ = + 0.43574	sum xy = 5909
sum x^2 = 6091	sum y^2 = 6055	$m_2(x)$ = 18.86400	cov.(xy) = + 18.31452
$m'_1(x)$ = + 0.47962		$m_2(y)$ = 18.79132	r_2 = + 0.973

SERIAL CORRELATION COEFFICIENT

for an 8-minute interval of PURPLE decometer-readings, daylight GERMAN chain.

	t	.12	.13	.14	.15	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30	.31	sum
	x	- 8	- 7	- 6	- 5	- 4	- 3	- 2	- 1	0	1	2	3	4	5	6	7	8	9	10	11	
t + 8	y																					
.34	14																				1	1
.33	13																					0
.32	12																					0
.31	11																					0
.30	10																		1	3		4
.29	9												1									1
.28	8															1	1					2
.27	7														2			1				3
.26	6											1		1		1		1	1	1		6
.25	5												1	1	3	4		1				10
.24	4											2	4	4	10		1					21
.23	3											5	3	3								11
.22	2										4	2	5	3		1						15
.21	1									4	5	4	2		1							16
.20	0							2	4	4	6			1								17
.19	- 1						1	4	6		3					1						15
.18	- 2					1		2	4	1												8
.17	- 3				4	3	5	1	1													14
.16	- 4			2	1	5	4	2														14
.15	- 5		1	2	3	2	2	1														11
.14	- 6			3		1																4
.13	- 7	2		1	2	1																6
.12	- 8				3																	3
.11	- 9			1																		1
sum		2	3	9	12	12	12	12	15	9	18	14	16	13	16	8	2	3	2	4	1	183

sum x = 107   sum y = 66 $m'_1(y)$ = + 0.36066   sum xy = 3096

sum x^2 = 3349   sum y^2 = 3312 $m_2(x)$ = 17.95868   cov.(xy) = + 16.70715

$m'_1(x)$ = + 0.58470 $m_2(y)$ = 17.96828 r_8 = + 0.930

SERIAL CORRELATION COEFFICIENT

for a 30-minute interval of PURPLE decometer-readings, daylight, GERMAN chain.

t+30	t / y \ x	.13	.14	.15	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30	sum
	x	- 7	- 6	- 5	- 4	- 3	- 2	- 1	0	1	2	3	4	5	6	7	8	9	10	
.29	9													1	1	1				3
.28	8																		1	1
.27	7												1		1					2
.26	6								1					2	1					4
.25	5								1			1	3	1		1				7
.24	4								2	1		3	1		1					8
.23	3								2			2		2			1			7
.22	2								2		2	2	2	1	1					10
.21	1						2			2	2	3	2	2			1	1		15
.20	0						1	1		4	3	1	2							12
.19	- 1				1	1		2	1	1	1	1								8
.18	- 2			1	1		4	3		1										10
.17	- 3	1		1	1	1	2	2		1										9
.16	- 4		2		1	2	2	1	1											9
.15	- 5	1		2	3	3	1													10
.14	- 6		2			1														3
.13	- 7			1	1	1														3
.12	- 8			1																1
sum		2	4	6	8	9	12	9	10	10	8	13	11	9	5	2	2	1	1	122

sum x = 73 sum y = 8

sum x^2 = 1805 sum y^2 = 1812

$m'_1(x)$ = + 0.59836

$m'_1(y)$ = + 0.06557

$m_2(x)$ = 14.43705

$m_2(y)$ = 14.84816

sum xy = + 11.71487

r_{30} = + 0.800

SERIAL CORRELATION COEFFICIENTS

for different intervals of RED decometer-readings on May 30th, 1952 (PM) GERMAN chain.

2-minute interval

t		.76	.77	.78	.79	.80	.81	sum	sum y	sum y^2		
	x	- 2	- 1	0	1	2	3					
t + 2	y										$m'_1(x)$	= + 0.5556
.81	3				1	2		3	9	27	$m'_1(y)$	= + 0.5463
.80	2			2	2	5	3	12	24	48	$m_2(x)$	= 0.9135
.79	1		3	9	21	6		39	39	39	$m_2(y)$	= 0.8960
.78	0		7	21	14			42	0	0	sum xy	= + 90
.77	- 1	1	1	9				11	- 11	11	cov.(xy)	= + 0.5298
.76	- 2			1				1	- 2	4	r_2	= + 0.586
sum		1	11	42	38	13	3	108	59	129		
sum x		- 2	- 11	0	38	26	9	60				
sum x^2		4	11	0	38	52	27	132				

4-minute interval

t		.76	.77	.78	.79	.80	.81	sum	sum y	sum y^2		
	x	- 2	- 1	0	1	2	3					
t + 4	y										$m'_1(x)$	= + 0.5714
.81	3				1	2		3	9	27	$m'_1(y)$	= + 0.5429
.80	2			1	6	4	1	12	24	48	$m_2(x)$	= 0.9306
.79	1		3	10	17	5	2	37	37	37	$m_2(y)$	= 0.9148
.78	0	1	6	19	13	2		41	0	0	sum xy	= + 80
.77	- 1		2	8	1			11	- 11	11	cov.(xy)	= + 0.4517
.76	- 2			1				1	- 2	4	r_4	= + 0.489
sum		1	11	39	38	13	3	105	57	127		
sum x		- 2	- 11	0	38	26	9	60				
sum x^2		4	11	0	38	52	27	132				

SERIAL CORRELATION COEFFICIENTS (cont.)

for different intervals of RED decometer-readings on May 30th, 1952 (PM) GERMAN chain.

8-minute interval

	t	.76	.77	.78	.79	.80	.81	sum	sum y	sum y^2			
	x	- 2	- 1	0	1	2	3						
t + 8	y										$m'_1(x)$	= +	0.5743
.81	3				2		1	3	9	27	$m'_1(y)$	= +	0.5149
.80	2				5	6	1	12	24	48	$m_2(x)$	=	0.9573
.79	1		4	14	9	5		32	32	32	$m_2(y)$	=	0.9428
.78	0	1	5	19	14	2	1	42	0	0	sum xy	= +	65
.77	- 1		2	4	5			11	- 11	11	cov.(xy)	= +	0.3479
.76	- 2				1			1	- 2	4	r_8	= +	0.366
sum		1	11	37	36	13	3	101	52	122			
sum x		- 2	- 11	0	36	26	9	58					
sum x^2		4	11	0	36	52	27	130					

12-minute interval

	t	.76	.77	.78	.79	.80	.81	sum	sum y	sum y^2			
	x	- 2	- 1	0	1	2	3						
t + 12	y										$m'_1(x)$	= +	0.5773
.81	3				2			2	6	18	$m'_1(y)$	= +	0.4948
.80	2				7	4		11	22	44	$m_2(x)$	=	0.9863
.79	1		4	13	9	4	3	33	33	33	$m_2(y)$	=	0.8892
.78	0	1	7	17	9	5		39	0	0	sum xy	= +	48
.77	- 1			5	6			11	- 11	11	cov.(xy)	= +	0.2092
.76	- 2				1			1	- 2	4	r_{12}	= +	0.223
sum		1	11	35	34	13	3	97	48	110			
sum x		- 2	- 11	0	34	26	9	56					
sum x^2		4	11	0	34	52	27	128					

SERIAL CORRELATION COEFFICIENTS (cont.)

for different intervals of RED-decometer readings on May 30th, 1952 (PM) GERMAN chain.

16-minute interval

	t	.76	.77	.78	.79	.80	.81	sum	sum y	sum y^2			
	x	- 2	- 1	0	1	2	3						
t + 16	y										$m'_1(x)$	=	+ 0.5699
.81	3				1			1	3	9	$m'_1(y)$	=	+ 0.3763
.80	2				2	4	1	7	14	28	$m_2(x)$	=	1.0193
.79	1		1	11	13	5	2	32	32	32	$m_2(y)$	=	0.7724
.78	0		7	22	9	2		40	0	0	sum xy	=	+ 53
.77	- 1	1	2	1	6	2		12	- 12	12	cov.(xy)	=	+ 0.3554
.76	- 2		1					1	- 2	4	r_{16}	=	+ 0.451
sum		1	11	34	31	13	3	93	35	85			
sum x		- 2	- 11	0	31	26	9	53					
sum x^2		4	11	0	31	52	27	125					

20-minute interval

	t	.76	.77	.78	.79	.80	.81	sum	sum y	sum y^2			
	x	- 2	- 1	0	1	2	3						
t + 20	y										$m'_1(x)$	=	+ 0.5843
.81	3							0	0	0	$m'_1(y)$	=	+ 0.3146
.80	2				4	1		5	10	20	$m_2(x)$	=	1.0519
.79	1		3	8	9	9	2	31	31	31	$m_2(y)$	=	0.6426
.78	0	1	7	15	14	3	1	41	0	0	sum xy	=	+ 40
.77	- 1		1	7	3			11	- 11	11	cov.(xy)	=	+ 0.2656
.76	- 2			1				1	- 2	4	r_{20}	=	+ 0.323
sum		1	11	31	30	13	3	89	28	66			
sum x		- 2	- 11	0	30	26	9	52					
sum x^2		4	11	0	30	52	27	124					

SERIAL CORRELATION COEFFICIENTS (cont.)

for different intervals of RED decometer-readings on May 30th, 1952 (PM) GERMAN chain.

24-minute interval

	t x	.76 - 2	.77 - 1	.78 0	.79 1	.80 2	.81 3	sum	sum y	sum y^2			
t + 24	y										$m'_1(x)$	= +	0.6000
.81	3							0	0	0	$m'_1(y)$	= +	0.2824
.80	2				2	2		4	8	16	$m_2(x)$	=	1.0870
.79	1		4	9	9	6	1	29	29	29	$m_2(y)$	=	0.6262
.78	0	1	6	17	11	4	1	40	0	0	sum xy	= +	20
.77	- 1		1	2	6	1	1	11	- 11	11	cov.(xy)	= +	0.0659
.76	- 2				1			1	- 2	4	r_{24}	= +	0.080
sum		1	11	28	29	13	3	85	24	60			
sum x		- 2	-11	0	29	26	9	51					
sum x^2		4	11	0	29	52	27	123					

28-minute interval

	t x	.76 - 2	.77 - 1	.78 0	.79 1	.80 2	.81 3	sum	sum y	sum y^2			
t + 28	y												
.81	3							0	0	0	$m'_1(x)$	= +	0.6420
.80	2				2			2	4	8	$m'_1(y)$	= +	0.2346
.79	1		4	9	10	4	1	28	28	28	$m_2(x)$	=	1.0940
.78	0	1	6	14	11	6	1	39	0	0	$m_2(y)$	=	0.5746
.77	- 1			2	5	3	1	11	- 11	11	sum xy	= +	5
.76	- 2				1			1	- 2	4	cov.(xy)	= —	0.0889
sum		1	10	25	29	13	3	81	19	51	r_{28}	= —	0.112
sum x		- 2	- 10	0	29	26	9	52					
sum x^2		4	10	0	29	52	27	122					

SERIAL CORRELATION COEFFICIENTS

for different intervals of PURPLE decometer-readings on 30th, 1952 (PM) GERMAN chain.

2-minute interval

t + 2	t x \ y	.18 - 5	.19 - 4	.20 - 3	.21 - 2	.22 - 1	.23 0	.24 1	.25 2	.26 3	.27 4	.28 5	sum	sum y	sum y^2
.28	5									2	1	7	10	50	250
.27	4									4	2	1	7	28	112
.26	3								4	4	2	2	12	36	108
.25	2							1	5	4	1		11	22	44
.24	1						1		2		1		4	4	4
.23	0				2	6	10	2					20	0	0
.22	- 1				4	8	9						21	- 21	21
.21	- 2			3	1	6	1	1					12	- 24	48
.20	- 3			3	4								7	- 21	63
.19	- 4	1		1	1								3	- 12	48
.18	- 5		2										2	- 10	50
sum		1	2	7	12	20	21	4	11	14	7	10	109	52	748
sum x		- 5	- 8	- 21	- 24	- 20	0	4	22	42	28	50	68		
sum x^2		25	32	63	48	20	0	4	44	126	112	250	724	sum xy = + 680	

$m'_1(x) = +\ 0.6239$ $m'_1(y) = +\ 0.4771$ cov.(xy) $= +\ 5.9408$

$m_2(x) = 6.2529$ $m_2(y) = 6.6348$ $r_2 = +\ 0.922$

SERIAL CORRELATION COEFFICIENTS (cont.)

for different intervals of PURPLE decometer-readings on May 30th, 1952 (PM) GERMAN chain.

4-minute interval

t + 4	t x	.18 - 5	.19 - 4	.20 - 3	.21 - 2	.22 - 1	.23 0	.24 1	.25 2	.26 3	.27 4	.28 5	sum	sum y	sum y^2
	y														
.28	5									2	3	5	10	50	250
.27	4								1	2	1	2	6	24	96
.26	3							1	2	6		3	12	36	108
.25	2								4	3	3		10	20	40
.24	1					1			2	1			4	4	4
.23	0				1	7	8	2	2				20	0	0
.22	- 1			1	4	5	9	1					20	- 20	20
.21	- 2			3	1	5	3						12	- 24	48
.20	- 3			1	4	2							7	- 21	63
.19	- 4		1	1	1								3	- 12	48
.18	- 5			1	1								2	- 10	50
sum			1	7	12	20	20	4	11	14	7	10	106	47	727
sum x			- 4	- 21	- 24	- 20	0	4	22	42	28	50	77		
sum x^2			16	63	48	20	0	4	44	126	112	250	723	sum xy = + 628	

$m'_1(x) = +\ 0.7264$ $\quad m'_1(y) = +\ 0.4434$ $\quad cov.(xy) = +\ 5.6024$

$m_2(x) = 6.2931$ $\quad m_2(y) = 6.6619$ $\quad r_4 = +\ 0.865$

TABLE 8 (cont. 2).

SERIAL CORRELATION COEFFICIENTS (cont.)

for different intervals of PURPLE decometer-readings on May 30th, 1952 (PM) GERMAN chain.

8-minute interval

	t	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28			
	x	- 5	- 4	- 3	- 2	- 1	0	1	2	3	4	5	sum	sum y	sum y^2
t + 8	y														
.28	5									4	4	2	10	50	250
.27	4								3	1		2	6	24	96
.26	3							1	2	6		3	12	36	108
.25	2								1	1	2	3	7	14	28
.24	1					1				1	1		3	3	3
.23	0				2	5	6	1	5	1			20	0	0
.22	- 1				3	7	9	1					20	- 20	20
.21	- 2			2	1	3	5	1					12	- 24	48
.20	- 3			2	2	3							7	- 21	63
.19	- 4			1	2								3	- 12	48
.18	- 5			1		1							2	- 10	50
sum				6	10	20	20	4	11	14	7	10	102	40	714
sum x				- 18	- 20	- 20	0	4	22	42	28	50	88		
sum x^2				54	40	20	0	4	44	126	112	250	650	sum xy = + 562	

$m'_1(x) = + 0.8627$ $m'_1(y) = + 0.3922$ cov.(xy) $= + 5.1714$

$m_2(x) = 5.6282$ $m_2(y) = 6.8462$ $r_8 = + 0.833$

SERIAL CORRELATION COEFFICIENTS (cont.)

for different intervals of PURPLE decometer-readings on May 30th, 1952 (PM) GERMAN chain.

12-minute interval

	t	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28			
	x	- 5	- 4	- 3	- 2	- 1	0	1	2	3	4	5	sum	sum y	sum y^2
t+12	y														
.28	5								2	5	2	1	10	50	250
.27	4								2	1	1	1	5	20	80
.26	3							1	2	3	1	3	10	30	90
.25	2									2	1	3	6	12	24
.24	1					1						2	3	3	3
.23	0				3	5	4		3	3	2		20	0	0
.22	- 1					8	10		2				20	- 20	20
.21	- 2				4	3	4	2					13	- 26	52
.20	- 3				1	2	2	1					6	- 18	54
.19	- 4			2	1								3	- 12	48
.18	- 5			2									2	- 10	50
sum				4	9	19	20	4	11	14	7	10	98	29	671
sum x				- 12	- 18	- 19	0	4	22	42	28	50	97		
sum x^2				36	36	19	0	4	44	126	112	250	627	sum xy = + 460	

$m'_1(x) = +\ 0.9898$ $m'_1(y) = +\ 0.2959$ $cov.(xy) = +\ 4.4010$

$m_2(x) = 5.4183$ $m_2(y) = 6.7593$ $r_{12} = +\ 0.727$

SERIAL CORRELATION COEFFICIENTS (cont.)

for different intervals of PURPLE decometer-readings on May 30th, 1952 (PM) GERMAN chain.

16-minute interval

t		.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	sum	sum y	sum y^2
	x	- 5	- 4	- 3	- 2	- 1	0	1	2	3	4	5			
t + 16	y														
.28	5								5	4	1		10	50	250
.27	4							1	1	1	1		4	16	64
.26	3									4	2	2	8	24	72
.25	2										1	4	5	10	20
.24	1						1			1		1	3	3	3
.23	0				1	4	6		1	4	1	3	20	0	0
.22	- 1				2	6	7	2	2		1		20	- 20	20
.21	- 2					5	5		2				12	- 24	48
.20	- 3				2	3	1	1					7	- 21	63
.19	- 4			1	1	1							3	- 12	48
.18	- 5				2								2	- 10	50
sum				1	8	19	20	4	11	14	7	10	94	16	638
sum x				- 3	- 16	- 19	0	4	22	42	28	50	108		
sum x^2				9	32	19	0	4	44	126	112	250	596	sum xy = + 380	

$m'_1(x) = + 1.1489$ $m'_1(y) = + 0.1702$ $cov.(xy) = + 3.8471$

$m_2(x) = 5.0205$ $m_2(y) = 6.7582$ $r_{16} = + 0.660$

SERIAL CORRELATION COEFFICIENTS (cont.)

for different intervals of PURPLE decometer-readings on May 30th, 1952 (PM) GERMAN chain.

20-minute interval

	t x	.21 - 2	.22 - 1	.23 0	.24 1	.25 2	.26 3	.27 4	.28 5	sum	sum y	sum y^2
t + 20	y											
.28	5				1	4	3	1		9	45	225
.27	4					1	2	1		4	16	64
.26	3					1	3	1		5	15	45
.25	2						1	2	2	5	10	20
.24	1		1						2	3	3	3
.23	0	1	4	5	1	1	3		5	20	0	0
.22	- 1	2	3	8	1	3	1	1	1	20	- 20	20
.21	- 2	1	4	3	1	1	1	1		12	- 24	48
.20	- 3		3	4						7	- 21	63
.19	- 4	1	2							3	- 12	48
.18	- 5		2							2	- 10	50
sum		5	19	20	4	11	14	7	10	90	2	586
sum x		- 10	- 19	0	4	22	42	28	50	117		
sum x^2		20	19	0	4	44	126	112	250	575	sum xy = + 269	

$m'_1(x) = + 1.3000$ $m'_1(y) = + 0.0222$ $cov.(xy) = + 2.9600$

$m_2(x) = 5.6989$ $m_2(y) = 6.5106$ $r_{20} = + 0.486$

TABLE 8 (cont. 6)

SERIAL CORRELATION COEFFICIENTS (cont.)

for different intervals of PURPLE decometer-readings on May 30th, 1952 (PM) GERMAN chain.

24-minute interval

t		.21	.22	.23	.24	.25	.26	.27	.28			
	x	- 2	- 1	0	1	2	3	4	5	sum	sum y	sum y^2
t+24	y											
.28	5				1	2	3	1		7	35	175
.27	4					1	1			2	8	32
.26	3					2	2	1		5	15	45
.25	2					1	3	1		5	10	20
.24	1		1					1	1	3	3	3
.23	0	1	2	5	2	1	2	2	5	20	0	0
.22	- 1	1	5	5		4	2	1	2	20	- 20	20
.21	- 2	2	3	4			1		2	12	- 24	48
.20	- 3		4	3						7	- 21	63
.19	- 4	1		2						3	- 12	48
.18	- 5			1	1					2	- 10	50
sum		5	15	20	4	11	14	7	10	86	- 16	504
sum x		- 10	- 15	0	4	22	42	28	50	121		
sum x^2		20	15	0	4	44	126	112	250			

sum xy = + 172

$m'_1(x) = + 1.4070$
$m'_1(y) = - 0.1860$
$m_2(x) = 4.6599$
$m_2(y) = 5.8259$
$cov.(xy) = + 2.2617$
$r_{24} = + 0.434$

SERIAL CORRELATION COEFFICIENTS (cont.)

for different intervals of PURPLE decometer-readings on May 30th, 1952 (PM) GERMAN chain.

28-minute interval

t		.21	.22	.23	.24	.25	.26	.27	.28	sum	sum y	sum y^2			
x		- 2	- 1	0	1	2	3	4	5						
t+28	y												$m'_1(x)$	=	+ 1.4878
.28	5				1	1	1	1		4	20	100	$m'_1(y)$	=	— 0.5366
.27	4					1	1			2	8	32	$m_2(x)$	=	4.6889
.26	3					2	2			4	12	36	$m_2(y)$	=	4.8341
.25	2					2	2	1		5	10	20	cov.(xy)	=	+ 2.2130
.24	1	1					1	1		3	3	3	r_{28}	=	+ 0.465
.23	0	1	2	3	1	3	2	3	5	20	0	0			
.22	- 1	1	2	6	1	2	3	1	4	20	- 20	20			
.21	- 2		5	4			2		1	12	- 24	48			
.20	- 3		4	3						7	- 21	63			
.19	- 4	1	1	1						3	- 12	48			
.18	- 5		1	1						2	- 10	50			
sum		4	15	18	3	11	14	7	10	82	- 44	420			
sum x		- 8	- 15	0	3	22	42	28	50	122					
sum x^2		16	15	0	3	44	126	112	250	566	sum xy = + 116				

32-minute interval

$m'_1(x) = + 1.6026$ $m'_1(y) = — 0.6667$ sum xy $= + 61$ cov.(xy) $= + 1.851$

$m_2(x) = 4.6240$ $m_2(y) = 3.8632$ $r_{32} = + 0.438$

TABLE 9

TABLE OF RED-AND GREEN DECOMETER-READINGS TAKEN AT ROTTERDAM APRIL 21, 1952.

(Sequence numbers 1-30 from 09.15 - 10.15 and 31-90 from 13.00 - 15.00 GMT)

SEQUENCE NUMBER	RED	GREEN	SEQUENCE NUMBER	RED	GREEN	SEQUENCE NUMBER	RED	GREEN
1	08.30	40.82	31	08.19	40.86	61	08.36	40.87
2	.30	.80	32	.19	.87	62	.33	.81
3	.30	.80	33	.19	.86	63	.31	.80
4	.34	.80	34	.19	.86	64	.31	.87
5	.30	.78	35	.19	.86	65	.32	.79
6	.30	.81	36	.19	.87	66	.31	.79
7	.30	.80	37	.19	.86	67	.36	.77
8	.30	.80	38	.19	.86	68	.35	.79
9	.36	.80	39	.19	.86	69	.36	.79
10	.31	.80	40	.19	.86	70	.36	.75
11	.30	.82	41	.19	.86	71	.25	.78
12	.30	.80	42	.19	.86	72	.34	.76
13	.34	.80	43	.19	.86	73	.34	.79
14	.30	.81	44	.19	.86	74	.33	.77
15	.30	.79	45	.20	.86	75	.33	.75
16	.44	.93	46	.20	.86	76	.33	.79
17	.35	.92	47	.19	.88	77	.28	.84
18	.42	.87	48	.20	.88	78	.28	.83
19	.30	.83	49	.20	.88	79	.31	.80
20	.42	.81	50	.20	.88	80	.29	.80
21	.39	.80	51	.24	.86	81	.31	.77
22	.39	.80	52	.33	.83	82	.25	.77
23	.40	.80	53	.33	.80	83	.27	.76
24	.42	.81	54	.35	.79	84	.29	.77
25	.40	.81	55	.33	.73	85	.25	.78
26	.39	.80	56	.32	.77	86	.25	.77
27	.40	.81	57	.33	.77	87	.33	.73
28	.40	.82	58	.28	.77	88	.35	.73
29	.40	.80	59	.32	.69	89	.19	.85
30	.40	.80	60	.30	.76	90	.19	.85

The units used in the following results of calculations are Units of 0.01 lane.

m'_1 Red = — 1.27 m'_2 Red = 52.58 m_2 Red = 50.97

m'_1 Green = + 0.37 m'_2 Green = 19.23 m_2 Green = 19.09

$$\frac{\text{sum x.y}}{90} = -14.00$$

covariance Red-Green = — 13.53 Corr. coefficient Red-Green = — 0.43

Table 10

CORRELATION COEFFICIENTS BETWEEN SIMULTANEOUS MEANS

of 10 observations (1-minute interval) of ENGLISH and GERMAN chains.

(the values given are differences from the total mean, expressed in standard units)

RED

	English x	x^2	German y	y^2	xy
	+ 1.62		— 1.77		
	+ 1.05		— 1.18		
	+ 0.57		— 0.73		
	+ 0.43		+ 0.27		
	— 1.57		+ 1.32		
	— 1.81		+ 1.18		
	— 0.43		+ 0.91		
	+ 0.14		— 0.14		
	— 0.67		+ 0.59		
	0		+ 0.50		
	— 0.57		+ 0.23		
	+ 0.52		— 0.41		
sum	— 0.72	11.2264	+ 0.77	9.9327	— 9.7651

$m'_1(x) = -0.060$
$m'_1(y) = +0.064$
$m_2(x) = 1.0170$
$m_2(y) = 0.8989$
$cov.(xy) = -0.8100$
$r_{red} = -0.847$

GREEN

	English x	x^2	German y	y^2	xy
	+ 0.68		— 1.36		
	+ 0.71		— 1.31		
	+ 1.68		— 0.88		
	+ 1.36		— 0.50		
	— 0.98		+ 1.06		
	— 1.36		+ 0.75		
	+ 0.10		— 0.13		
	+ 0.88		+ 0.25		
	— 0.37		+ 0.37		
	— 0.54		+ 1.69		
	— 1.41		+ 1.50		
	— 0.68		+ 0.06		
sum	+ 0.07	12.1402	+ 1.48	11.6570	— 9.0840

$m'_1(x) = +0.006$
$m'_1(y) = +0.123$
$m_2(x) = 1.1037$
$m_2(y) = 1.0446$
$cov.(xy) = -0.7577$
$r_{green} = -0.706$

TABLE 10 (cont.)

CORRELATION COEFFICIENTS BETWEEN SIMULTANEOUS MEANS
of 10 observations (1-minute interval) of ENGLISH and GERMAN chains.
(the values given are differences from the total mean, expressed in standard units)

PURPLE

	English x	English x^2	German y	German y^2	xy
	+ 0.19		+ 1.23		
	+ 0.68		— 0.05		
	+ 0.26		+ 1.50		
	+ 1.22		+ 0.91		
	— 0.22		+ 0.86		
	+ 0.35		+ 0.09		
	+ 1.42		— 1.42		
	— 0.97		+ 0.34		
	— 0.94		— 0.18		
	— 1.42		— 0.36		
	— 1.61		— 0.57		
	+ 0.94		— 0.84		
sum	— 0.10	11.5584	+ 1.51	8.6657	+ 0.0045

$m'_1(x) = -0.008$
$m'_1(y) = +0.126$
$m_2(x) = 1.0507$
$m_2(y) = 0.7719$
$cov.(xy) = +0.0027$
$r_{purple} = +0.003$

The curvi-linear regression line in the scatter-diagram of English-German observations has the equation

$$Y = 0.49 - 0.12\,X - 0.51\,X^2$$

The percentage of seapath from the transmitters of the English chain to the monitor is:

Master	68.5 %
Red	85.3 %
Green	70.1 %
Purple	47.7 %

TABLE 11

CORRELÀTION COEFFICIENTS BETWEEN SIMULTANEOUS FICTITIOUS MEANS
of 10 observations (1-minute interval) of ENGLISH and GERMAN chains.
(the values given are differences from the mean, expressed in standard units)

3/4 RED - GREEN - WHITE			GREEN — 3/5 PURPLE = BLUE		
English x	German y		English x	German y	
— 0.55	— 0.03		+ 0.60	— 1.64	
— 0.32	+ 0.27		+ 0.40	— 0.58	
— 1.71	+ 0.27		+ 1.60	— 1.61	
— 1.41	+ 1.27		+ 0.82	— 0.94	
+ 0.50	+ 0.55		— 0.90	— 0.15	
+ 0.85	+ 0.73	r_{white} = — 0.468	— 1.58	+ 0.33	r_{blue} = — 0.605
— 0.30	+ 1.64		— 0.50	+ 1.12	
— 0.97	— 0.55		+ 1.35	— 0.12	
+ 0.15	+ 0.36		+ 0.05	+ 0.33	
+ 0.68	— 1.54		+ 0.05	+ 1.12	
+ 1.47	— 1.73		— 0.70	+ 1.21	
+ 1.09	— 0.64		— 1.12	+ 0.73	

TABLE 12

VALUES OF THE FIRST 190 TERMS OF THE ARTIFICIAL SERIES

$u_t = 5 \sin\left(\frac{360°.t}{100} + e_1\right) + 2 \sin\left(\frac{360°.t}{52} + e_2\right) + e_3$ where e_1 and e_2 are rectangular random variables with range — 9 to + 9 and e_3 is a normally distributed random variable with r.m.s. error of 0.01 lane. Values rounded off to the nearest unit.

-1,	1,	3,	2,	3,	3,	4,	5,	6,	6,	5,	7,	5,	8,	5,	5,	6,	4,
6,	6,	6,	7,	6,	4,	6,	4,	5,	5,	4,	3,	4,	3,	3,	3,	3,	1,
3,	3,	0,	1,	2,	-1,	1,	1,	0,	-1,	1,	-1,	2,	-1,	-1,	1,	-1,	-2,
1,	-1,	0,	-1,	-2,	-2,	-1,	-1,	-3,	-3,	0,	-4,	-3,	-4,	-4,	-2,	-2,	-6,
-4,	-4,	-4,	-3,	-5,	-4,	-6,	-5,	-8,	-6,	-6,	-5,	-7,	-7,	-4,	-6,	-5,	-6,
-3,	-4,	-7,	-4,	-3,	-4,	-2,	-2,	-1,	-2,	-2,	2,	1,	4,	2,	4,	2,	3,
2,	5,	4,	5,	6,	6,	6,	7,	8,	7,	6,	6,	7,	6,	7,	6,	6,	6,
7,	4,	5,	4,	4,	5,	3,	6,	3,	2,	3,	2,	0,	2,	2,	-1,	-2,	2,
1,	-1,	1,	-1,	0,	-2,	1,	0,	-2,	-2,	-2,	-2,	-2,	-3,	-1,	-2,	-2,	-2,
-2,	-3,	0,	-2,	-2,	-5,	-3,	-4,	-4,	-2,	-3,	-3,	-4,	-4,	-2,	-4,	-5,	-5,
-5,	-5,	-5,	-5,	-6,	-4,	-5,	-6,	-5,	-3.								

For histogram of this series, see DRG No. 9.

used between successive readings increases to a certain value. Beyond that value serial correlation of opposite sign may be expected and the shape of the histogram will move away from normality again. This Gaussian distribution of readings with zero mutual correlation is not yet firmly proved, but seems fairly probable (see par. 4.2.1).

d. The shape of the histograms will be subject to false interpretation if a whole number of mean periods of the main oscillation is not comprised in the sample of readings used. This is only true for finite series. For an infinite series or very long series this point is of no interest. If, however, different samples are taken from one parent group of readings it is thought that the phases of the various oscillations at the beginning and end of the sample will influence the shape of the resultant histograms (see par. 5).

e. There is no marked difference between relative frequency histograms expressed in standard units obtained at long and at medium ranges, apart perhaps from a slight tendency to zero-difference from the mean at longer ranges because of reduced torque.

f. Simultaneous observations of patterns of different chains will be related insofar as the radio signals materializing these readings travel along the ground or through the air at the same moment. If the conditions affecting the radio speed of propagation, or affecting phase-shifts along the ground or in the ionosphere, are the same for the signals radiated by both chains the readings will show a more or less marked positive correlation. If these conditions are the opposite for one chain of what they are for the other, a negative correlation between readings of different chains will be found. This is the case for readings taken at the beginning of June 1952 of the English and German chains at The Hague. Furthermore an indication was found for a discontinuity in the ionosphere over the coastline of England in summertime (see par. 4. 2. 3).

Short Description of Trials

2.1 *Long Range Trials.*

On board HNMS « Karel Doorman » readings were taken at large distances from the transmitters of the English chain. These comprise the observations at Algiers, Gibraltar and Lisbon. The ship was moored alongside the docks in all three cases. The respective positions were

ALGIERS	36-46-37.5 N	03-04-03.0 E
GIBRALTAR	36-07-27.6 N	05-21-16.1 W
LISBON	38-42-30.4 N	09-11-08.6 W

These positions were fixed in the nautical charts and consequently they are only correct within the local triangulations. The various distances from the observation points to the transmitters will be subject to substantial errors caused by shifts in latitude and longitude of one triangulation relative to the others. The magnitude of these shifts is unknown.

The readings at Algiers were taken from March 7 to March 10, 1952, and consist of 248 daylight observations. Those at Gibraltar were taken from March 14 to March 16, 1952, and comprise 24 daylight and 180 night observations. The observations at Lisbon were obtained from March 21 to March 25 and include 300 daylight and 455 night readings. The Mark V Marine Receiver was used for all observations together with a 38-foot vertical antenna. The length of this antenna at Lisbon was 70 feet.

The time-interval between two readings was 2 minutes. The green decometer was always read 15 seconds after the red one and the purple decometer 15 seconds after the green one. Thus a fairly dense set of readings was obtained in an hour's time without tiring the observers too much. Such sets of 30 observations (1 hour) were obtained several times a day. The night readings were taken before midnight.

2.2 *The Rotterdam Trials.*

The medium range trials on the English chain were carried out from April 7 to April 27, 1952, when the ship was moored on buoys at Rotterdam. The ship's position according to the Netherlands nautical chart was 51-54-43.9 N 04-29-17.2 E. Normally the daylight observations were carried out from 10.00 to 11.00, 14.00 to 15.00 and from 16.00 to 17.00 M.E.T. (= G.M.T + 1 hour). The night readings were taken from 20.00 to 22.00 MET. The number of available readings is: day 1620, night 870. The time-interval and sequence of the readings are the same as for the long range trials described in par. 2.1. The Mark V Marine Receiver was used, and the length of antenna was 70 feet.

2.3 *The Hague Trials.*

During trial runs of the German chain Mr. J.Th. Verstelle of the Netherlands Hydrographic Office observed these transmissions at the monitor station in the Hydrographic Office, position 52-06-14.3 N 04-17-55.2 E. Only daylight readings are available and about 950 could be used for construction of histograms. The time-interval was 1 minute and the readings were taken from May 30 to June 6, 1952. In addition M. Verstelle observed the English chain in between sets of observations of the German chain. The observations of the English chain consist of sets of 10 uninterrupted 1 minute-interval readings.

3. *Results*

3.1 *Relative Frequency Histograms.*

Drawing No. 1 gives relative frequency histograms for daylight observations at Algiers of Red, Green and Purple. The three histograms are given in differences from the total daylight mean expressed in half standard units (1/2 r.m.s. error).

Drawing No. 2 gives similar histograms for daylight observations at Gibraltar. The number of night observations was too scanty to be used in a histogram.

Drawing No. 3A gives relative frequency histograms for daylight observations at Lisbon, using the differences from the total mean expressed in half r.m.s. error. The night observations at Lisbon were widely scattered, but brought together in the histograms of drawing No. 3B, using the differences from the daylight mean expressed in half standard units.

Drawings 4A and 4B give the histograms of the daylight observations at Rotterdam; differences from the total mean are again expressed in half standard units; as drawing No. 5 shows the histograms of the night observations at Rotterdam, differences are counted from the daylight mean.

All the foregoing histograms are of observations of the English chain. Drawing No. 6 gives the histograms of observations at The Hague monitor of the German chain in the same way as the above.

3.2 Correlation between simultaneous Decometer-Readings of the Same Chain.

It is a well-known phenomenon that the simultaneous decometer-readings of different patterns of the same chain are correlated, and as all three decometer-readings are the result of the difference in phase between the Master and respective Slave-signal as received at the observation-point, they all have the Master-signal in common. Consequently a change in phase of the Master-signal will affect all three decometers in the same way, causing a positive correlation to exist most of the time between simultaneous readings of different colours.

However, the Red histogram of drawing No. 4A and the Green histogram of drawing No. 4B resulting from the daylight observations at Rotterdam suggest the existence of a negative correlation at the edges of the histograms. Table No. 9 gives the reading of the Red and Green decometers on April 21, 1952, which were the cause of the wide skirts of the histograms. The first 30 readings were taken from 09.15 to 10.15 GMT, the following 60 from 13.00 to 15.00 GMT. This table also gives the variances of both sets of readings, the covariance and the correlation coefficient r which is equal to —0.43. The significance of this moderate negative correlation can be found by estimating the sampling variance of this correlation coefficient. For this purpose we use the equation

$$\text{variance } r = \frac{1}{n} (1 - R^2)^2$$

where r is the corelation coefficient of the sample of 90 Red and Green readings and R is the parent correlation coefficient. Strictly speaking this equation is only valid for normal distributions and a large n, but our order of approximation this is indifferent and for convenience we also can take $r = R$. We thus find var. $r = 0.0075$ and the standard error of $r = 0.086$, and it is improbable that the parent correlation coefficient lies outside the range -0.43 ± 0.17 and very improbable that it lies outside the range -0.43 ± 0.26, which means that the negative correlation found is slightly significant.

3.3 Serial Correlation.

A set of decometer readings is in fact a fine specimen of a time series in which the observed values are an unknown function of time. To state this more accurately it can be said that the observed readings are known functions of values assumed by variables at different points of time. This time-variation can hardly be regularly functional owing to physical influences and consequently will involve random variables, thus being of the stochastic type.

Of the three parts of which these series are generally thought to be composed, i.e. a trend or long-term movement, an oscillation about the trend and a random component, the trend can be considered non-existent in decometer-readings. In fact monitoring of a chain is mainly done to prevent trend from entering into the patterns.

But oscillating as well as random components are the two parts that caracterize a set of decometer-readings. If however the oscillatory movement is slow and the sample of readings small or rather covering a short interval of time, the false inference could be made of trend being present where only part of a slow oscillation was observed. In the following all trend will be considered to be absent from the parent group and sets of decometer-readings of sufficient scope will be considered to present fluctuations of a more or less regular kind. In case short samples of readings have to be used, it may be necessary to eliminate trend first.

It is clear that the observed values in a set of decometer-readings are *not* random in the sense that they could occur *in the observed order,* by random sampling from a homogeneous group. The systematic part of the series is the oscillatory movement which perhaps could be represented as a function of time, on which a purely random part is superposed.

From the very nature of random variation we cannot expect to find any formula, however approximate, which will measure the random component at any given point of the series. The only way to proceed is to determine or to eliminate the non-random components and to obtain a random residual unaccounted for by these components. One of the major difficulties is to find out to what extent the methods used for determinaton or elimination of the systematic components have affected the random residual.

The more or less regular fluctuations of the time-series were called oscillations and not cycles. By a cyclical component is meant one that is a strictly periodic (sine or cosine) function of time. Considering the many physical irregularities which influence a set of readings it is clear that our decometer observations will never produce a time series of the cyclical kind. An oscillatory movement, however, has no sharply defined period but fluctuates more or less regularly about a mean value.

The assumed oscillatory movement of the observed values of a sample of decometer-readings makes necessary the introduction of the time interval between two successive readings. It is clear that the length of this interval will influence the discrepancy to be expected between two successive readings taken at the same observation point. The change in reading during a few minutes is likely to be smaller than the change over half an hour or more. Here the calculation of serial correlation coefficients for different time intervals will allow us to find the interval which must elapse between readings before they can be treated as mutually uncorrelated observations. Moreover the correlogram, i.e. the graph showing the change of the serial correlation coefficient against the time interval might reveal something of the structure of the oscillatory movements involved. Further details are given in par. 5.

Tables No. 1, 2 and 3 give serial correlation coefficients of the Red decometer-readings of the German chain observed at The Hague for a 2-minute, 8-minute and 30-minute interval. Tables No. 4, 5 and 6 give the same for Purple readings. These tables were calculated by using pairs of observations separated by the correct time interval. As the complete set of observations, however, was not continuous but consisted of several sets of unequal length, the normal way to calculate serial correlation coefficients could not be followed, e.g. after using the pair u_t, u_{t+a} it was not always possible to obtain the pair u_{t+1}, u_{t+a+1}. So the number of available pairs grew progressively smaller with growing interval a. But the pairs of values used in tables No. 1 to 6 cover all the days on which observations were made.

In order to find the possible exstence of short-term oscillations the serial correlation coefficients were also calculated for 2, 4, 8, 12, 16, 20, 24, 28 and 32-minute intervals of the Red and Purple decometer-readings taken on May 30, 1952. On this day a sample of 130 uninterrupted observations was available which showed no trend caused by long-term oscillations and thus made it possible to avoid trend-elimination. It has been stated already that to find short-term oscillations by serial correlation coefficients from a relatively short sample the latter must not be disturbed by false trend due to long-term oscillations, as a positive or negative trend would cause the calculated correlation coefficients to diminish either too slowly or too quickly. For the short-term serial correlation coefficients see the calculations of tables No. 7 and 8.

The notations used in the tables No. 1 to 8 are:

$m'_1(x)$ = the first moment of x, i.e. the mean value of x = $\frac{\text{sum } x}{n}$, with regard to an arbitrary reference point, e.g. an arbitrarily chosen zero-point.

$m_2(x) = m'_2(x) - (m'_1(x))^2$ is the second moment of x, called the variance of x with regard to the *mean*, $m'_2(x)$ being equal to $\frac{\text{sum } x^2}{n}$ with regard to the arbitrary zero-point. The same notations apply to y. The covariance of x and y is found from cov.(xy) = $\frac{\text{sum } xy}{n} - m'_1(x).\ m'_1(y)$. From this covariance the serial correlation coefficient can be calculated with the equation

$$r = \frac{\text{cov. }(xy)}{(m_2(x).\ m_2(y))^{1/2}}$$

3.4 *Correlation between Simultaneous Readings of the Same Colour of Different Chains.*

At the monitor station at The Hague not only were decometer-readings of the German chain taken, but small sets of observations of the English chain were also collected for control purposes. As only one Mark V Marine Receiver was available, the sets of readings could not be exactly simultaneous but had to be taken one after the other.As each set of observations of the English chain consisted of 10 uninterrupted 1-minute interval readings, the nearest set of the same weight of German observations was used for comparison, giving a time interval between the respective means of at least 10 minutes with a maximum of 13 minutes. These means were used for determination of the amount of correlation between them and had the following values expressed as differences from their total means in standard units :

RED		GREEN		PURPLE		
German	English	German	English	German	English	Time of the day
— 1.77	+ 1.62	— 1.38	+ 0.68	+ 1.23	+ 0.19	11.55
— 1.18	+ 1.05	— 1.31	+ 0.71	— 0.05	+ 0.68	15.35
— 0.73	+ 0.57	— 0.88	+ 1.68	+ 1.50	+ 0.26	10.00
+ 0.27	+ 0.43	— 0.50	+ 1.36	+ 0.91	+ 1.22	08.55
+ 1.32	— 1.57	+ 1.06	— 0.98	+ 0.86	— 0.22	14.02
+ 1.18	— 1.81	+ 0.75	— 1.36	+ 0.09	+ 0.35	15.01
+ 0.91	— 0.43	— 0.13	+ 0.10	— 1.42	+ 1.42	09.51
— 0.14	+ 0.14	+ 0.25	+ 0.88	+ 0.34	— 0.97	12.06
+ 0.59	— 0.67	+ 0.37	— 0.37	— 0.18	— 0.94	14.12
+ 0.50	0	+ 1.69	— 0.54	— 0.36	— 1.42	11.11
+ 0.23	— 0.57	+ 1.50	— 0.68	— 0.57	— 1.61	14.27
— 0.41	+ 0.52	+ 0.06	— 0.68	— 0.84	+ 0.94	10.40

Table No. 10 gives the calculation of the various correlation ceofficients. Though the Purple means appear to be uncorrelated their scatterdiagram given in Drawing No. 7 shows the probable existence of a regression curve. A first approximation of this curve was found by fitting a second-order polynomial to the observed points by means of the equation

$$\begin{vmatrix} Y & 1 & X & X^2 \\ )Y(& n &)X(&)X^2(\\ )YX(&)X(&)X^2(&)X^3(\\ )YX^2(&)X^2(&)X^3(&)X^4(\end{vmatrix} = 0,$$

where the reversed brackets)(stand for summation of all the terms between them. This determinant results in : $Y = 0.49 - 0.12\,X - 0.51\,X^2$

As is seen in the scatterdiagram this curve fits only loosely and it remains to be proved whether or not curvi-linear regression has much significance. Scatter-diagrams for Red and Green are not given here; they show a definite linear regression.

In order to eliminate the influence of the Master signal from both readings of a pair, fictitious readings were formed:

White = 3/4 Red - Green and
Blue = Green - 3/5 Purple. As the three decometer readings Red, Green and Purple are formed by the following equations:
Red = 4.ϕ Master — 3.ϕ Red
Green = 3.ϕ Master — 2.ϕ Green
Purple = 5.ϕ Master — 6.ϕ Purple, where ϕ stands for the phase of the Master and respective slave signals as they are received at the observation point, the fictitious readings White and Blue are no longer influenced by the phase of the Master signal. These readings could be found directly from:

White = 2.ϕ Green — 9/4.ϕ Red
Blue = 18/5.ϕ Purple — —2.ϕ Green.

The correlation coefficients of the fictitious White means for both chains as well as of the Blue means are given in Table No. 11.

4. *Analysis of Results*

4.1 *Histograms.*

In some publications on Decca and also in one or two of the Memos of the Planning Section of the Decca Navigator Company it is stated that the distribution of decometer-readings is not normal but peaky with wider « skirts » than those of the normal curve.

To be able to compare different histograms that are the result of samples of readings of various sizes and different r.m.s. error, all histograms should give relative frequencies of the differences from the total mean expressed in standard units, i.e. the r.m.s. error as unit of difference. Drawings No. 1 to 6 show different histograms constructed accordingly, with the exception that half standard units are used as units of difference.

Indeed these histograms indicate most clearly that the frequency distribution of a sample of decca readings is not of the normal type. Moreover the histograms vary in appearance: some are peaky, others are skew or both. But apart from the Rotterdam daylight trials there is little evidence the « skirts » being wider than in normal frequency distributions. And even the large differences from the total mean observed at Rotterdam on April 21, 1952 (see table No. 9 and drawings No. 4A and 4B) occurred only once, and it has been shown from the negative correlation between Red and Green on that day that something uncommon happened there.

Except for this unaccounted for phenomenon, it may be said that the frequency distribution of decometer-readings is about as wide as the normal distribution. But its tendency to skewness and peaks as well as its generally larger number of variates around the zero value show that the decca frequency distribution is not Gaussian. Moreover, no other classic frequency functions show histograms resembling those of samples of decca readings. The shape of these histograms will be subject to some further investigations in a following chapter.

4.2 *Correlation*

4.2.1 Serial correlation.

In drawing No. 8 a correlogram is given showing the graphs of the calculated serial correlation coefficients against the time interval for Red Purple of the German chain, using all days of observation, thereby including long-term oscillatory movements. The other correlogram in this drawing was calculated from the sample of May 30, 1952 only, so as to exclude long-term but still include short-term oscillations. These correlograms show clearly the presence of long-term as well as short-term oscillatory influences.

The available samples of uninterrupted observations are far too short to give any significant results with regard to long-term fluctuations, but long enough to reveal something about short-term oscillations, at least as far as the Red decometer-readings are concerned.

As is indicated in drawing No. 8 the Red correlogram intersects the abscissa in the neighbourhood of a time interval of 25 minutes. This means that Red decometer-readings on May 30, 1952 taken with an interval of 25 minutes are no longer influenced by systematic components from short-term fluctuations. It should be borne in mind, however, that this absence of serial correlation does not mean that readings thus spaced are mutually independent. Generally, absence of correlation in one particular case out of many similar ones in which correlation is present only means that regression lines intersect at right angles. The case in which two uncorrelated variates are mutually independent only occurs when both variates are normal.

It would be interesting to know whether or not this absence of serial correlation for a 25-minute interval is consistent in samples of readings taken on other days than May 30. For this purpose a number of 70 readings lying 25 minutes apart was randomly chosen out of all the available observations of the Red decometer of the German chain. Care was taken not to overlap the various 25-minute intervals. In order not to let long-term oscillatory movements enter the result every observation chosen was considered as a difference from the mean of the sample in which it occurred. The differences were expressed in units of 0.01 lane. The frequency distribution of these 70 variables was:

Difference from sample means	Observed numbers expressed in: units of 0.01 lane	standard units	Normal distribution in standard units
+ 3	0	0	0.3
+ 2	3	2	3.7
+ 1	17	16	16.9
0	27	31	27.9
— 1	16	15	16.9
— 2	4	5	3.7
— 3	3	1	0.3

The third column gives the observed differences reduced to standard units and the fourth column the normal frequency distribution of 70 variables expressed in standard units. The closeness of fit between the observed frequency distribution and the Gaussian one is surprising even if the slight central tendency is considered. The discussion of what this might prove will be postponed till the end of this paragraph.

As the uninterrupted sets of observations at Rotterdam are too short to be used for the construction of similar correlograms of sufficient scope, the time interval of 25 minutes found for the Red decometer-readings of the German chain was also used on the available observations of the Red decometer of the English chain at Rotterdam, in order to find the frequency distribution of these readings when taken 25 minutes after each other. The same method as for the observations at The Hague was applied with the one exception that the differences from the sample means are given in standard units only.

The results were:

ROTTERDAM RED DECOMETER-READINGS

Differences from sample means expressed in standard units

Daylight observations Diff.	observed	normal	*Night observations* Diff.	observed	normal
+ 4	2	0.01	+ 3	1	0.4
+ 3	3	0.7	+ 2	8	4.7
+ 2	5	8.8	+ 1	14	21.1
+ 1	18	39.2	0	38	34.7
0	94	64.6	— 1	16	21.1
— 1	28	39.2	— 2	7	4.7
— 2	9	8.8	— 3	3	0.4
— 3	3	0.7			

Here the discrepancy between the observed distribution and the normal one is considerable especially for the daylight observations. In both cases there are too many zero values and too few variates in the + 1 or — 1 class, which could partly be caused by friction of the fractional pointer, but another reason will be given in par. 5. Moreover it should be borne in mind that the 25-minute interval was found in the case of observations at The Hague and used also for the readings taken at Rotterdam of a different chain.

But with some certainty it can be said that the random component in a sample of decometer-readings is normally distributed, though further confirmation of this will be necessary.

It is clear that the time interval of 25 minutes found to exist between mutually uncorrelated observations is about a quarter of the mean period of the main short-term oscillation of the sample investigated. But the irregularities of the Red and Purple correlograms suggest the existence of smaller « ripples » with shorter periods which could only be found if the frequency of observations were appreciably higher.

4.2.2. The negative correlation found at Rotterdam between the Red and Green decometer-readings of the English chain

As no change of the phase of the received Master signal can be responsible for this phenomenon, an explanation of it can only be found by assuming a temporary negative correlation between the Red and Green slave signals. This negative correlation could hardly be accounted for by interference from transmitters on board.

But as the azimuths of the incoming Red and Green slave signals differ by rather a large amount, about 38°, it is not impossible that some meteorological influence is responsible for this negative correlation, e.g. a passing front with warm and cold air in the frontal zone. However, nothing definite can be said about this.

4.2.3 The correlation between patterns of different chains

Table No. 10 gives the correlation coefficients between simultaneous means for all three patterns of the English and German chain. Especially the high negative correlation for Red and Green, but also the absence of correlation for Purple, are of interest, though in the Purple case traces of curvi-linear regression have been found.

In Table No. 11 the same negative correlation appears between fictitious means of both chains, proving that the Master signal is not solely responsible for this negative correlation. It is a fair assumption to consider all incoming signals of the same colour but from different chains to be negatively correlated.

The attention is drawn to the fact that all signals from the German chain travel completely over land, whereas the signals from the English chain all have to cross the North Sea. The percentage of their seapath is given in Table No. 10 and is: Master 68.5 % Red 85.3 % Green 70.1 % and Purple 47.7 %. This means that the « single-hop » skywave components of all the English signals except the Purple one have their reflection-points over the North Sea. For Purple this reflection-point is still inland but nearly over the coastline.

The absence of correlation in the Purple case again does not mean that the English and German Purple decometer-readings are mutually independent. The scatterdiagram of drawing No. 7 illustrates this point; the variates are obviously dependent. In fact for negative values of the English readings there is a rather marked positive correlation which changes into a less dominant negative correlation for positive values of those readings. From the times given in par. 3.4 it appears that negative values for English Purple only occur between about 11.00 and 14.45 MET or between 10.10 and 13.50 Local Mean Time at the reflection-point of the single-hop skywave component of the Purple slave signal, which point lies about 5 nautical miles inland from a position on the coast 2 miles north of Orfordness lighthouse.

A negative difference from its mean of the Purple readings indicates either a negative difference from its mean of the phase of the Master signal, or a positive difference of the phase of the Purple slave signal, as the Purple reading is equal to $5.\varphi$ master — $6.\varphi$ purple where φ stands for the phase of the incoming signals. It is improbable that φ Master is responsible for the negative difference of Purple readings as all other readings show a far less marked correlation with time. It is more likely therefore that φ Purple has a value above normal in the four hours around noon. That this is caused by the groundwave is not probable as this wave is rather stable and its intensity and phase mainly a function of the distance and soil conductivity. Lateral refraction at the coast line has nearly a constant value and will moreover be practically absent in the Purple case, as the slave signal crosses the coast nearly at right angles. Skywave intensity and phase, however, are dependent on the effective height of the reflective layer, the reflection coefficient, the convergence factor and path-length. But whether the skywave intensity and phase have to increase or decrease to give φ Purple (which is the resultant of the phases of the received groundwave and skywave) a value above normal is not known as this depends on the unknown relative phase of the skywave with respect to the groundwave.

We see, however, that the Purple readings of the English chain behave like the other two regarding the negative correlation with the German chain, except during the four hours around noon, as the below-normal Purple readings, giving positive correlation, were only found during these four hours.

The readings were taken during mid-summer, and the suggestion might be made that a discontinuity in the effective height of the ionosphere occurs over a coastline when the difference of temparature between soil and water is large. This is only a hypothetical statement of doubtful significance, but further experiments in the summer of 1953 will perhaps prove there is some truth in it.

The marked negative correlation of Red and Green between both chains can only be accounted for by assuming different conditions prevailing on the ground- and air paths of signals coming from different chains, an assumption which is not altogether impossible when one path is lying completely over land while the major part of the other lies over the sea. It can be said that the numerical values which express these conditions will be negatively correlated too.

5. *Discussion*

The normally distributed random component in a sample of decometer-readings as found in par. 4.2.1 by serial correlation must be considered as being caused mainly by observational fluctuations.

It would be of importance to know which type of oscillation was eliminated when the serial correlation was reduced to zero in the case of the sample of Red decometer-readings of the German chain. Three schemes will be considered briefly which can account for the typical oscillatory movements usually observed.

a. *Moving averages.* In trend elimination there is a method of moving averages along the sample of observed values in order to find a series with the same oscillatory properties as the original one, but free from trend. In practice it is found that the random component superposed on the oscillatory movement is affected by this method and shows a marked oscillation after trend is eliminated. This is a spurious oscillation, induced by the method of moving averages, and it was studied by Slutzky and Yule and called after them the « Slutzky-Yule effect ». But even when trend and oscillation are absent and the series is purely random, the application of this method will still induce spurious oscillations in the new series found. It is at least possible that some of oscillations observed in time series may be generated in this way.

b. *Sums of cyclical components.* An attempt could be made to represent the oscillations as the sum of a number of cyclical components by Fourier analysis or general harmonic analysis.

c. *Autoregression equations.* Under certain conditions a series will be of the oscillatory type, e.g. the series

$$u_{t+2} = - a.u_{t+1} - b.u_t + e_{t+2}$$

where a and b are constants and e is a random component.

From the nature of radiopropagation it seems improbable that the schemes a and c would account for the observed oscillations in a sample of decometer-readings. Scheme b seems more promising. But before going on, it should be mentioned that the three types of series considered above, however similar their graphs may be to the eye, will have distinct types of correlograms provided the series are long enough for the observed correlations to approach the expected values for an infinite series.

The correlogram of a series generated by moving averages, though it may oscillate in the beginning, will vanish after a certain point; that of a series of harmonic terms will oscillate, but will not vanish or be damped; that of the auto-regressive scheme will oscillate and will not vanish, but it will be damped. Without proving these statements mathematically, it can be said that the correlogram offers a theoretical basis for discriminating between the three types of oscillatory series.

Unfortunately the series with which we had to work were far too short to enable a decisive distinction to be made. If long-term oscillatory movements are included, an uninterrupted sample of decometer-readings of about 120 hours would be necessary to make that distinction with some confidence. Such a long series (of 1- or 2-minute intervals) will require the calculation of serial correlation up to the 120th order or even higher. Such arithmetical calculations are extremely tedious and though further experiments are planned for the summer of 1953, an attempt has already been made to generate a series of type b using some of the information found in the Red correlogram of the German chain in order to see whether or not such artificial series could be used as mathematical models.

It was found from drawing No. 8 that the quarter of the mean period of the main Red oscillation was 25 minutes with a minor oscillatory movement, a ripple, having a quarter period of 13 minutes, superposed. Now the following artificial series was calculated in an attempt to represent the Red decometer-readings taken of the German chain during daylight :

$$u_t = 5.\sin \left(\frac{360^\circ.t}{100} + e_1\right) + 2.\sin \left(\frac{360^\circ.t}{52} + e_2\right) + e_3$$

where the amplitude 5 and 2 are given in units of 0.01 lane, and for convenience were considered to have a constant value. The unit of time t is 1 minute. e_1 and e_2 are rectangular random variables with a range of -9° to $+9^\circ$. e_3 is the normally distributed random observational variable with a r.m.s. error of 0.01 lane.

The range of e_1 and e_2 was arbitrarily chosen so as to compensate more or less for the obvious fact that if the fluctuations in readings, caused by oscillatory movements in the phases of groundwave and skywave, can be represented by sums of cyclical components, these fluctuations will never be of a pure sine form but will be physically disturbed.

The values for the amplitudes of 0.05 and 0.02 lanes as well as the r.m.s. error of 0.01 lane for e_3 are possible values that have been observed in practice.

The first 190 terms of this artificial series are given in table No. 12 and its histogram in drawing No. 9. This histogram seems quite acceptable as a representation of a number of decometer-readings and in fact resembles very much the histogram of the observed Red decometer-readings at The Hague in drawing No. 6 except for the larger r.m.s. error. The graph of these 190 terms seems to be more irregular than most decometer-graphs are, but a possible explanation for this will be given at the end of this paragraph. The artificial series was started with both sine functions having phase zero, which is not the general case but can be done without loss of generality as the periods are not commensurable. However, small variations in the histogram could be obtained by choosing at random one of the phases at the beginning of the series. This fact is a possible explanation for observed differences in histograms constructed from different parts of the same sample of readings.

If it could be proved that the oscillatory motion of a decometer time series consists mainly of probably disturbed cyclical functions of time, the interval of 25 minutes found for the Red decometer-readings of the German chain (par. 4.2.1) should be the quarter period of one of those cyclical movements. This cyclical function can be represented by a sine function of the appearance $y = a.\sin(x+E)$, where a is the amplitude in appropriate units and probably varying with time more or less regularly, x a variable increasing uniformly with time and E a random component of normal, rectangular or other distribution. Let $x = 360^\circ.t/w$; then it can be shown that the correlogram of this function $y = a.\sin(360^\circ.t/w + E)$ will oscillate according to the equation $r_t = a.\cos(360^\circ.t/w + F)$ approximately, where F is a random component different from E. The complication of a varying amplitude may be left aside for the moment. From the equation of r_t it follows that when $t = 1/4.w$, $r_t = 0$ approximately.

If we have one finite series composed of c cycles and choose an arbitrary starting point at the beginning of the series, call the phase of this point ϕ and from this point on start counting the points interspaced $t = 1/4.w$, it will be seen that only 4 points of each cycle will be used and that of all 4.c available points the phase will be either ϕ or $\phi + 90^\circ$ or $\phi + 180^\circ$ or $\phi + 270^\circ$. The ordinates of these points will be $a.\sin(\phi + E)$ or $a.\cos(\phi + E)$ or $- a.\sin(\phi + E)$ or $-a.\cos(\phi + E)$. This means that the set of these 4.c points forms a discontinuous rectangular histogram with a relative probability of occurrence of 0.25 for each variate.

But if, as is the case with the 70 randomly chosen readings interspaced 1/4.w in par. 4.2.1, several samples are available and used, and if we may assume that all samples have a length of about one cycle and that the starting point in each sample will be points with mutually different and uncorrelated phases, the problem is slightly more complicated. Of the available samples s_1 to sn we call the phases of the respective starting point ${}_1$ to n. If ϕ_1 to ϕn are more or less regularly distributed along one quarter cycle, it can be proved that the sample of points thus found (in case of complete regularity of distribution of the initial phases) forms a discontinuous nearly rectangular histogram with a relative probability of occurrence for each variate of 1/2n for n equal or greater than 2. Only the border variate $\pm$ a has a relative probability of occurrence of 1/4n.

This means that the random observational variable which makes the observations of these equally probable variates fluctuate should nearly disappear from the histogram of readings lying 1/4.w apart. This histogram should be very flat-topped over the whole range of $+a$ to $-a$ and from there disappear smoothly along a curve slightly resembling the edges of a normal distribution histogram.

But nothing of this kind was found in the three observed frequency distributions given in par. 4.2.1; which does not mean that the initial phases ϕ_1 to ϕn are irregularly distributed, for even then a marked flatness should have occurred instead of a central tendency. Nor does it show that the interval of 25 minutes should not be consistent. The only reason for this central tendency, especially in daylight-readings, can be a very irregular value of amplitude a. This in fact has been observed in practice, as several of the used one-hour samples gave only one unvarying reading, with consequently a sample mean of the same value and all differences zero. This need not necessarily mean reduced torque or absence of it, for very stable propagation conditions, mostly during daylight, may also be the cause. No reason was found for leaving these samples out of consideration as they too show a kind of variation, if only with zero amplitude. If moreover the fractional pointer is very quiet, the influence of even the slightest friction will be such that the r.m.s. error of the random observational variable is reduced to zero, thereby causing too large a number of zero values to enter the histograms.

A correlogram analysis of the artificial series was not carried out for several reasons. First of all the oscillations used would recur more or less regularly, depending on the length of the series. Secondly it is only worthwhile in our present stage of knowledge to analyse correlograms of considerably large series of observed values in order to obtain decisive information regarding the type of oscillations that are involved.

If this information should indicate sums of cyclical components, as it well may do, then will be the time to attempt the construction of more elaborate artificial series to be used for prediction purposes. It is the writer's opinion that in this direction some, at least, of the answers to questions regarding histograms, correlograms, oscillations, variations in skywave and in groundwave will be found.

Biak, New Guinea,
December 23rd, 1952.

Lieutenant Commander
W. Langeraar R.Neth.N.

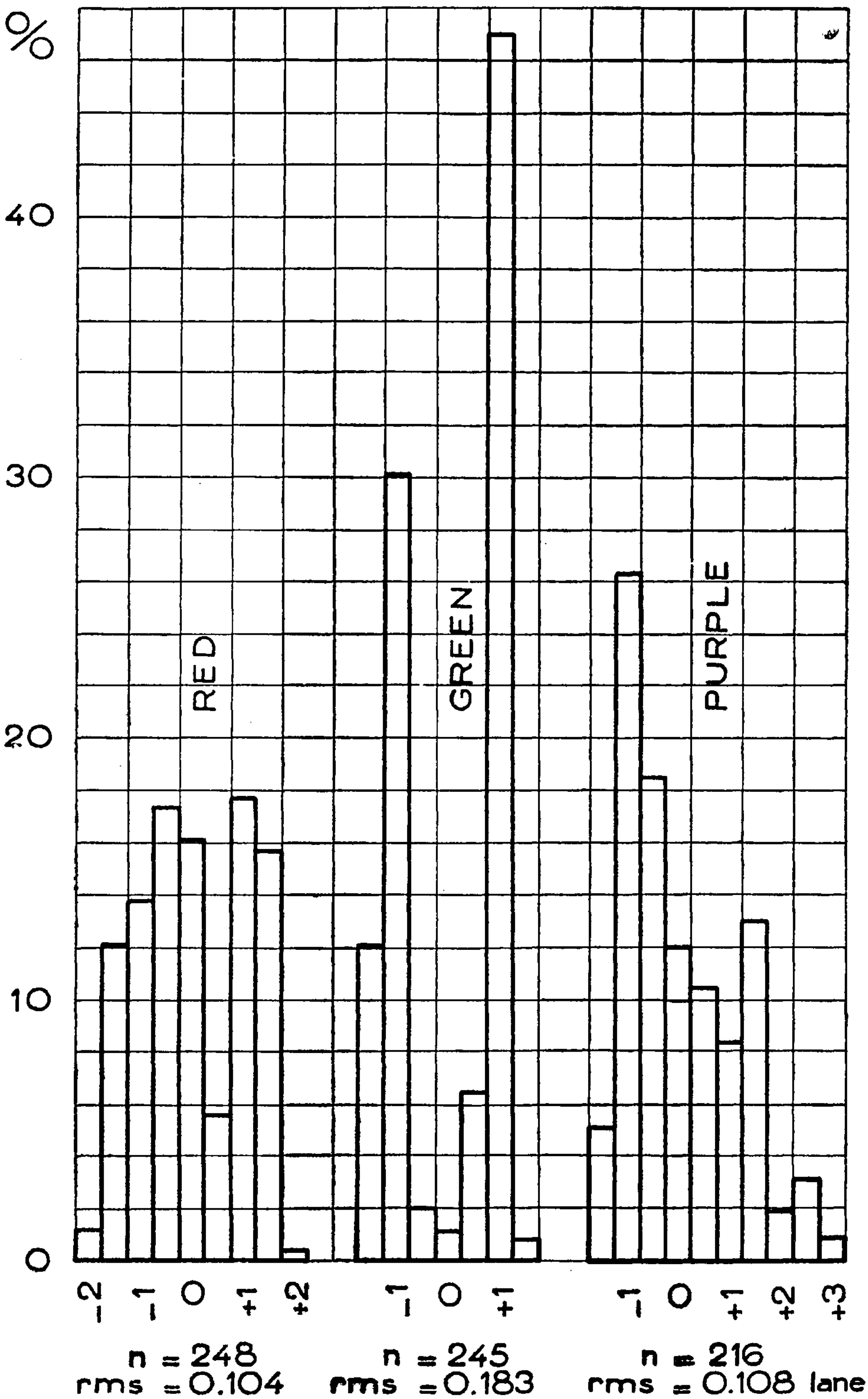

Plate No. 1 : ALGIERS. — Differences from total mean expressed in half standard units (1/2 rms)

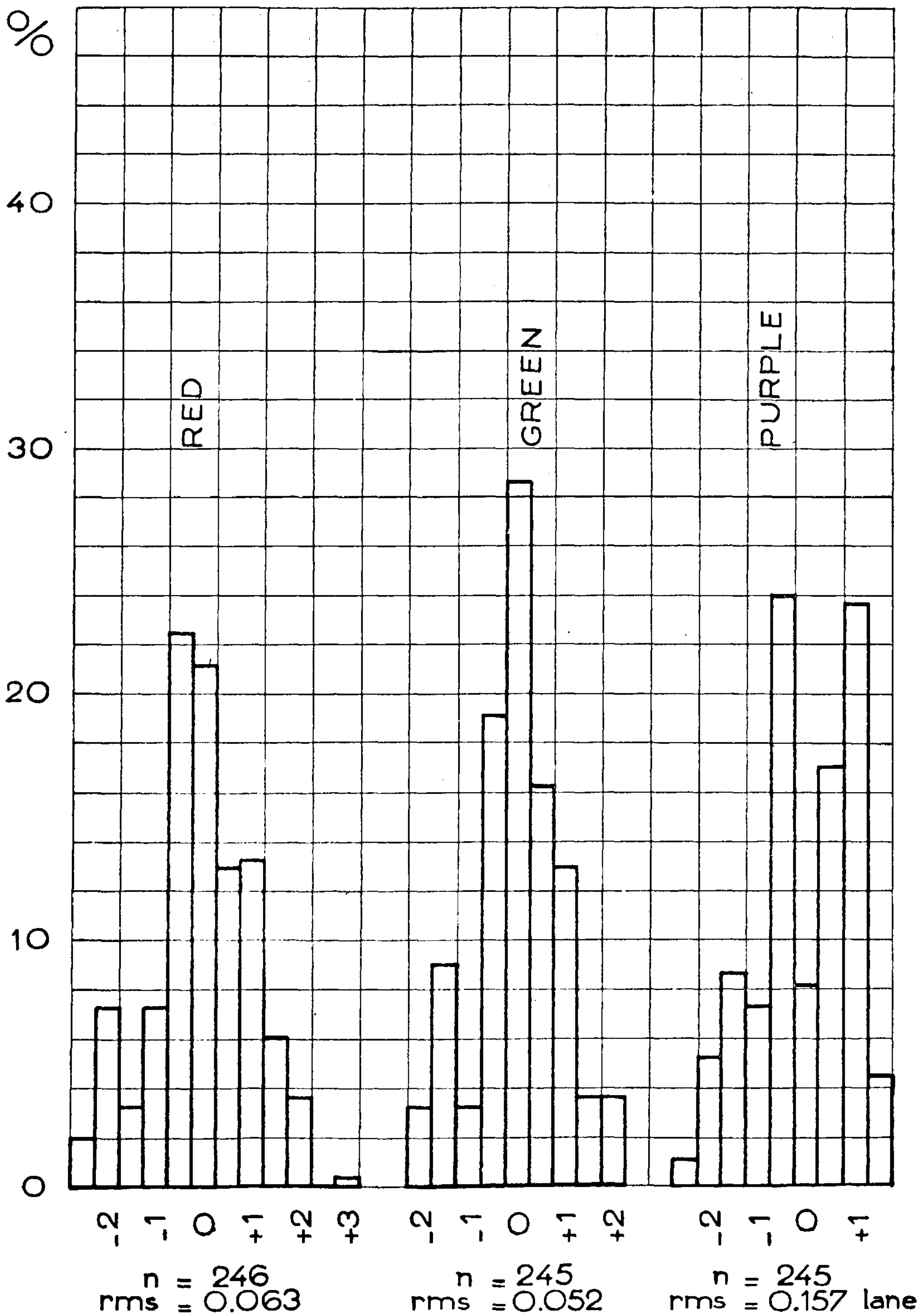

Plate No. 2 : GIBRALTAR. — Differences from total mean expressed in half standard units (1/2 rms).

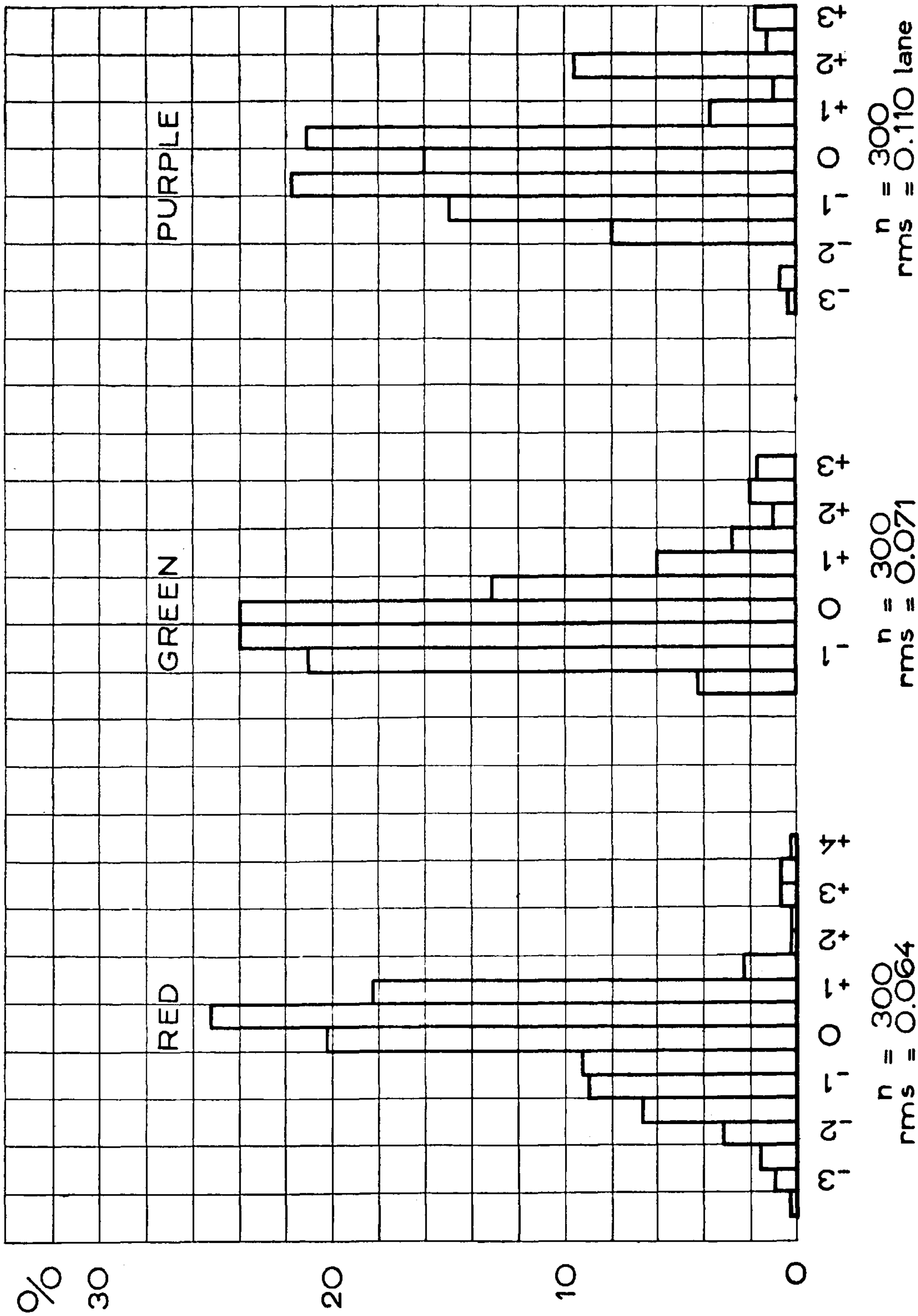

Plate No. 3A : LISBON. — Daylight.

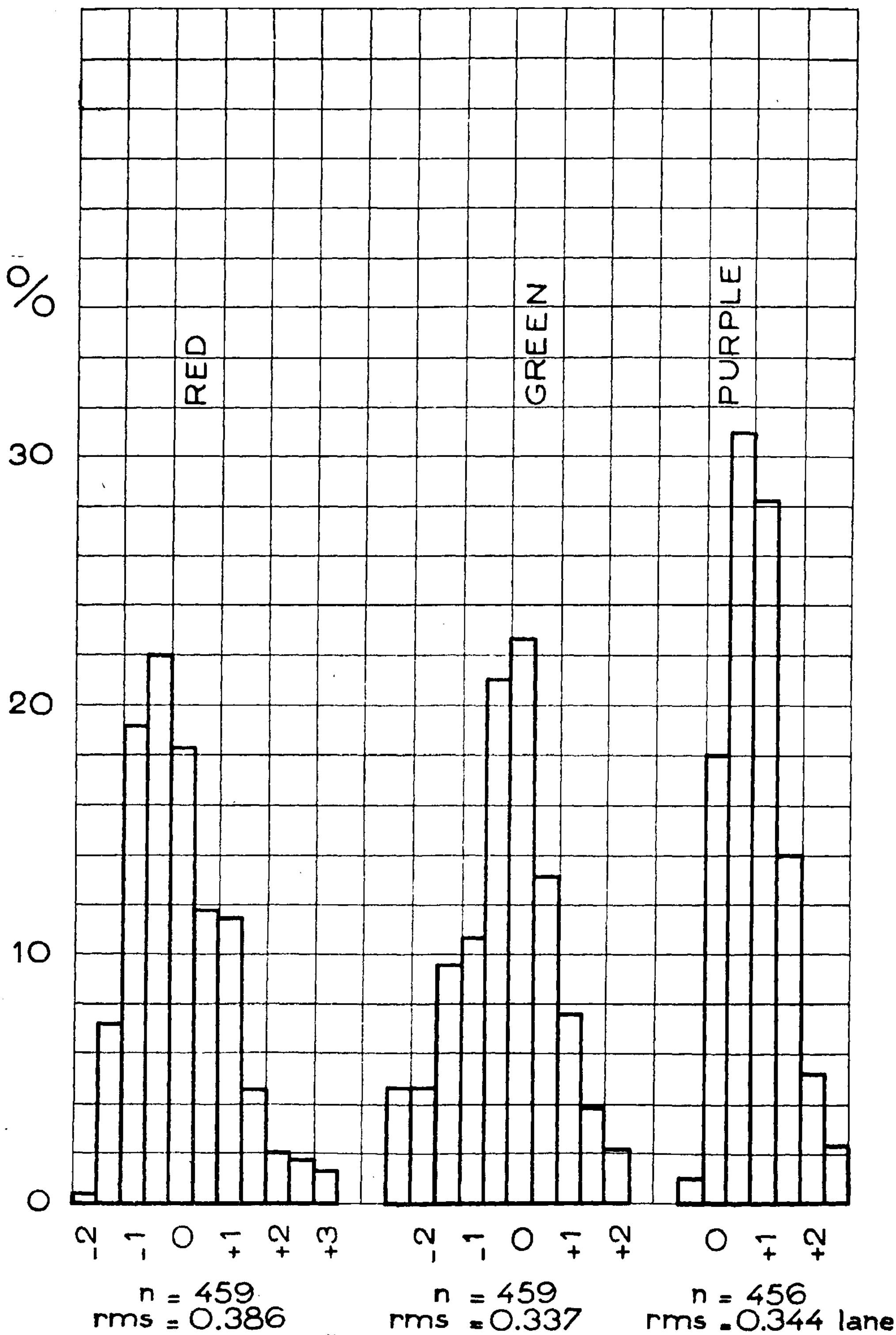

Plate No. 3B : LISBON. — Night Obs. — Differences from total daylight — mean given in half standard units (1/2 rms).

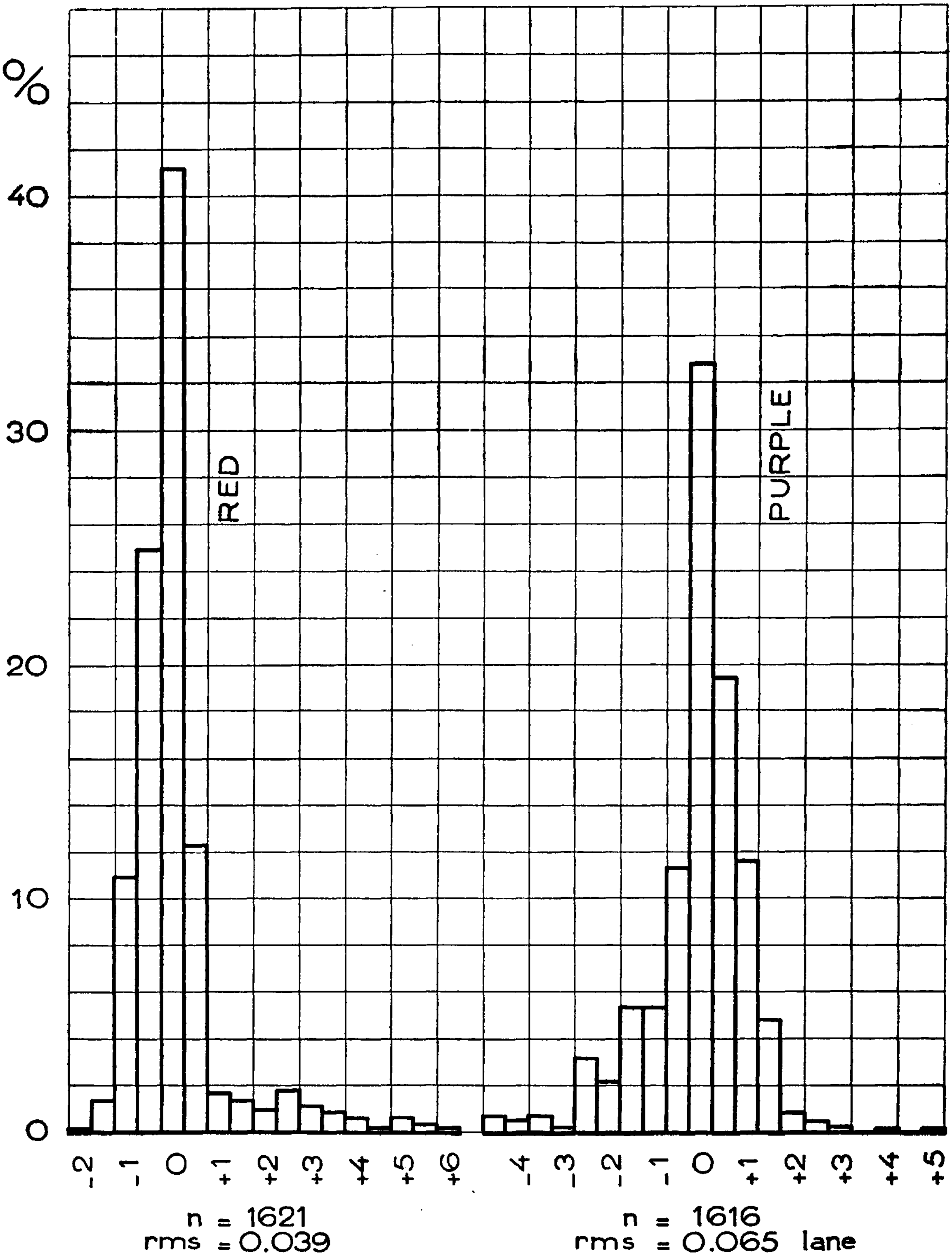

Plate No. 4A : ROTTERDAM. — Daylight. — Differences from total mean expressed in half standard units (1/2 rms).

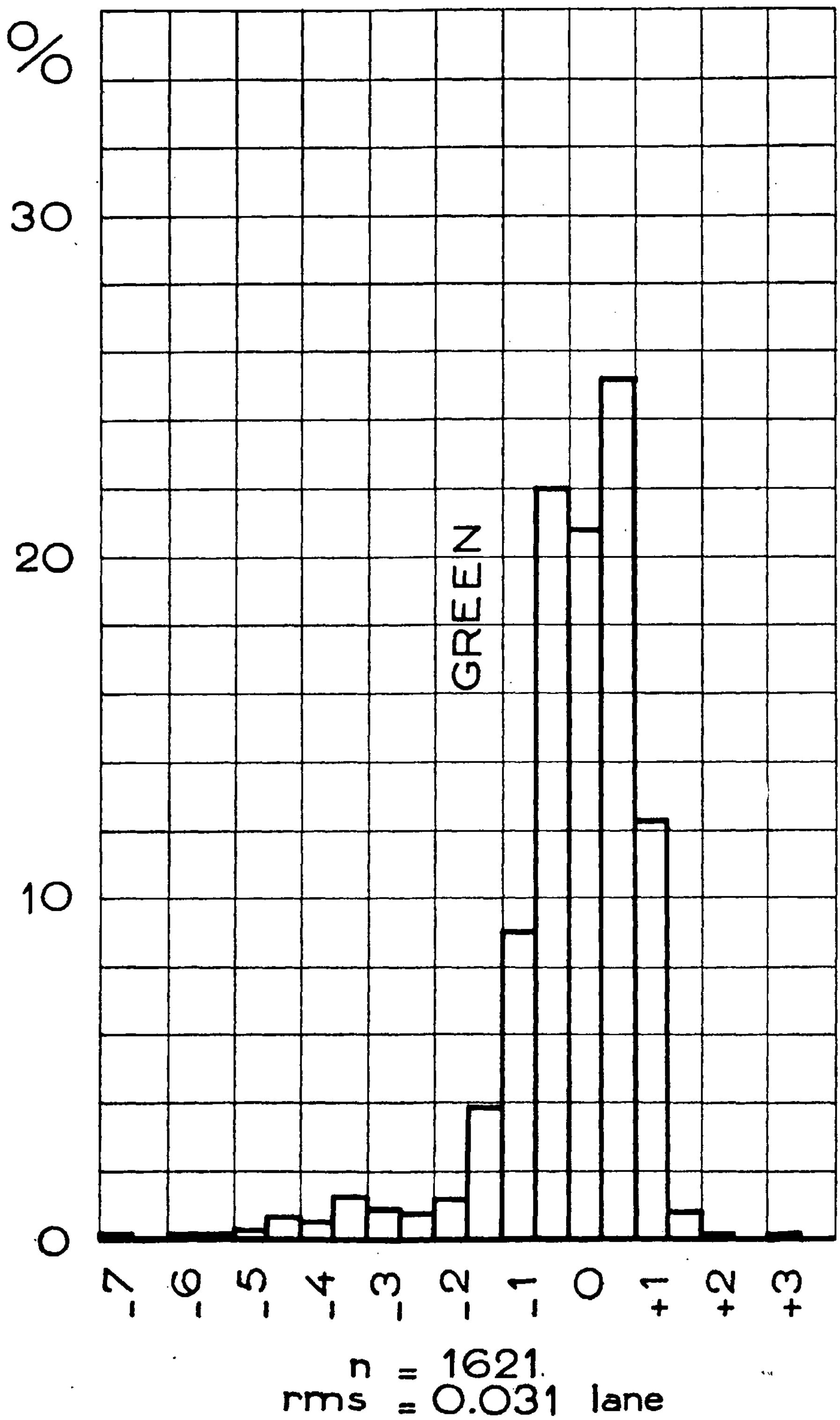

Plate No. 4B : ROTTERDAM. — Daylight.

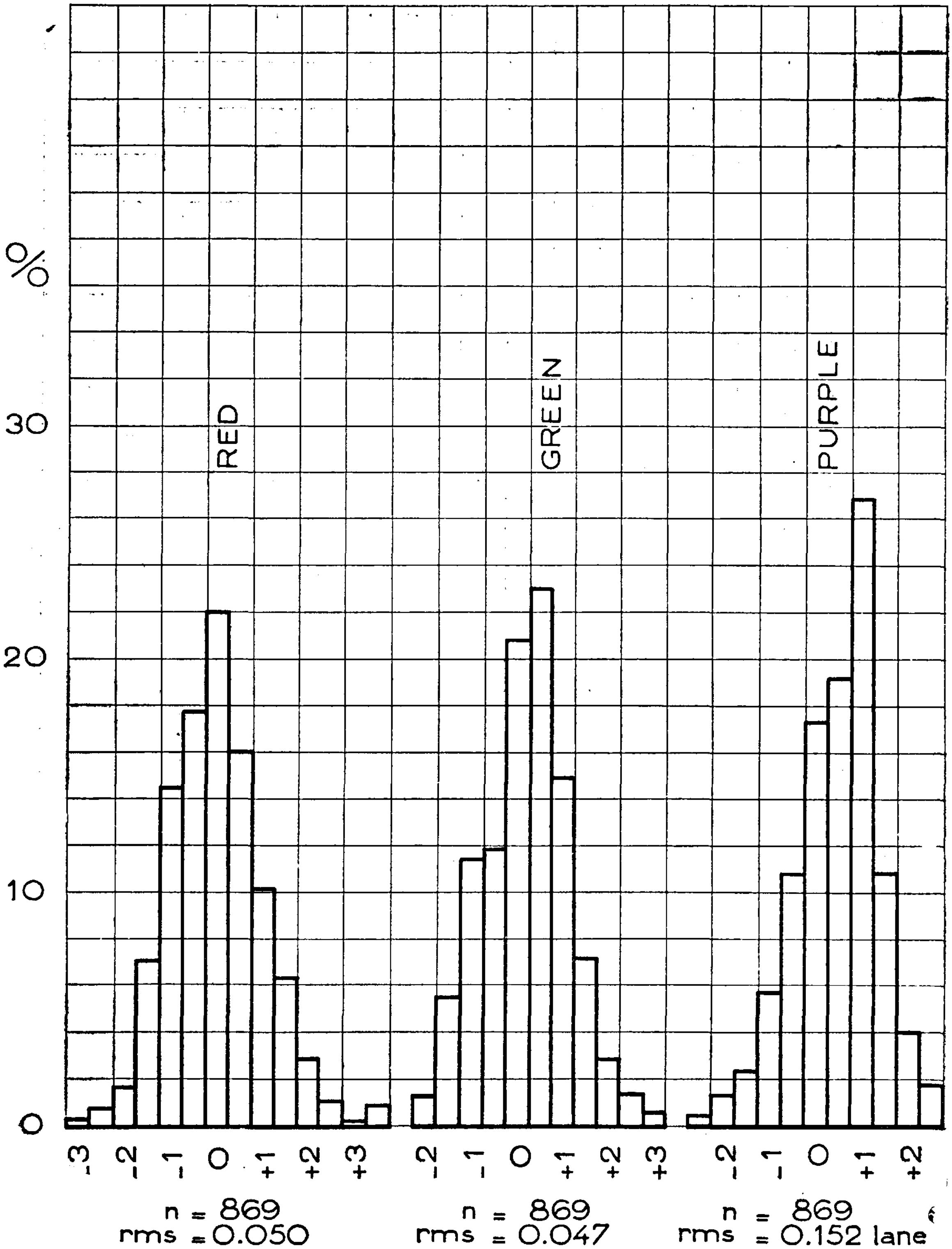

Plate No. 5 : ROTTERDAM. — Night Obs. — Differences from total daylight — mean given in half standard units (1/2 rms).

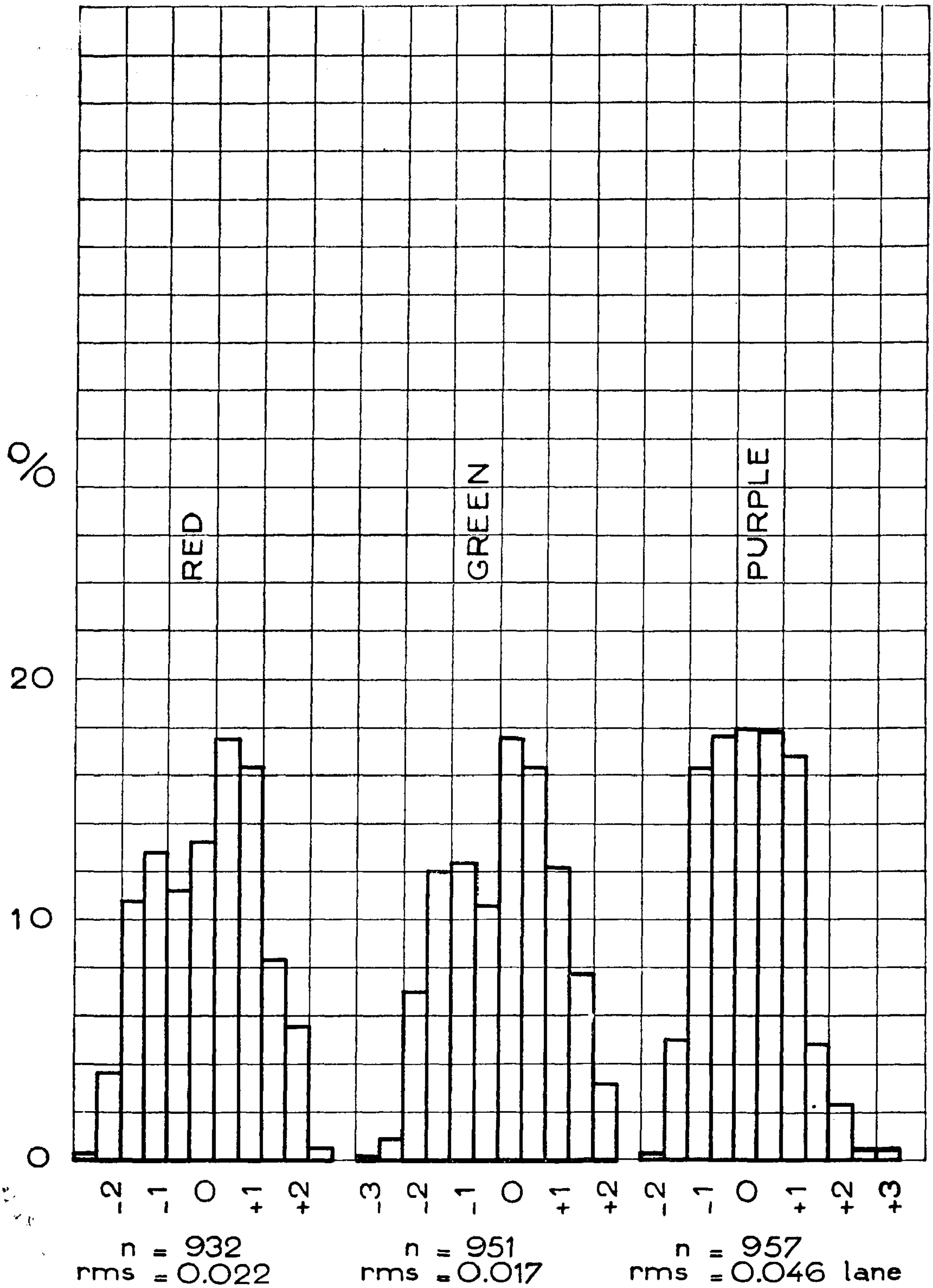

Plate No. 6 : THE HAGUE. — Monitor. — Differences from total mean expressed in half standard units (1/2 rms).

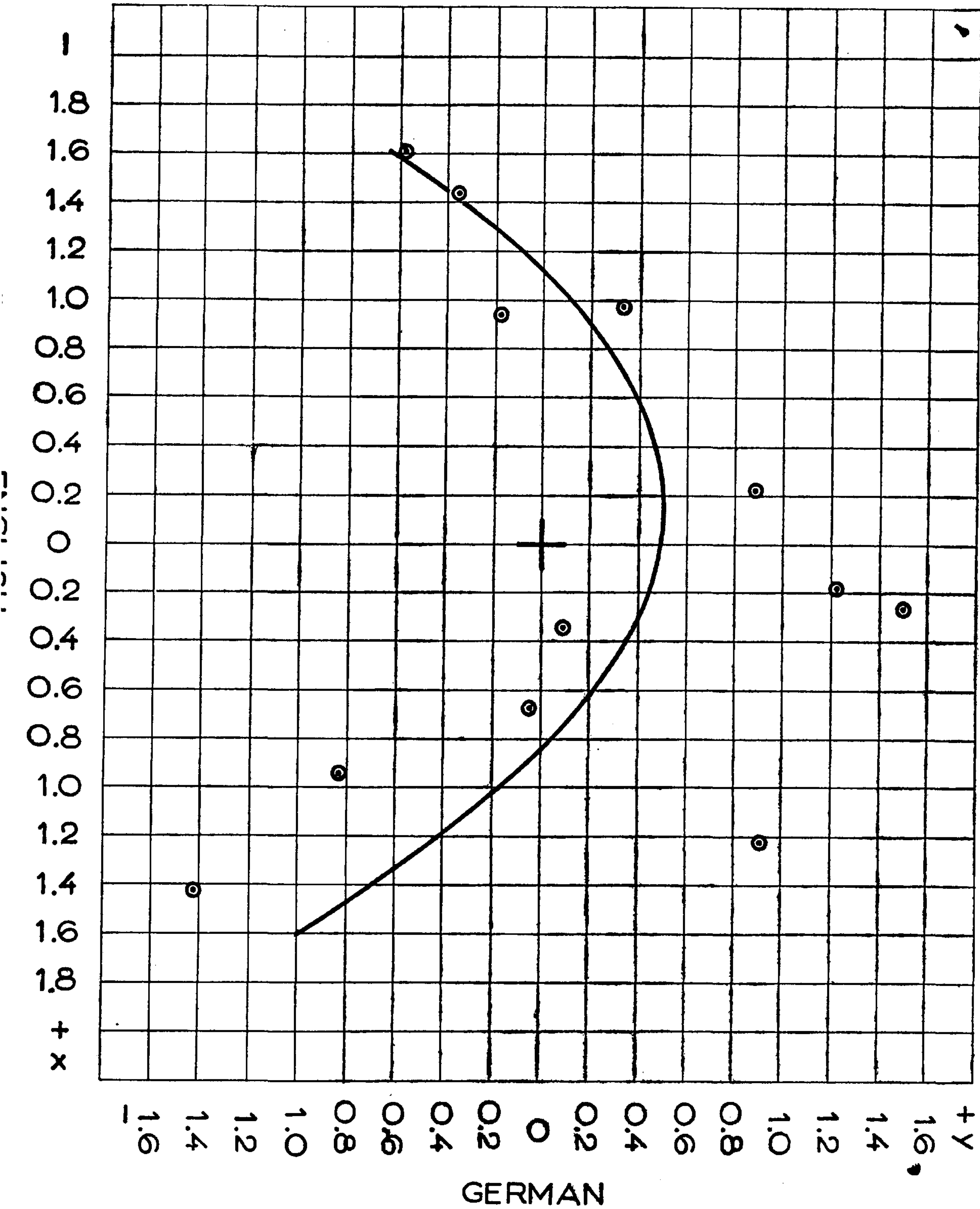

Plate No. 7 : Scatter Diagram PURPLE German-English the Hague — regression curve : $y = 0.49 - 0.12 \times - 0.51 \times^2$

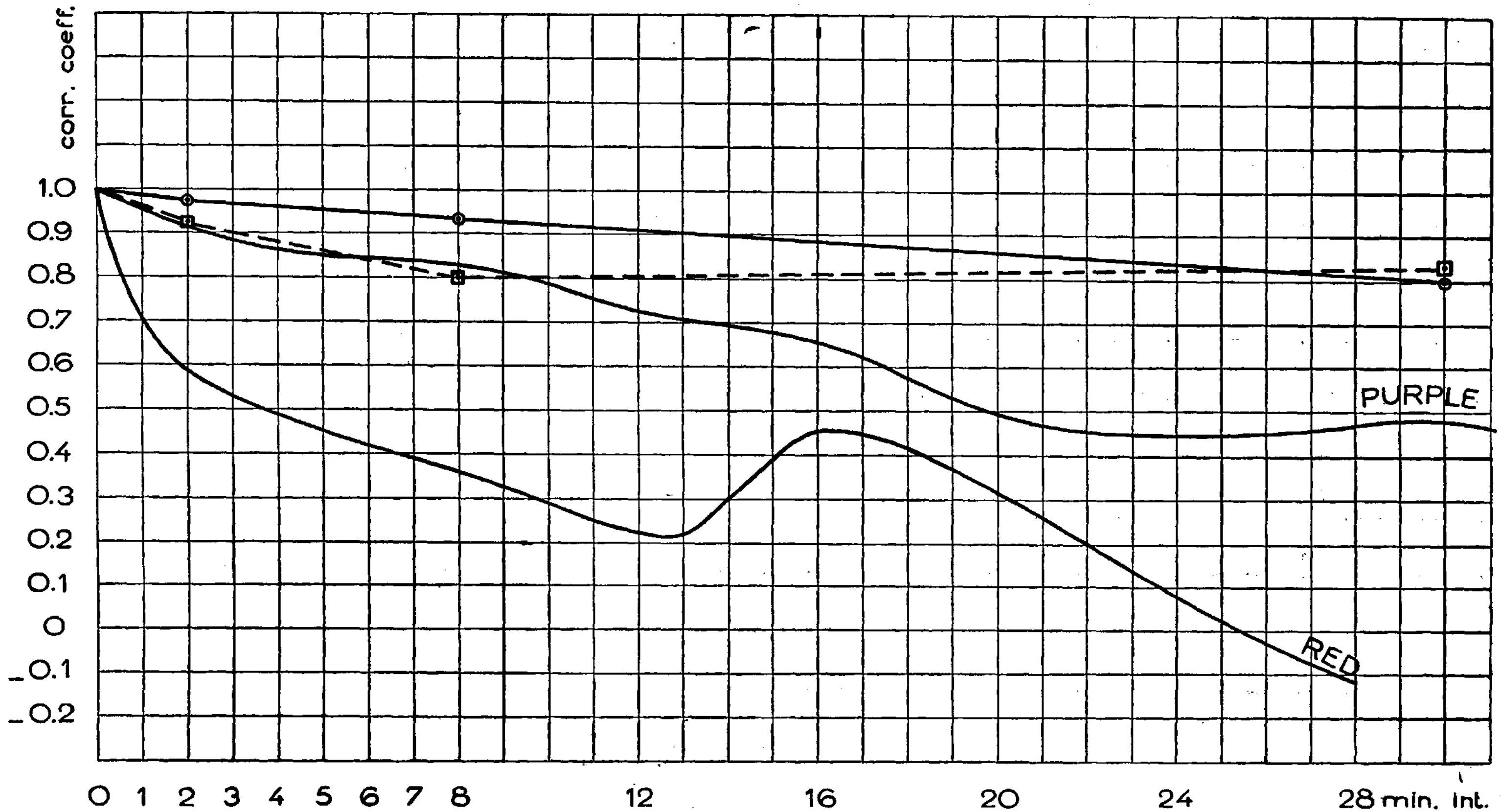

Plate No. 8 : Correlogr. German Chain. — Red May 30, 1952 (P.M.).
Purple May 30, 1952 (P.M.).
Purple May 30, — June 6.
Red May 30 — June 6.

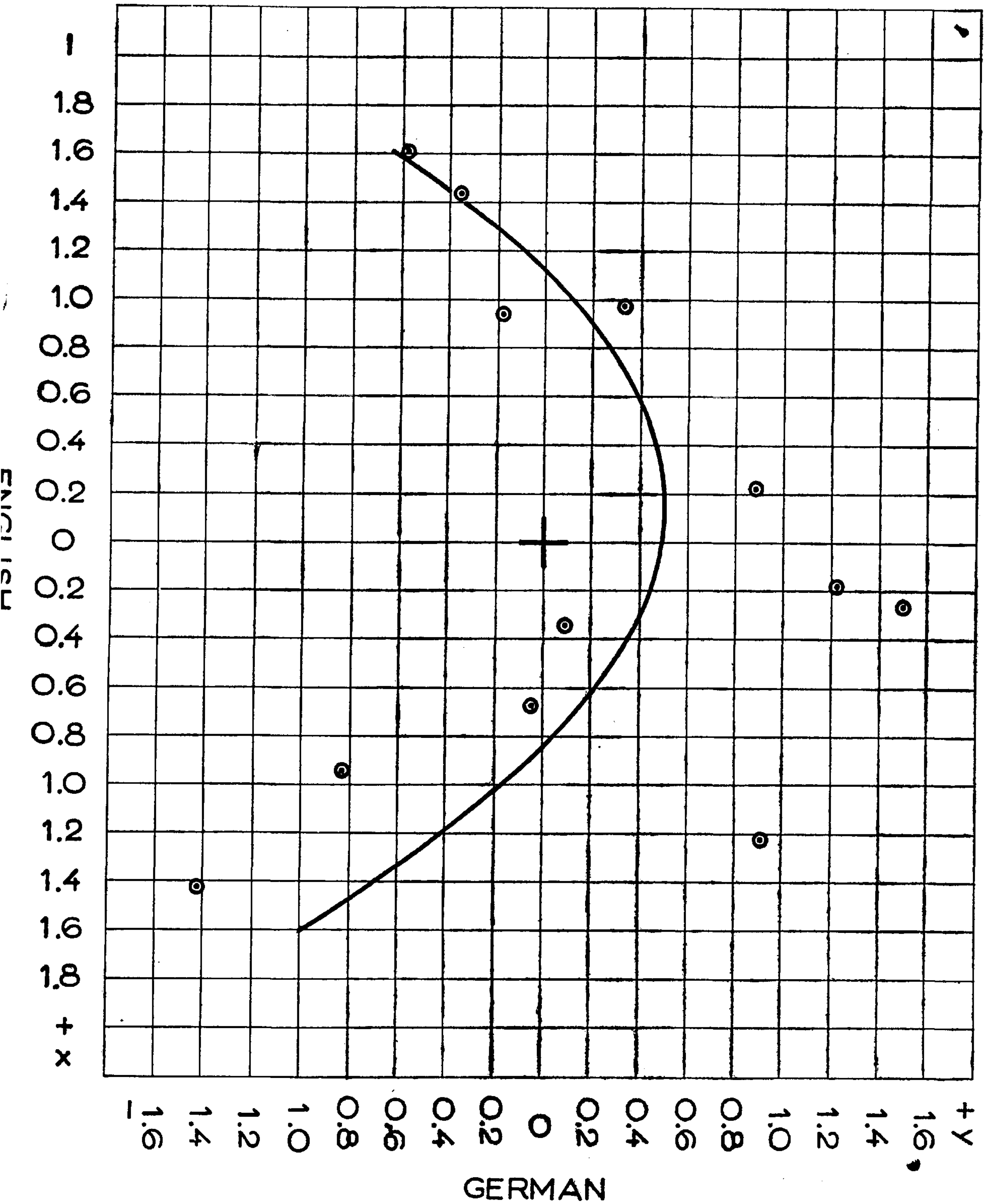

Plate No. 7 : Scatter Diagram PURPLE German-English the Hague — regression curve : $y = 0.49 - 0.12\,x - 0.51\,x^2$

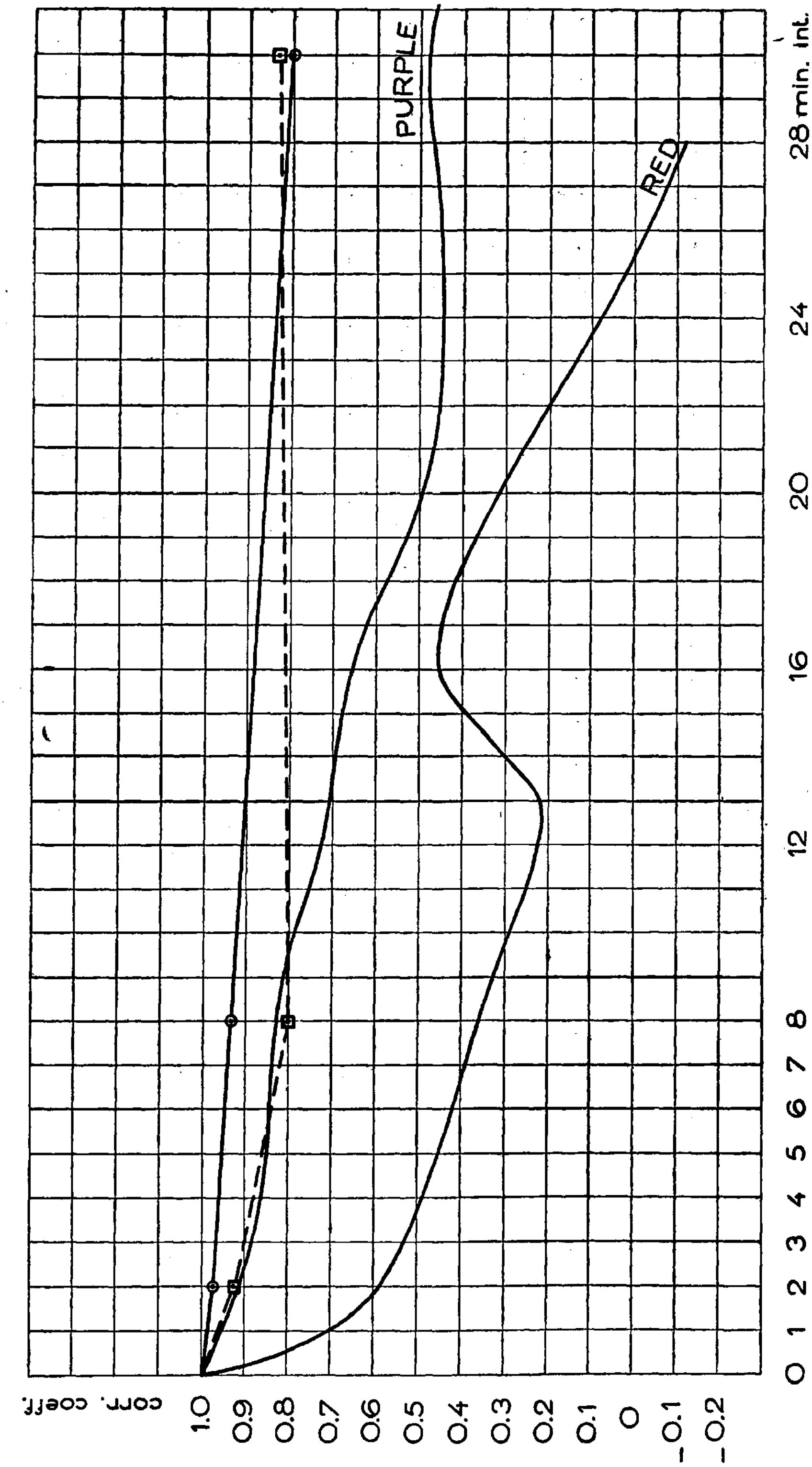

Plate No. 8 : Correlogr. German Chain. — Red May 30, 1952 (P.M.).
Purple May 30, 1952 (P.M.).
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Red May 30 — June 6.

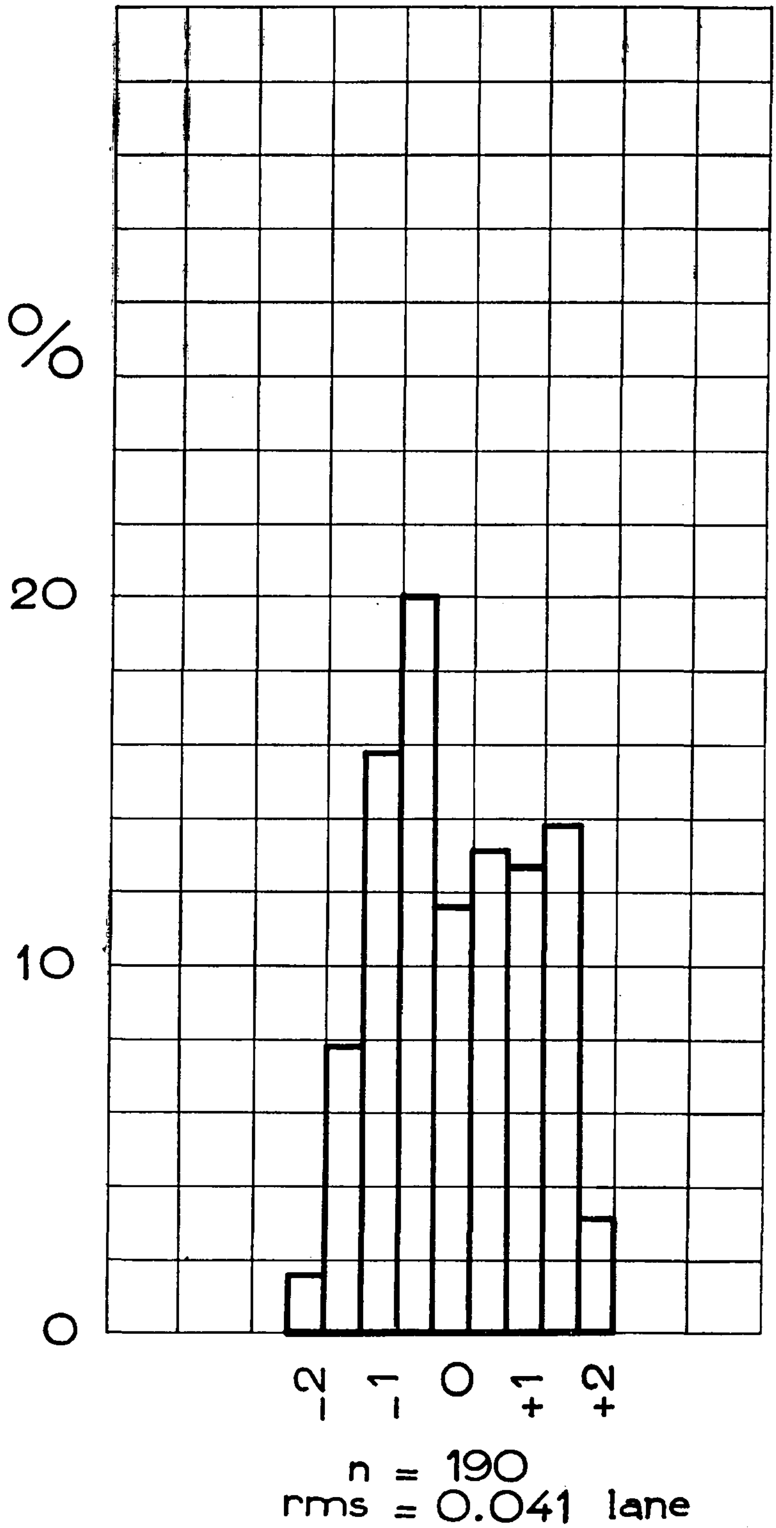

Plate No. 9 : Histogram of first 190 terms of the artificial series :

$$U_t = 0.05 \sin \left(\frac{360^\circ t}{100} + e_1\right) + 0.02 \sin \left(\frac{360^\circ t}{52} + e_2\right) + e_3$$

differences from mean expressed in half standard units (1/2 rms)