HYPERBOLIC POSITIONING SYSTEMS FOR HYDROGRAPHIC SURVEYS

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INTRODUCTION

At the conclusion of World War II it was apparent that several electronic systems developed for other purposes could also be applied to control problems in hydrographic and geodetic surveys. Shoran, a war time radar ranging system, has been employed extensively in the United States, Canada and Australia for hydrographic control and geodetic surveys. The United States Navy Hydrographic Office at that time adopted and used a standard Navy radar transponder beacon system as interim equipment on its hydrographic survey vessels such as those shown by figure I. A detailed explanation of the use of this equipment was published in Volume XXV, No. 2, of the International Hydrographic Review, in an article titled « Use of Radar Beacons in Hydrographic Surveying ». While this radar beacon equipment represented a step forward in hydrographic control, compared to non-electronic systems, it has certain serious limitations. These limitations were mainly: limited range, bulky transmitters, frequency drift, and lack of desired accuracy at all times due to varying response time of the transponders. In 1949, therefore, a study was made to determine the main characteristics of a nearly ideal positioning system for the type of surveys conducted by the Hydrographic Office. As a result of this study the following characteristics were considered the most important:

a. *Accuracy:* Accuracy when applied to an electronic positioning system is somewhat undefinable in that it varies with position in the area of coverage. Generally speaking, accuracy should be adequate for hydrographic purposes and preferably adequate for controlling air photography for mapping of land areas adjacent to the coast line.

b. *Range:* A desirable range was tentatively set at 100 nautical miles. A greater design range than this would require that the earth's curvature be considered in preparing work sheets and a lesser range would require more frequent shifting of shore installations with a consequent loss of work time.

c. *Frequency Requirements:* Due to the frequency allocation demands on the radio spectrum this a very important consideration, since permanent assignment of a radio frequency for survey purposes on an international basis is a practical impossibility.

d. *Ambiguity:* The system should be free from ambiguity in position determination. Or, if ambiguity is present, as in the case of systems having recurring zones or lanes of identical readings, these ambiguous zones should be broad enough to permit positive lane identification by normal methods of navigation.

e. *Internal Transmission Interference :* Transmission interference within the area of coverage by the operating units is very undesirable as it leads to special operating techniques such as time sharing. From this consideration, a system should be such that transmission from mobile units is not required, permitting an unlimited number of users to operate in the same area at the same time.

f. *Portability :* The equipment should be portable in the strictest sense of the word. To the Hydrographic Office survey groups this means that all units should be of such size and weight as to be transportable by a small helicopter.

g. *Field Sheet Requirements :* The preparation of field sheets where none of the transmitting stations fall within the limits of the sheet can be a problem. In the case of ranging systems the necessary curves can be plotted easily after a few position computations have been made and circular templates constructed. In the case of a hyperbolic system, computation and construction of lines of position is more complicated.

h. *Summary :* Endless debate could be entered into regarding the relative importance of the above factors and others not listed depending on such considerations as size and composition of survey party, geographical location, and prevailing operating conditions in the survey area, etc. From the standpoint of the U.S. Navy Hydrographic Office, however, the factors enumerated are pertinent.

Applying the above criteria to available electronic positioning systems, it soon became obvious that an ideal system did not exist and probably could not be designed at the present time. The radar-type ranging systems fail in the category of range and internal interference, while the radio frequency hyperbolic systems are less desirable in the categories of ambiguity and field sheet requirements. The choice, however, narrowed down to a hyperbolic system. Three such systems have been commercially advertised: Decca, manufactured in Great Britain, and Raydist and Lorac of United States manufacture. In 1950, a set was obtained from one of these manufacturers for experimental purposes and similar equipment has since been employed for hydrographic survey control in the Bahama Islands area, the Persian Gulf and the Arctic regions. The following is intended as a brief discussion on experience with this equipment as a service to the member nations of the International Hydrographic Bureau and does not imply endorsement by the United States Navy of any commercial product as compared to any other.

PRINCIPLE OF OPERATION

Figure 2 shows the cycle of operation and frequencies used. It should be noted that reference-phase stations instead of synchronized slave stations are used. A separate reference station is not required, however, since the system makes use if a switching arrangement which causes each slave station to periodically function as a reference station for the other slave station and the master station. In order to operate in this manner the master-slave functions are split between two radio frequency channels which are not critically related. Two transmitters are located at the master station, one on each of the two frequencies. One slave is operated on a frequency near that of one master transmitter and the other slave is operated on a frequency near that of the other master transmitter. At each slave station there is located a receiver which is tuned to the frequency channel of the other

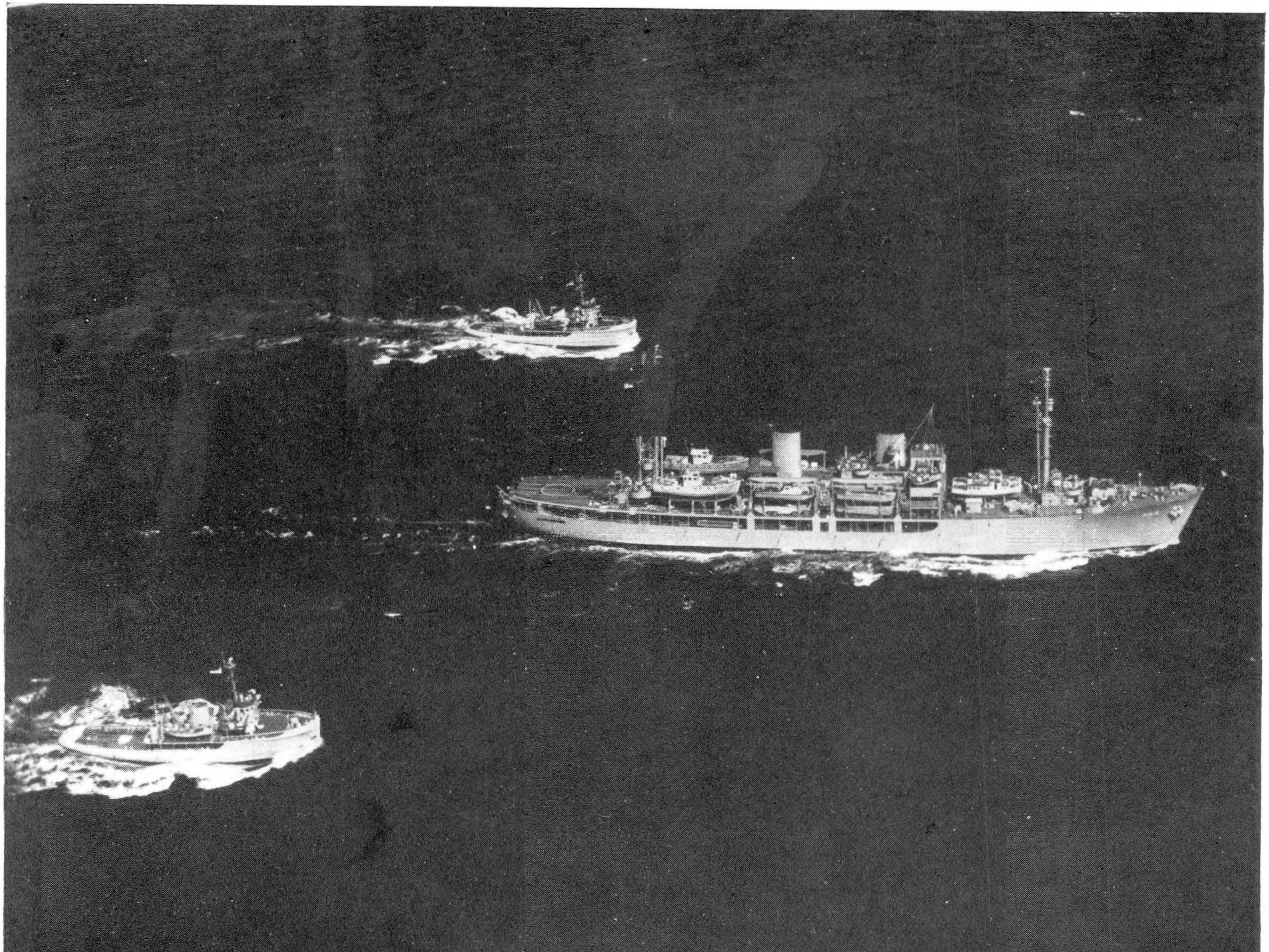

Fig. 1
Hydrographic Survey Group ONE

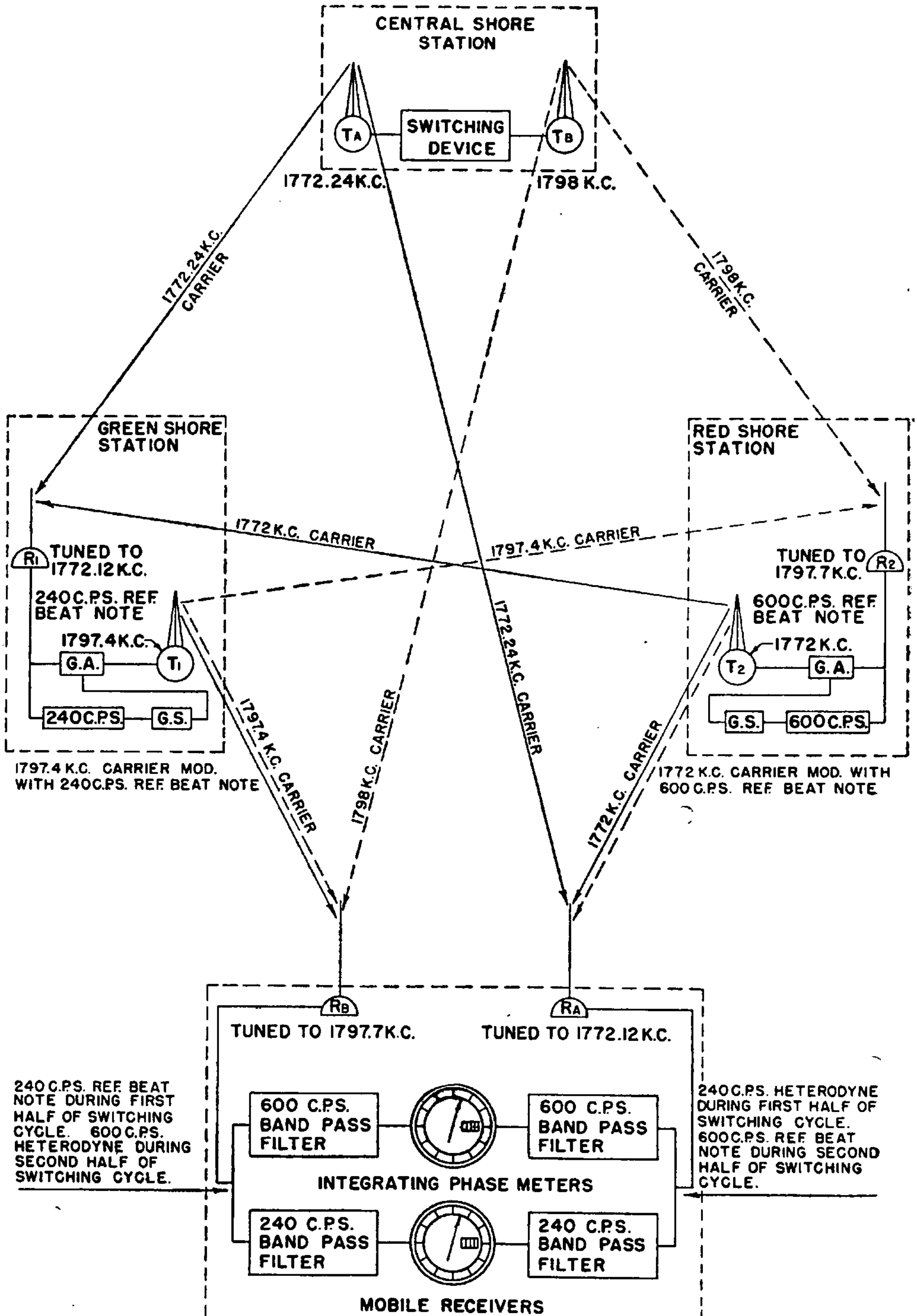

Fig. 2

Complete Cycle of Operation

master-slave station pair. The beat frequency signal output of this receiver is used to modulate the local slave transmitter.

In operation the two master transmitters are switched on and off, alternately. The two slave transmitters are on at all times. When transmitter 1 at the master station is on it produces a beat frequency signal in its channel with its slave station. This is the phase interference pattern in space from this master-slave combination. The receiver at the other slave station receives these signals, detects the beat frequency and modulates the local slave transmitter, which is on another radio frequency. This produces the constant-phase reference signal. After an appropriate relatively short interval, the transmitter cycling switch at the master station operates to turn off transmitter 1 and turn on transmitter 2, and the process is repeated in reverse order from the other master-slave combination. The system navigation receiver contains two radio frequency receivers, appropriate audio filters for separating the beat frequency signals and phase meters similar to those used in the Decca system. Although phase measurements are made alternately on the two lines-of-position, the switching rate is sufficiently high so that the data is effectively continuous. Power requirements for transmitters are 60 cycle, 115 volts A.C. Two 3kw gasoline or diesel engine driven generators are standard equipment at both master and slave stations, one generator acting as a standby to permit uninterrupted power supply. Power requirements of the portable receivers are also 60 cycle 115 volt A.C. consuming about 220 watts.

PREPARATION OF WORK SHEET

The plotting and construction of plane hyperbolic curves (lanes) to form a control lattice on field survey sheets may be accomplished either by graphic or computational methods. The graphic method becomes cumbersome and less accurate for laying out large scale field sheets where all, or part, of the shore stations plot off the sheet. The graphic method is useful, however, in quickly laying out index sheets showing the areas and limits covered by the large scale working sheets.

The U.S. Navy Hydrographic Office has developed a computational method employing punched card computing machines which will compute and list Universal Transverse Mercator (U.T.M.) grid coordinates of uniformly selected points on the hyperbolae. The listing (see figure 3) shows system identification (red or green), lane number, hyperbolic sectorial angle (B) and U.T.M. coordinates of computed points on each lane on both sides of the base line. Forty points on each of the base lines are normally computed for each lane and grid coordinates listed in numerical order of lane. Coordinates for areas in which no hydrographic work is intended are not computed.

The mathematical steps preceding the above tabulation are as follows: The U.T.M grid positions, azimuths, and distances required are first computed for the selected transmitting stations and the mid-points of the connecting base lines. The central transmitter is called the master and the outer transmitters red and green slaves. The mid-points of the red and green base-lines become local origins of their respective hyperbolic systems with semi-major axis equal + 1 at the master and —1 at the slaves, and Kr and Kg the respective scale factors of the base-lines (see figure 10). The red and green radio systems form their respective hyperbolic interference patterns with lane widths of half a wave length. The number of lanes in each base line is the length of the base divided by the lane width. Lanes

UTM GRID COORDINATES

SYSTEM	LANE NO.	β SECTORIAL ANGLE	(+b) EASTING	(+b) NORTHING	(−b) EASTING	(−b) NORTHING
RED	8	0.0	700378.5	2045465.7	700378.5	2045465.7
RED	8	0.1	700177.7	2045800.7	700662.6	2045197.7
RED	8	0.2	700058.3	2046206.2	701033.0	2044994.0
RED	8	0.3	700019.1	2046686.1	701493.3	2044852.7
RED	8	0.4	700059.7	2047245.3	702048.2	2044772.3
RED	8	0.5	700180.5	2047889.4	702703.2	2044752.0
RED	8	0.6	700382.6	2048624.7	703464.8	2044791.6
RED	8	0.7	700668.2	2049458.7	704340.7	2044891.6
RED	8	0.8	701040.1	2050399.8	705339.6	2045052.8
RED	8	0.9	701502.0	2051457.2	706471.5	2045276.9
RED	8	1.0	702058.5	2052641.7	707747.8	2045566.2
RED	8	1.1	702715.2	2053965.1	709181.3	2045923.5
RED	8	1.2	703478.7	2055440.5	710786.3	2046352.5
RED	8	1.3	704356.6	2057082.8	712578.8	2046857.4
RED	8	1.4	705357.7	2058908.5	714576.8	2047443.3
RED	8	1.5	706492.0	2060935.8	716800.2	2048116.0
RED	8	1.6	707770.9	2063184.9	719271.4	2048882.3
RED	8	1.7	709207.1	2065678.4	722015.1	2049749.9
RED	8	1.8	710815.1	2068441.2	725058.7	2050727.3
RED	8	1.9	712610.9	2071501.1	728432.7	2051824.5
RED	8	2.0	714612.6	2074888.6	732170.9	2053052.4
RED	8	2.1	716840.1	2078637.6	736310.7	2054423.2
RED	8	2.2	719315.7	2082785.6	740893.4	2055950.8
RED	8	2.3	722064.2	2087374.2	745965.0	2057650.3
RED	8	2.4	725113.2	2092449.3	751576.3	2059538.8
RED	8	2.5	728493.1	2098061.7	757783.3	2061635.3
RED	8	2.6	732237.8	2104267.5	764648.3	2063960.6
RED	8	2.7	736384.7	2111128.9	772239.8	2066538.0
RED	8	2.8	740975.3	2118714.5	780634.0	2069393.4
RED	8	2.9	746055.7	2127100.2	789914.8	2072555.4
RED	8	3.0	751676.6	2136370.0	800175.1	2076055.5
RED	8	3.1	757894.2	2146616.7	811517.5	2079928.8
RED	8	3.2	764771.0	2157942.8	824055.7	2084214.1
RED	8	3.3	772375.5	2170461.6	837915.0	2088954.2
RED	8	3.4	780784.0	2184298.5	853234.3	2094196.6
RED	8	3.5	790080.6	2199591.9	870166.7	2099993.8
RED	8	3.6	800358.4	2216494.9	888881.9	2106403.8
RED	8	3.7	811720.2	2235176.7	909567.0	2113490.7
RED	8	3.8	824279.7	2255824.2	932429.1	2121325.5
RED	8	3.9	838162.6	2278644.1	957697.1	2129986.5
RED	8	4.0	853507.9	2303864.8	985623.8	2139560.5

Fig. 3
Tabulation of hyperbolic coordinates

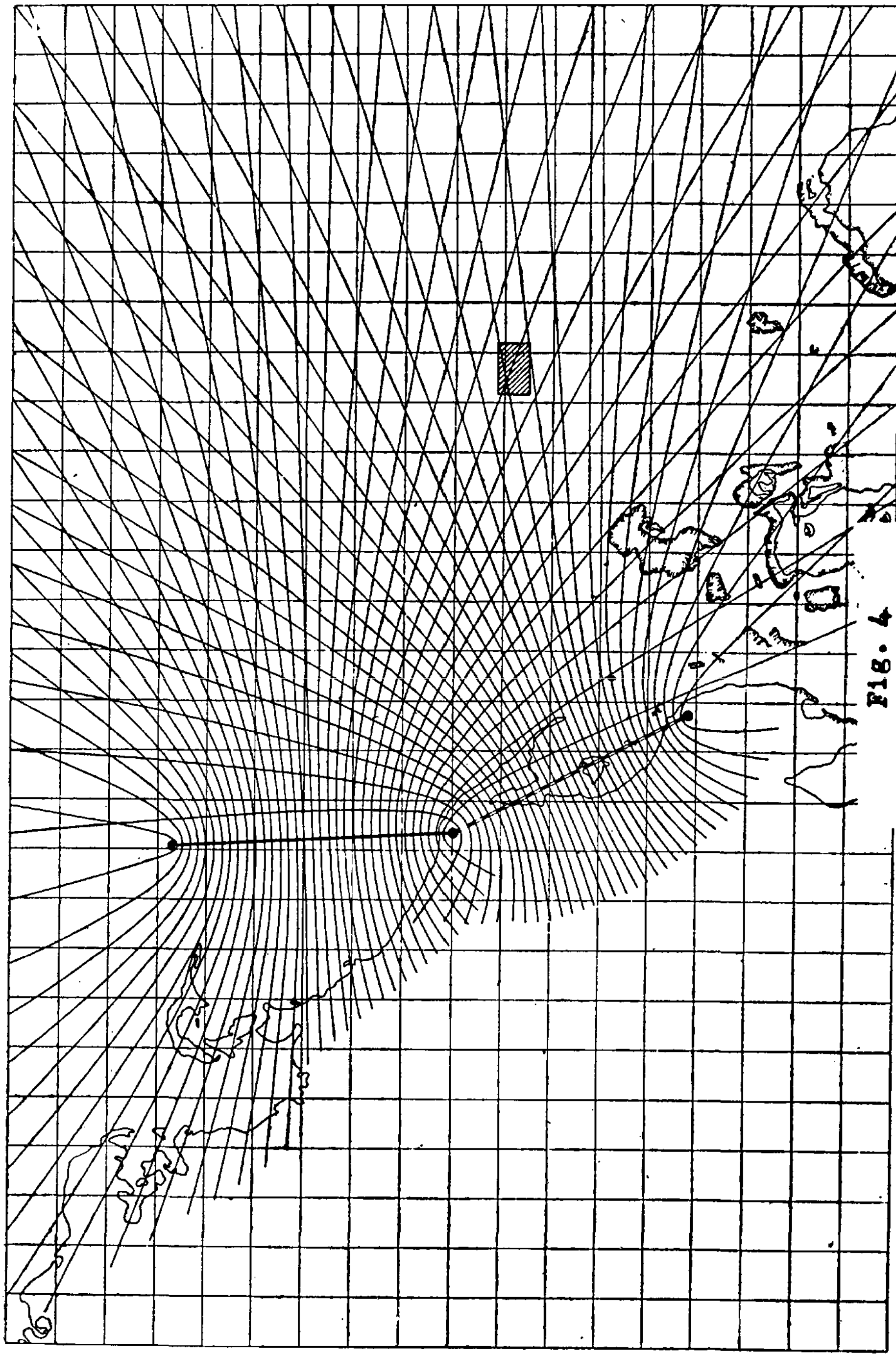

Fig. 4
Sample field sheet

are numbered zero at the master and increase toward the slaves. In the local systems the parametric equations of a particular hyperbola are given by:

$x' = (Ka)\cos h\beta$ where $a = 1 - L(\Delta a)$, $a^2 + b^2 = 1$, $-1 \leq a \leq +1$
$y' = (Kb)\sin h\beta$ L = lane number, $\beta = 0$, $1n$, where $n = 0, 1, 2 \ldots 40$

For the purpose of plotting, the local coordinates are rotated and translated into the U.T.M. coordinate system by the usual transformation equation:

$$x = +x'\cos\psi + y'\sin\psi + h$$
$$y = -x'\sin\psi + y'\cos\psi + k$$

Occasionally it is desirable to find the exact mathematical position corresponding to the red and green phase meter readings This involves the simultaneous solution of the two hyperbolic systems which is most conveniently done by polar coordinates owing to the fact that the radius vector is common to both systems and the polar angles are related in a simple manner. One way of expressing the solution is:

$$x_p = x_m + R\cos(\theta_g - \psi_g)$$
$$y_p = y_m + R\sin(\theta_g + \psi_g) \text{ where:}$$

$$\theta_g = \varphi \cos^{-1}\left[(Ka_g - a_r)\frac{\sin\varphi}{\sin\omega}\right]$$

$$a_g = 1 - L_g(\Delta a_g)_1 \qquad a_r = 1 - L_r(\Delta a_r)$$

$$K = \frac{K_r(1 - a_r)^2}{K_g(1 - a_g)^2}\left(\varphi = \tan^{-1}\frac{\sin\omega}{K - \cos\omega}\right) 0 \leq \varphi \leq (180 - \omega)$$

$$R = \frac{K(1 - a_g^2)}{a_g - \cos\theta_g}$$

* The proper quadrant of θ_g is easily determined by reference to a diagram and familiarity with the area under consideration.

Setting phase meters from the known geodetic position necessitates the converse solution of finding the lorac lane numbers when given the U.T.M. position of a station. The solution is simple as it involves only one hyperbola system at a time and is as follows:

$$L_i = \frac{2 + D_i - \sqrt{D_i^2 + 4(1 - D_i\cos\theta_i)}}{2(\Delta a_i)} \text{ where: } i = \text{« r » or « g »}$$

$$D_i = \frac{R}{K_1} \qquad R = \sqrt{(y_p - y_m)^2 + (x_p - x_m)^2}$$

$$\theta_i = \gamma_i + \lambda, \qquad \lambda = \tan^{-1}\left(\frac{y_p - y_m}{x_p - x_m}\right)$$

Quantity	Description	Dimensions (Inches)	Cubic Feet Each	Cubic Feet Total	Weight in Pounds Each	Weight in Pounds Total
2	End Station Transmitters	22×25×47	14.959	29.918	400	800
1	Center Station Transmitter......	22×25×47	14.959	14.959	340	340
2	Reference Receivers	20×18×11	2.29	4.38	70	140
3	Generators	23×29×46	17.75	53.25	285	865
3	Battery Boxes with Batteries	20×12×12	1.666	4.998	70	210
2	Loop Antennae	12×38×40	10.55	21.10	30	60
2	Loop Tripods	12×12×72	6.00	12.00	40	80
3	Transmitting Antenna Base and Tuner	26×26×18	7.04	21.12	40	120
3	Transmitting Antennae	3×3×79	.41	1.23	20	60
3	Large Spare Parts Boxes packed with parts	12×16×37	4.11	12.33	150	450
1	Mobile Receiver-Indicator	17×30×8	2.36	2.36	100	100
2	Small Spare Parts Boxes packed with parts	13×13×19	1.858	3.716	30	60
1	Mobile Receiving Antenna Base and Matching Section	6×6×36	.750	.750	30	30

182.111 cubic feet 3315 pounds

Fig. 5. — Table of weights and dimensions.

After the computation and tabulation are completed, points on the hyperbolae are plotted on standard plastic gridded sheets to the desired scale (see figure 4). The curves are drawn by using splines and weights. When time permits the two sets of hyperbolae are printed in red and green respectively with the UTM and geographic coordinate values indicated by tick marks.

FIELD PERFORMANCE

Establishing a Shore Station: No particular difficulty has been encountered in setting up a shore station. From figure 5 the maximum weight of any single item is 400 pounds. To this list, of course, must be added generators, radio equipment, personnel, food and camp gear. Where the country permits truck transport a single truck load will suffice for each station and in some areas landings from seaward can be made by amphibious vehicle. In several cases stations were established by helicopter air lift in less than one day's operation. One great advantage of a medium radio frequency system is that there is no requirement for elevation of station and the geographical site is normally flexible within 5-10 miles, provided the exact geographic position can be determined from nearby triangulation stations. The ideal site would be nearby a coastal village where landings would be facilitated and where electric power could be obtained and thus eliminate one of the major problems in station operation. The geometric consideration will generally prevail, however, in the selection of station sites. Where the coast to be surveyed lies in a straight line, the master transmitter should be positioned a distance inland, preferably on a river bank. Whenever practicable the transmission path from slave to slave should be over water. Figure 7 shows a typical station installation in the Persian Gulf area.

Setting the Phase Meters: When all the transmiters are operating, the first step is the correct setting of the lane counters and phase meters. This operation consists of two steps; the manual setting of the pointers of the phase meter and the setting of the correct lane numbers on the counters. This operation requires an exact position for the receiver antenna relative to the transmitters. This requirement has not been any great problem so far, since each survey area has contained at least one pier or dock where the ship's antenna could be positioned accurately by triangulation. Several other methods, however, have been used to solve the problem. One receiver has been transported by helicopter, amphibious vehicle or truck to an exact geographic position for proper setting. Once the phase meter has been accurately set no further check is normally necessary till the stations are moved. The lane counters, however, require constant checking in case of interruption of transmission or after being disturbed by excessive night effect.

Lane Recovery: One of the war-time comments regarding the lane determination devices was that « you knew exactly what Pew you were in but never in what Church ». While this situation has been corrected in such systems, it is true with this equipement in the sense that position within the lane is known after interrupted service, but that one or more lanes may have been lost due to this same cause. In the case of the survey launches operating close to shore, a series of three-point sextant locations will permit lane recovery. In the case of the survey ship operating far from land the most practical solution has been by means of anchored buoys equipped with radar reflectors. (The type of buoy used for this purpose is shown in figure 6). In survey areas where the depths of water make anchored buoys impractical, lane checks have been accomplished readily

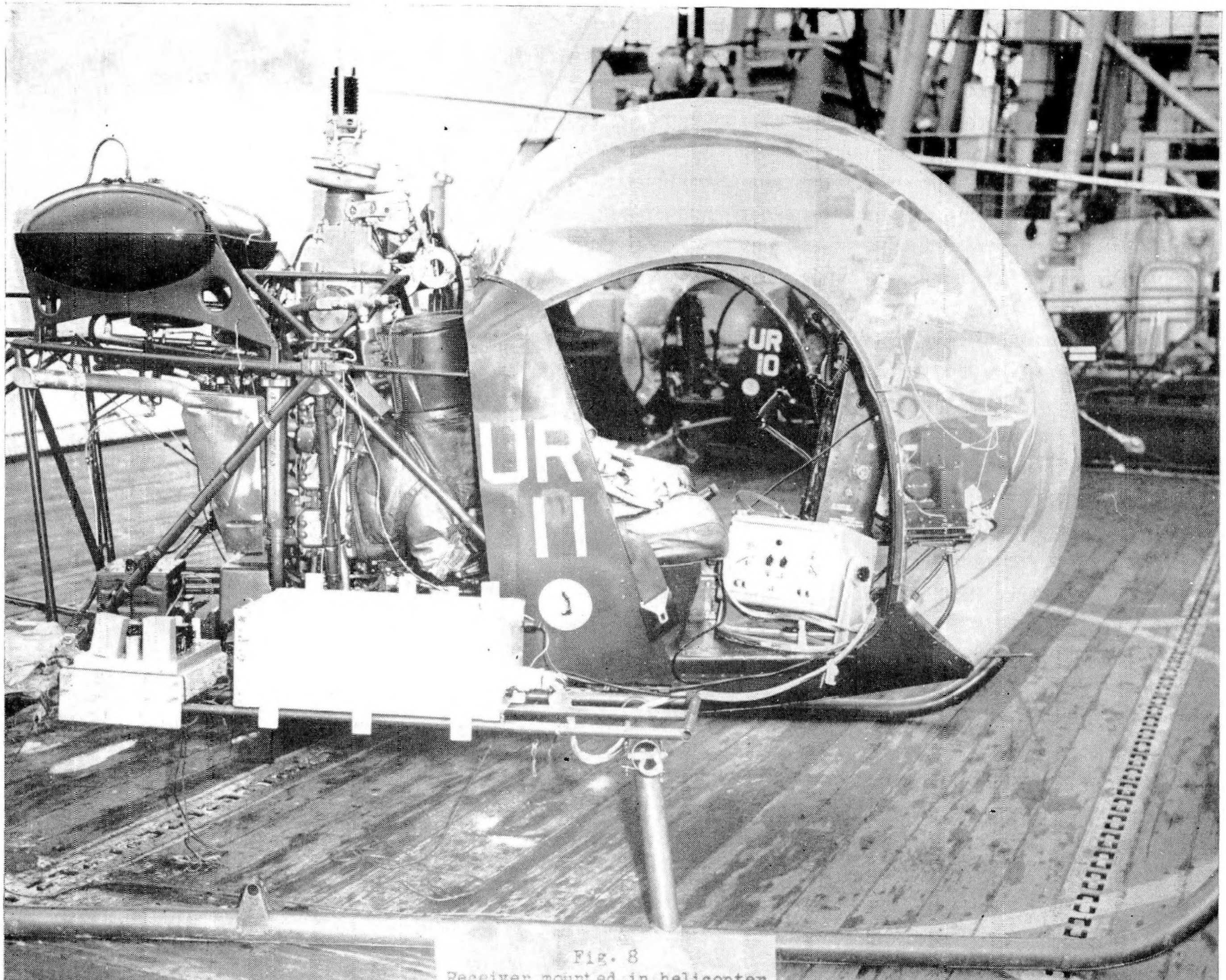

Fig. 8
Receiver mounted in helicopter

Fig. 7
Typical shore station

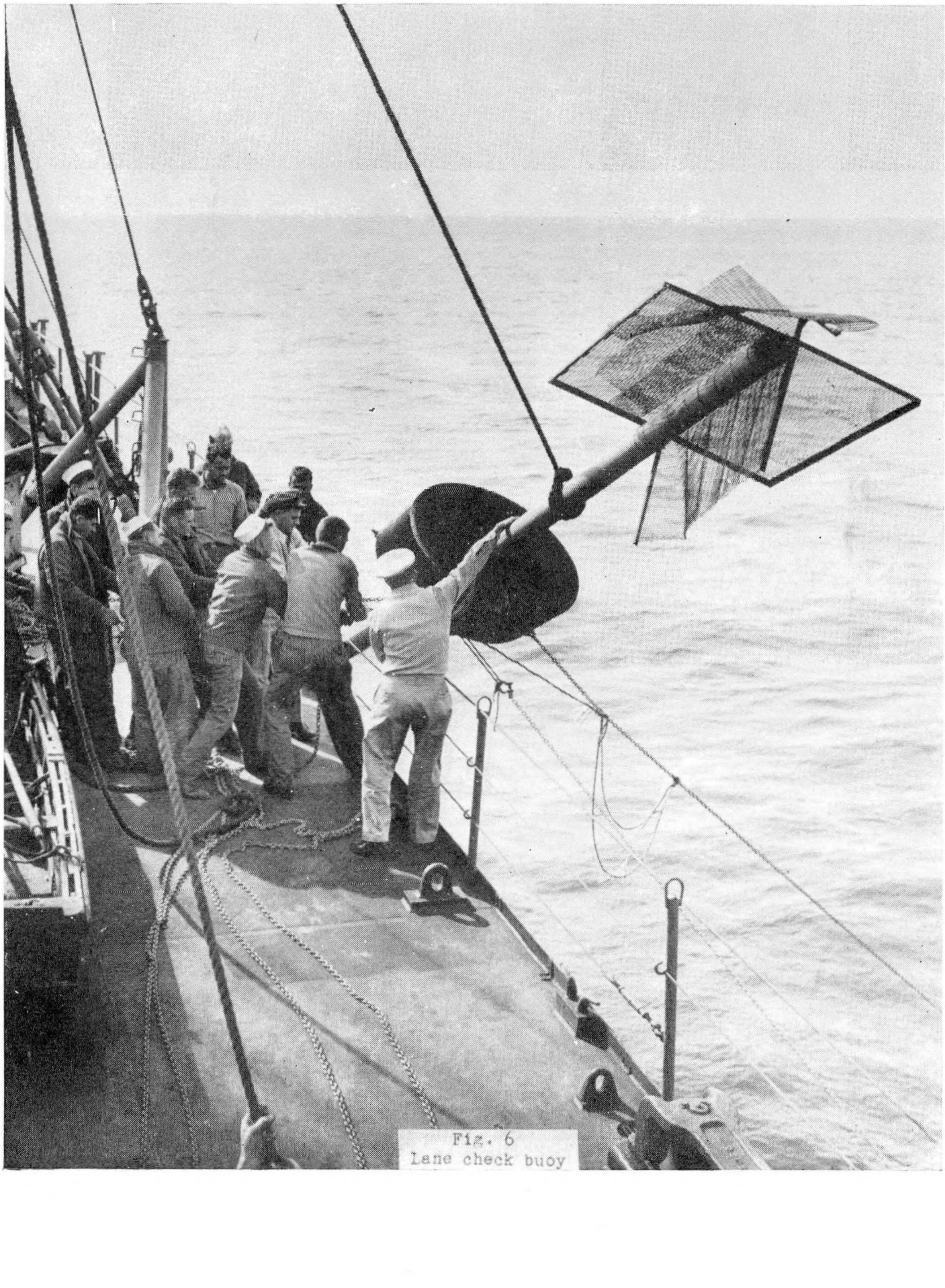

Fig. 6
Lane check buoy

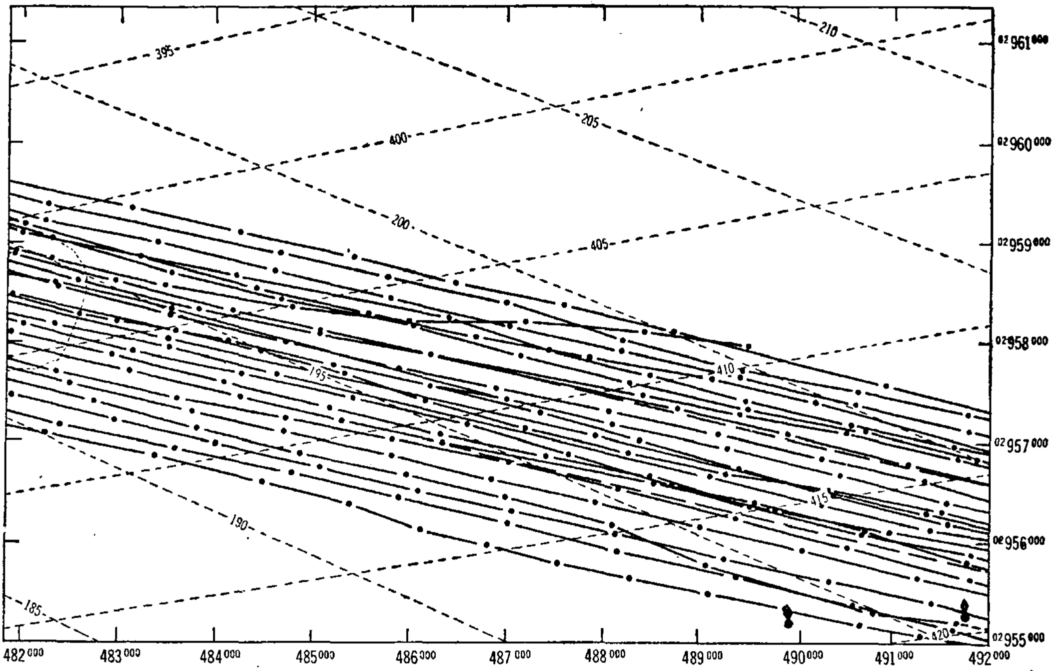

Fig. 9. — Section of work sheet showing ship positions.

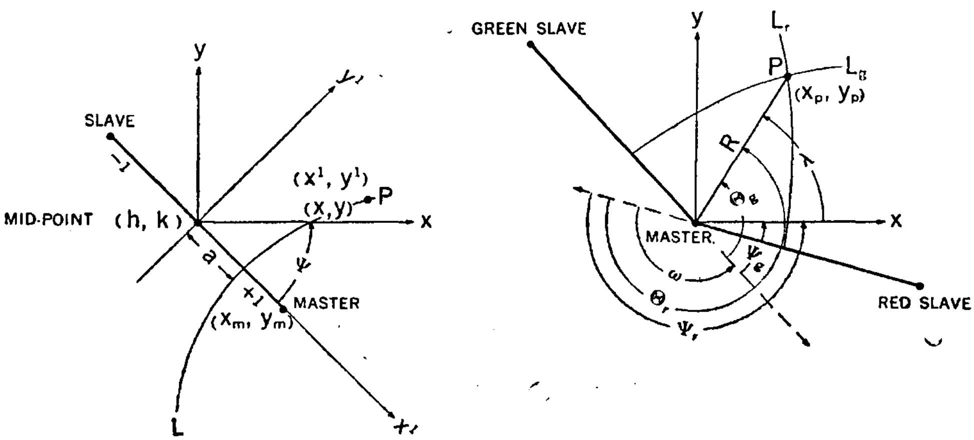

Fig. 10.

DEFINITIONS

x_p y_p, x_m, y_m, h, k	= U.T.M. coordinates of point P, master station, and mid-point of base line respectively.
x^1, y^1	= Local coordinates of point P
K_r K_g	= 1/2 of red and green base lines respectively
Δa_r Δa_g	= 1/2 number of lanes in red and green base lines respectively
L_r L_g	= Red and green lorac Lane numbers of a hyperbola respectively
Ψ_r Ψ_g	= CCW angle between positive red or green axis and the positive x axis respectively
R	= Radius vector from master to point P
θ_r θ_g	= CCW angle between positive red or green axis and the radius vector R
ω	= CCW angle between red and green base lines
λ	= CCW angle between positive x axis and the radius vector R

by means of a receiver in a helicopter flown to a check point on shore. Figure 8 shows receiver and phase meter installation in a helicopter.

Effective Range and Interference Problems: Experience up to the present time indicates remarklably little interference with or by other radio installations, and no survey time has been lost to this cause. The range of the equipment has also proven to be greater than the specified 100 nautical miles. Both in the Bahama Islands and Persian Gulf survey areas sounding lines have been positioned up to distances of 135 nautical miles from the most distant transmitter without effective loss of signal strength. In the Arctic areas north of the polar circle present indications are that range is lessened appreciably by atmospherics or static.

Night Effect: Sky wave interference effectively limiting the survey to daylight operation have been encountered on two occasions. One in the Bahamas area where the distance between end stations was 115 nautical miles and one in the Persian Gulf area of 78 nautical miles. In the Persian Gulf and other middle latitude areas, experience has indicated 55 miles as a maximum separation of end stations when it is desired to operate both day and night.

Accuracy: Limited experience has indicated that the relative accuracy of ship fixation is greater than the possible accuracy of graphical plotting on standard survey scales of 1 :25 000 to 1 :100 000. Figure 9 shows a ship channel survey about 50 miles from control stations where the survey ship was able to run sounding lines 50 yards apart on a scale of 1 :50 000 without difficulty. In this case even slight changes in ship's speed and course were immediately reflected by the position fix. Data on absolute accuracy of fixation over water and over land as compared with triangulation are still lacking. It is expected, however, to obtain more factual data in the near future.

SUMMARY AND CONCLUSION

It should be realized that experience with this equipment for positioning hydrographic survey vessels is still very incomplete and that the inherent possibilities have not yet been fully explored. Present conclusions, however, are that the equipment is « of great value to the hydrographic survey parties, » to quote from a report by the Commanding Officer of one of the Hydrographic Survey Groups. This comment pertains to surveys in the Persian Gulf area where radio reception conditions are considered fair. In the far north where radio static is high, the value to the survey parties would be reduced accordingly. Present disadvantages of the hyperbolic systems, which are the advance preparation of work sheets and possibility of loss of lane count, can probably be overcome in the future by changes in design and the development of an automatic hyperbolic plotting device.