THE ANALYSIS OF TIDAL OBSERVATIONS FOR 29 DAYS

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It is generally recognised that the harmonic analysis of tidal observations is best effected from a set of hourly heights of tide for a span of time nearly equal to a solar year. The reason for this is that certain groups of constituents are such that the component constituents may require a span of a whole year for their phases to become separated by 360°. Unless such a phase separation can be obtained it is not easy to analyse the observations so as to obtain the harmonic constants for all the constituents. If shorter intervals of time are taken for the observations special assumptions have to be made so as to obtain usable results. Under certain circumstances it is necessary to use only a month's observations, as would be the case when a survey is being made of a coastal region remote from a standard port, or which might be obtained by an expedition, say, into Arctic waters, or, again, which might be made during the summer months in harbours which are ice-bound in winter. Other reasons for monthly analyses may occur to the reader, but one of special importance arises where observations, even in a harbour, are intermittent for one reason or another, so that while sundry sets of monthly observations may be obtainable from the records there are no continuous records which can be submitted to the standard method of analysis. In this latter case it is sometimes very important to obtain the maximum of accuracy in the monthly analyses so as to facilitate the combination of the results for one month with the results for other months.

The word « accuracy » may be used in two senses, and is it desirable to examine both senses. It is obvious that the fewer the number of observations the greater will be the error due to casual, or non-tidal, causes. The most elaborate method which may be utilised will not in any way reduce this casual error beyond a certain point, which depends upon the degree of error in the individual observations and upon the number of the observations. There are well-established rules for estimating the « standard error » in the results of analysis. It might be argued that in such circumstances there is no valid argument for using an elaborate method which will nominally give results far more accurately than is warranted by the known or estimated errors. That may be so for analytical results which are not likely to be combined with other results, but when a number of months of analysis have been done, then the question arises as to whether the combination will free the results from systematic error. Thus in a monthly analysis a constituent nominally obtained may be perturbed by another constituent. When results are combined for the purpose of increasing the real accuracy, the casual errors are diminished, but the systematic ones may remain with but little diminution. It is therefore regarded as a principle that all systematic perturbations of analytical results should be removed as far as is possible in each analysis.

There are in existence many methods of analysis for a month's observations of tides, and the present author has in his time devised many methods, but he has found that the simpler methods which were considered to be sufficient at one time have had to be revised repeatedly to include more constituents or to achieve more nominal accuracy.

There comes a time when the analyst must decide whether the extra work which may be involved in extending his processes is likely to yield results at all commensurate with the work done. An interesting example of this is found in the Admiralty Method of Analysis of Tides, which was devised by the present writer. As this was intended for use at sea computation had to be reduced to a minimum, and the method has found favour throughout the world, not only in connexion with ordinary tidal observations, but also for the purpose of reducing oceanographical observations and observations of ground tilt under the influence of tidal loads and forces. The results are well balanced in real accuracy with the span of observations available. But if tidal predictions are required it is often found necessary to extend the analyses when they come into the hands of the predicting authorities. Part of this paper will be devoted to this problem.

2. *Fundamental principles and formulae.*

The method of analysis here described is a revised version of one that has been extensively used at the Tidal Institute for over 25 years. The general principles of this and all other analyses devised by the author are to be found in his large memoir, « The Analysis of Tidal Observations », Phil. Trans. Roy. Soc., Vol. 127, pp. 223-279, 1928. The most fundamental principle is to combine the heights of tide daily in order to isolate as far as possible all the species of tides, whether diurnal, semi-diurnal, or higher species. The species of tide will be denoted by the species-number p, which gives the number of periods per day, and the combinations of the tides according to certain rules are denoted by X and Y, with appropriate suffixes.

The daily values of X and Y are obtained by multiplying the hourly heights by the small multipliers given in table I and summing the products, which is easily effected upon a simple adding machine. The multipliers are small integers with positive and negative signs and the combinations have been chosen so as to reduce to insignificance any contributions from constituents other than those of the one species indicated by the suffix. Thus, from X_1 and Y_1 we can readily obtain the diurnal tide for the successive days.

The daily values of any function, as may be seen from the example, table 3, are subject to constant terms, terms with monthly variations, terms with semi-monthly variations, and so on. For example, the function X_2 will have constant terms, which are due to S_2 and to which also K_2 and other constituents contribute largely. The monthly oscillations are due to L_2 and associated constituents, the semi-monthly oscillations are due to M_2 and $2SM_2$, and the third-monthly oscillations are due to N_2 and associated constituents, and so on.

The next stage of analysis is therefore to combine the daily values of each function by applying the multipliers given in table II and summing the 29 products for the month. We thus obtain functions such as X_{22} and X_{2b} where the first suffix indicates that the operations have been performed upon X_2 and the second suffix indicates that the combinations denoted by D_2 or D_b have been used.

The next important principle is that for each species the constituents and the corresponding functions should be reduced to a common and central time-origin. For all the theory and details of this operation recourse should be made to the memoir already mentioned. In effect, we deal with functions given by

$$A = X + Y \qquad B = X - Y$$

each being taken with the same suffixes. The memoir gives divisors which can be applied to the functions A and B to give, nominally, R cos r and R sin r for the constituents, where R is the amplitude and r is the phase lag appropriate to the time origin.

It will be noted that we have given emphasis to the word « nominally », and the reason for this is that the functions A and B are not entirely freed from other groups of constituents of the same species. The memoir gives all the data whereby the effects of all these processes can be computed once for all time, and the advantage of the central time origin, indeed a very great advantage over other methods, is that all the functions can be readily corrected by combination of themselves, without having to use tedious methods of computing angles and amplitudes and then combining results by vectorial methods. The functions already obtained are in two groups, one of which involves only cosines and the other involves only sines. One group has no effect upon the other.

The same substantial advantage applies also to the Admiralty Method of Analysis, but that method saves labour by using simpler multipliers which, however, are not so efficient as the multipliers used in the method here described, for they do not so efficiently separate the species and groups of constituents. The methods will give the same results but the system of end corrections for the Admiralty Method is more complex than the system for the Tidal Institute Method.

Labour is saved by a further combination of functions according to the typical formulae

$$C_{12} = A_{12} + B_{1b} \qquad D_{12} = -B_{12} + A_{1b}$$
$$D_{1b} = -B_{1b} + A_{12} \qquad C_{1b} = A_{1b} + B_{12}$$

but in actual practice it is found convenient to express the functions C and D direct in terms of the functions X and Y, and the combinations are indicated in table III.

Finally the functions C and D are combined so that each combination gives a result depending only on one major constituent, and the combinations for this are in table IV, yielding values of R cos r and R sin r for the constituents named, subject, however, to other corrections. These will be dealt with firstly in the manner which has been practised hitherto and later on other corrections will be described, which are new to ordinary analysis. The new corrections can thus be made to any existing analysis if desired.

Note: The tables of multipliers for X, Y, and the combinations are written out by the computer to suit his computation forms, so that the multiplier sheets may be folded to permit the column of multipliers to be placed alongside the column of figures with which it is to be used.

It was indicated in the introduction that certain constituents are so nearly equal in speed that they cannot be separated from observations covering only one month. For example, the diurnal constituents K_1 and P_1 are inseparable by direct analysis of 29 days' observations, but fortunately tidal theory permits the use of the relations indicated by the forces, and it may be assumed that both constituents have the same phase-lag g and that their amplitudes H have a constant relationship. On this basis we are able to give in table V values of factors and angles, denoted respectively by (1 + W) and w. These can be applied to the values of R and r which may have been obtained nominally for K_1. Tables of this kind are used in all methods and full instructions for their use are found for the Admiralty

Method. Table V gives three sections for correcting apparent results for K_1, S_2, and N_2, and the same tables can be applied to the shallow water tides.

The harmonic constants which are the end-results of the analysis are called H and g, and the formulae for calculating them are

$$H = R \text{ divided by } f\ (1 + W)$$

$$g = r + V + u + \Delta + w$$

where V is the astronomical argument at zero hour of the central day, Δ is a correction according to the time-origin of the formulae used for X and Y for the different species, w and $(1 + W)$ have just been explained, and f and u are factors and angles which change with the longitude of the moon's node, in a period of about 19 years.

As a general rule computers will use standard tables for determining V, u, and f, but formulae for these quantities are given in tables VI and VII so that this paper may be completely furnished with all necessary data. Values of Δ come from the memoir, and are given in table VII.

When all these operations have been completed the values of H and g for the principal constituents will have been obtained, according to the methods which have hitherto been deemed sufficient, and which are sufficient for all practical purposes unless it is desirable to obtain maximum accuracy for purposes mentioned in the introduction.

3. *Additional refinements.*

It has been supposed that for practical purposes the constituent P_1 may be considered in the analytical processes in exactly the same way as the constituent K_1 so that correction multipliers designed for the elimination of K_1 may be supposed to be correct also for P_1, but this is not strictly true. The functions X_1 and Y_1 have not quite the same multiples of contributions from P_1 and K_1 and the same may be said of functions such as X_{11} and Y_{11} where the second suffix is an integer, but for functions such as X_{1a} and Y_{1a}, where the second suffix is literal, the contributions of P_1 and K_1, are opposite in sign. It follows that the combinations of functions C and D which serve to eliminate K_1 will not wholly eliminate P_1.

Similarly, the combinations which eliminate S_2 will not wholly eliminate K_2 and T_2, those which eliminate N_2 will not wholly eliminate ν_2, and those which eliminate μ_2 will not wholly eliminate $2N_2$.

The constituents μ_2 and $2N_2$ require special consideration because they are not related to one another according to the theoretical indications of the tide-generating forces, except in very deep water. If $2SM_2$ has an appreciable amplitude there will be a term $2MS_2$ which has an appreciable amplitude, and this term has a speed equal to that of μ_2. The complex terms just mentioned arise from the effects of shallow water conditions and they are associated in their generation with the sixth-diurnal constituents. Thus to the sixth-diurnal constituents whose speeds may be denoted by

$$3M_2, \quad 2M_2 + S_2, \quad 2S_2 + M_2, \quad 3S_2$$

we have the corresponding semi-diurnal constituents whose speeds are denoted by

$$2M_2 - M_2, \quad 2M_2 - S_2, \quad 2S_2 - M_2, \quad 2S_2 - S_2$$

which are shallow-water contributions to

$$M_2, \quad \mu_2, \quad 2SM_2, \quad S_2.$$

It is possible, however, to infer P_1, K_2, T_2, ν_2, and $2N_2$ from theoretical relationships, so that their effects may be fully allowed for in the analysis. The method of inference from the analytical results will now be explained on the assumption that for two constituents whose speeds are very nearly equal they will take the same values of Δ and also for g. For the principal constituent we have

$$r + V + u + \Delta + w = g$$

and for the subsidiary constituent we have

$$r' + V' + u' + \Delta = g$$

whence

$$r' = r + w + (u - u') + (V - V')$$
$$= \text{value of } r \text{ for the subsidiary constituent.}$$

From this formula are obtained the results given in table VIII and from the resulting values of R cos r and R sin r the combinations in table IX will give the corrections to the values of H and g for the principal constituents.

After all corrections have been made then the values of H for the subsidiary constituents may be obtained by using the middle column of table VIII, but ignoring symbol f. The values of g may be taken as the same as those of the corresponding principal constituents. For $2N_2$, however, we use the formula

$$(2g \text{ of } N_2) - (g \text{ of } M_2).$$

In addition to the above constituents, which are sufficiently large to have an appreciable effect on other constituents there are some small diurnal constituents which may be considered to have effect only on K_1. These constituents are called π_1, ψ_1, φ_1, and their relations to K_1 are given in table VIII.

It is not practicable to analyse direct for the constituent $2Q_1$ with any certainty of getting a trust-worthy result, because of the constituent σ_1 whose amplitude is theoretically greater than that of $2Q_1$. Tables of quantities such as $(1 + W)$ and w would be useless when the constituents were in opposite phases. Analytical results for $2Q_1$ are therefore only of value if they are obtained for a number of months so as to enable the contributions for σ_1 to be eliminated. No simple rules can be given for this process as usually the months are scattered fortuitously. Under such circumstances, if these constituents are required, they should be inferred from the formulae

$$(g \text{ of } \sigma_1) = (g \text{ of } 2Q_1) = (2g \text{ of } Q_1) - (g \text{ of } O_1)$$
$$(H \text{ of } \sigma_1) = 0.031 \ (H \text{ of } Q_1)$$
$$(H \text{ of } 2Q_1) = 0.025 \ (H \text{ of } O_1).$$

The inference of additional constituents for the shallow-water species and for special constituents of the semi-diurnal species may be effected from a consideration of the formulae given in the Admiralty Manual of Tides, para. 8.3.

With regard to the inference of harmonic constants the question is sometimes asked as to whether it is sufficiently accurate to take the same values of g for a principal and subsidiary constituent. The purist anxious for meticulous treatment of the results may use his dicretion as to how the phase-lags for constituents of the same species change with speed. If, for instance, there is a change of 24° in phase-lag for S_2 compared with M_2 then the phase-lag for K_2 will be 2° more than that for S_2. Again, the question is sometimes asked as to whether regional differences may not be used, so that a change in phase-lag for one place may be adopted for an adjacent place. Seeing, however, that there are always some errors in any analysis it is usually safer to use the methods outlined above, but in the event of there being very abnormal changes in phases, owing to nearness to a amphidromic point of the system, methods of inference may not be trustworthy except for constituents very close together in speed.

The long-period tides are largely influenced by meteorological perturbations of sea level, so that the results for one month are not trustworthy. The forthnightly constituents MSf and Mf are not separable by direct analysis or by inference, but for all the long-period constituents a series of monthly analyses will give results which may be satisfactorily analysed.

TABLE I : HOURLY MULTIPLIERS FOR X, Y.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
X_0	1	.	1	.	.	1	.	1	1	.	2	.	1	1	.	2	1	1	2	.	2	1	1	2
	.	1	1	.	2	.	1	1	.	1	.	.	1	.	1	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.		.	.	-1	.	-1	.	-1
X_1	.	-2	.	-1	.	-1	.	1	.	2	.	2	.	4	.	2	.	2	.	1	.	-1	.	-1
	.	-2	.	-1	.	-1	.	-1																
Y_1	-1	.	-1	.	-1	.	-2	.	-1	.	-1	.	1	.	2	.	2	.	4	.	2	.	2	.
	1	.	-1	.	-1	.	-2	.	-1	.	-1	.	-1	.	.	.	.	.	.	.	.	.	.	.
X_2	1	.	2	.	1	.	-2	.	-4	.	-2	.	2	.	4	.	2	.	-2	.	-4	.	-2	.
	1	.	2	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Y_2	.	.		1	.	2	.	1	.	-2	.	-4	.	-2	.	2	.	4	.	2	.	-2	.	-4
	.	-2	.	1	.	2	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X_3	1	.	.	.	-1	.	.	.	1	1	.	.	-2	-1	.	.	2	1	1	.	-2	-2	-1	.
	1	2	1	.	-1	-2	-2	.	1	1	2	.	.	-1	-2	.	.	1	1	.	.	.	-1	.
	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X_4	.	1	.	.	-1	.	.	2	.	.	-2	.	.	2	.	.	-2	.	.	2	.	.	-2	.
	.	1	.	.	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Y_4	.	.	.	1	.	.	-1	.	.	2	.	.	-2	.	.	2	.	.	-2	.	.	2	.	.
	-2	.	.	1	.	.	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
X_6	1	.	-1	.	2	.	-2	.	3	.	-3	.	3	.	-3	.	3	.	-3	.	3	.	-3	.
	2	.	-2	.	1	.	-1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.

The stencil hole for the date must appear on the same line as the symbol X,Y. Enter multipliers with negative sign in red ink.

TABLE II: DAILY MULTIPLIERS

L	d_0	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_a	d_b	d_c	d_d	d_e	d_f	d_g
-14	1	-2	2	-2	2	-2	1	-1	0	-1	1	-1	1	-2	2
-13	1	-2	2	-1	1	0	-1	1	1	-1	2	-1	2	-1	1
-12	1	-2	1	0	-2	2	-2	1	1	-2	2	-2	1	1	-1
-11	1	-1	0	1	-2	1	0	-1	1	-2	1	0	-1	2	-1
-10	1	-1	-1	2	-1	0	2	-2	2	-2	0	1	-2	0	1
- 9	1	-1	-2	2	0	-2	1	1	2	-1	-1	2	-1	-2	2
- 8	1	0	-2	1	2	-1	-1	2	2	-1	-2	1	1	-1	-1
- 7	1	0	-2	-1	2	1	-2	-1	2	1	-2	0	2	1	-2
- 6	1	1	-2	-1	1	2	1	-2	2	1	-1	-2	0	2	1
- 5	1	1	-1	-2	-1	1	2	1	2	2	-1	-2	-2	0	2
- 4	1	1	0	-2	-2	-1	1	2	1	2	1	-1	-2	-2	-1
- 3	1	1	1	-1	-2	-2	-2	0	1	2	2	1	0	-1	-2
- 2	1	2	1	1	0	-1	-2	-2	1	1	2	2	2	1	0
- 1	1	2	2	2	1	1	1	0	1	1	1	2	2	2	2
0	1	2	2	2	2	2	2	2	0	0	0	0	0	0	0
1	1	2	2	2	1	1	1	0	-1	-1	-1	-2	-2	-2	-2
2	1	2	1	1	0	-1	-2	-2	-1	-1	-2	-2	-2	-1	0
3	1	1	1	-1	-2	-2	-2	0	-1	-2	-2	-1	0	1	2
4	1	1	0	-2	-2	-1	1	2	-1	-2	-1	1	2	2	1
5	1	1	-1	-2	-1	1	2	1	-2	-2	1	2	2	0	-2
6	1	1	-2	-1	1	2	1	-2	-2	-1	1	2	0	-2	-1
7	1	0	-2	-1	2	1	-2	-1	-2	-1	2	0	-2	-1	2
8	1	0	-2	1	2	-1	-1	2	-2	1	2	-1	-1	1	1
9	1	-1	-2	2	0	-2	1	1	-2	1	1	-2	1	2	-2
10	1	-1	-1	2	-1	0	2	-2	-2	2	0	-1	2	0	-1
11	1	-1	0	1	-2	1	0	-1	-1	2	-1	0	1	-2	1
12	1	-2	1	0	-2	2	-2	1	-1	2	-2	2	-1	-1	1
13	1	-2	2	-1	1	0	-1	1	-1	1	-2	1	-2	1	-1
14	1	-2	2	-2	2	-2	1	-1	0	1	-1	1	-1	2	-2

TABLE 111: COMBINATIONS OF X AND Y FOR C AND D.

	C_{po}	C_{p1}	D_{pa}	D_{po}	D_{p1}	C_{pa}
X_{po} , Y_{po}	1 , 0	...	...	0 , 1	...	...
X_{p1} , Y_{p1}	...	1 , 1	1 , 1	...	-1 , 1	1 ,-1
X_{pa} , Y_{pa}	...	1 ,-1	-1 , 1	...	1 , 1	1 , 1

Interpretation: $C_{po} = X_{po}$.
$C_{p1} = X_{p1} + Y_{p1} + X_{pa} - Y_{pa}$.

Similar combinations are taken for functions where the second suffixes are changed to 2, b or 3, c, etc.

TABLE 1V: COMBINATIONS TO GIVE R cos r and R sin r

COMBINATIONS for 10^6 R cos r.

	S_0	M_m	MS_f
X00.	1150	....	....
01.	56	1156	-78
02.	20	-4	1085

	Q_1	O_1	M_1	K_1	J_1	OO_1
C10.	31	-43	82	1150	-95	86
11.	11	-15	643	-23	-87	21
12.	-49	594	41	-6	-8	-173
13.	583	104	-31	16	-31	35
14.	65	-14	-7	-18	11	-33
D1a.	-58	26	-90	84	686	-157
1b.	11	-92	23	-40	136	744

	μ_2	N_2	M_2	L_2	S_2	$2SM_2$
C20.	....	....	....	....	1016	....
21.	28	-13	25	510	-49	-12
22.	5	0	476	-2	9	-22
23.	-21	493	49	-11	5	6
24.	495	-34	31	-34	14	9
D2b.	3	-9	3	0	9	496
C12.	-1	2	34	-3	2	15

	MO_3	M_3	MK_3
X32.	-2	30	1072
33.	28	1061	24
34.	1012	-29	22

	MN_4	M_4	SN_4	MS_4
C42.	36	6	1	755
43.	-66	-31	793	77
44.	6	803	-43	51
45.	812	-53	-48	-6

	$2MN_6$	M_6	MSN_6	$2MS_6$	$2SM_6$
X62.	12	71	65	23	905
64.	7	-20	-23	936	77
65.	89	-177	1048	58	-29
66.	11	1018	-42	46	78
67.	1108	-42	34	40	-84

COMBINATIONS for 10^6 R sin r.

	M_m	MS_f
X0a.	1175	-145
0b.	5	1127

	Q_1	O_1	M_1	K_1	J_1	OO_1
D10.	57	-60	113	1493	-118	102
11.	1	-5	634	3	124	-26
12.	-45	585	49	-19	39	92
13.	595	102	-26	14	-28	27
14.	65	-8	-11	-15	12	-31
C1a.	74	-27	-45	-85	-688	149
1b.	-28	-11	-34	45	-141	-727

	μ_2	N_2	M_2	L_2	S_2	$2SM_2$
D20.	....	....	....	....	1016	....
21.	28	-13	26	509	-48	-11
22.	5	0	476	-2	9	4
23.	-22	493	48	-10	4	4
24.	497	-31	30	-33	12	7
C2b.	-3	9	22	0	-9	-495
D12.	-2	4	33	-2	1	-14

	MO_3	M_3	MK_3
X3b.	-35	120	1063
3c.	22	1073	45
3d.	1103	57	36

	MN_4	M_4	SN_4	MS_4
D42.	44	-9	-5	747
43.	-55	-32	782	79
44.	-2	789	-63	52
45.	823	-16	-29	-7

	$2MN_6$	M_6	MSN_6	$2MS_6$	$2SM_6$
X6b.	104	18	36	-3	935
6d.	62	45	19	1009	57
6e.	7	-192	996	-159	7
6f.	22	1065	-160	44	-6
6g.	1058	132	66	-50	1

TABLE V: CALCULATION of w and W.

	S_2, MS_4, $2MS_6$		K_1, MK_3		N_2, MN_4, $2MN_6$		
Angle	w/f	W/f	wf	Wf	w	1 + W	Angle
000°	0·7°	-0·214	0·0°	0·331	0·0°	1·184	000°
010	-6·6	-0·192	-2·5	0·327	1·6	1·182	010
020	-12·3	-0·131	-4·9	0·316	3·1	1·174	020
030	-15·5	-0·046	-7·3	0·297	4·6	1·163	030
040	-16·5	0·047	-9·6	0·271	5·9	1·147	040
050	-15·6	0·134	-11·8	0·239	7·2	1·127	050
060	-13·4	0·207	-13·8	0·201	8·3	1·104	060
070	-10·3	0·258	-15·6	0·157	9·2	1·077	070
080	-6·6	0·284	-17·1	0·107	9·9	1·048	080
090	-2·6	0·284	-18·3	0·053	10·4	1·017	090
100	1·6	0·256	-19·1	-0·003	10·6	0·984	100
110	5·6	0·204	-19·3	-0·060	10·4	0·953	110
120	9·2	0·131	-19·0	-0·118	10·0	0·922	120
130	12·0	0·041	-17·8	-0·173	9·1	0·893	130
140	13·7	-0·058	-15·9	-0·224	7·8	0·867	140
150	13·6	-0·157	-13·1	-0·268	6·2	0·846	150
160	11·2	-0·245	-9·3	-0·302	4·3	0·830	160
170	6·0	-0·307	-4·9	-0·323	2·2	0·819	170
180	-0·9	-0·330	0·0	-0·331	0·0	0·816	180
190	-7·8	-0·308	4·9	-0·323	-0·2	0·819	190
200	-12·6	-0·247	9·3	-0·302	-4·3	0·830	200
210	-14·9	-0·163	13·1	-0·268	-6·2	0·846	210
220	-14·8	-0·067	15·9	-0·224	-7·8	0·867	220
230	-13·0	0·029	17·8	-0·173	-9·1	0·893	230
240	-9·8	0·115	19·0	-0·118	-10·0	0·922	240
250	-6·0	0·186	19·3	-0·060	-10·4	0·953	250
260	-1·8	0·236	19·1	-0·003	-10·6	0·984	260
270	2·6	0·263	18·3	0·053	-10·4	1·017	270
280	6·9	0·265	17·1	0·107	-9·9	1·048	280
290	10·8	0·241	15·6	0·157	-9·2	1·077	290
300	14·1	0·192	13·8	0·201	-8·3	1·104	300
310	16·5	0·124	11·8	0·239	-7·2	1·127	310
320	17·5	0·039	9·6	0·271	-5·9	1·147	320
330	16·8	-0·051	7·3	0·297	-4·6	1·163	330
340	13·7	-0·133	4·9	0·316	-3·1	1·174	340
350	8·0	-0·193	2·5	0·327	-1·6	1·182	350
360	0·7	-0·214	0·0	0·331	0·0	1·184	360
	Angle is (V+u) for K_1; f is f(K_2)		Angle is (2V+u) for K_1; f is f(K_1)		Angle is (3V for M_2) minus (3V for N_2)		

TABLE VI: ASTRONOMICAL DATA, f and u.

s, h, p, N are the mean longitudes of the moon, sun, moon's perigee, and moon's ascending node, respectively.

Y = the year.
D = the number of days elapsed since January 1 in the year Y.
L = the integral part of $\frac{1}{4}$(Y − 1901), equal to the number of leap years between 1900 and Y, excluding Y as the leap day in this year is counted in D.

$$\left.\begin{aligned} s &= 277^\circ{\cdot}025 + 129^\circ{\cdot}38481\,(Y - 1900) + 13^\circ{\cdot}17640\,(D + L) \\ h &= 280{\cdot}190 - 0{\cdot}23872\,(Y - 1900) + 0{\cdot}98565\,(D + L) \\ p &= 334{\cdot}385 + 40{\cdot}66249\,(Y - 1900) + 0{\cdot}11140\,(D + L) \\ N &= 259{\cdot}157 - 19{\cdot}32818\,(Y - 1900) - 0{\cdot}05295\,(D + L) \end{aligned}\right\}\text{ at zero hour of day D, G.M.T.}$$

	f: series of multiples of				u: series of multiples of		
	1	cos N	cos 2N	cos 3N	sin N	sin 2N	sin 3N
Mm :	1·0000	−0·1300	0·0013	...	...	...	...
Q_1, O_1 :	1·0089	0·1871	−0·0147	0·0014	10°·80	−1°·34	0°·19
K_1 :	1·0060	0·1150	−0·0088	0·0006	−8·86	0·68	−0·07
J_1 :	1·0129	0·1676	−0·0170	0·0016	−12·94	1·34	−0·19
OO_1 :	1·1027	0·6504	0·0317	−0·0014	−36·68	4·02	−0·57
μ_2, N_2, M_2:	1·0004	−0·0373	0·0002	...	−2·14	...	...
K_2 :	1·0241	0·2863	0·0083	−0·0015	−17·74	0·68	−0·04

L_2: $f \cos u = 1 - 0{\cdot}2505 \cos 2p - 0{\cdot}1102 \cos (2p - N) - 0{\cdot}0156 \cos (2p - 2N) - 0{\cdot}0370 \cos N.$
$\quad f \sin u = - 0{\cdot}2505 \sin 2p - 0{\cdot}1102 \sin (2p - N) - 0{\cdot}0156 \sin (2p - 2N) - 0{\cdot}0370 \sin N.$

M_1: $f \cos u = 2 \cos p + 0{\cdot}4 \cos (p - N)$
$\quad f \sin u = \sin p + 0{\cdot}2 \sin (p - N)$

M_3: $f = 1 + 1{\cdot}5\,(f - 1 \text{ of } M_2) = - 0{\cdot}5 + 1{\cdot}5\,(f \text{ of } M_2)$
$\quad u = 1{\cdot}5\,(u \text{ of } M_2)$

TABLE V11: VALUES OF V AND Δ , f AND u.

	V				Δ
	s	h	p	°	°
Mm	1	0	−1	...	10·3
Q_1	−3	1	1	270	207·7
O_1	−2	1	0	270	216·1
M_1	−1	1	0	90	224·6
K_1	0	1	0	90	233·1
J_1	1	1	−1	90	241·6
OO_1	2	1	0	90	250·2
μ_2	−4	4	0	...	73·5
N_2	−3	2	1	...	80·8
M_2	−2	2	0	...	89·2
L_2	−1	2	−1	180	97·7
S_2	0	0	0	...	105·0
M_3	−3	3	0	180	6·9

	V	Δ
Msf	$S_2 - M_2$	19·3
$2SM_2$	$2S_2 - M_2$	120·8
MO_3	$M_2 - O_1$	353·2
MK_3	$M_2 + K_1$	20·6
MN_4	$M_2 + N_2$	80·1
M_4	$2M_2$	88·5
SN_4	$S_2 + N_2$	95·8
MS_4	$M_2 + S_2$	104·2
$2MN_6$	$2M_2 + N_2$	306·1
M_6	$3M_2$	314·3
MSN_6	$M_2 + S_2 + N_2$	321·4
$2MS_6$	$2M_2 + N_2$	329·5
$2SM_6$	$2S_2 + M_2$	344·8

The values of u for the constituents on the left are given by table V1 or are zero.

For the compound constituents in the right half of the table the angles are given in terms of those of the generating constituents, and the same applies to u. The values of f are the products of the values of f for the generating constituents. In full, therefore, for example:

$$(V \text{ of } MSN_6) = (V \text{ of } M_2) + (V \text{ of } S_2) + (V \text{ of } N_2)$$
$$(f \text{ of } MSN_6) = (f \text{ of } M_2) \times (f \text{ of } S_2) \times (f \text{ of } N_2)$$
$$(V \text{ of } 2MS_6) = 2(V \text{ of } M_2) + (V \text{ of } S_2)$$
$$(f \text{ of } 2MS_6) = (f \text{ of } M_2) \times (f \text{ of } M_2) \times (f \text{ of } S_2)$$

TABLE VIII: VALUES OF R and r FOR SUBSIDIARY CONSTITUENTS.

Subsidiary constituent	Principal constituent	R	r
K_2	S_2	0·272 (f of K_2)(H of S_2)	(r + w of S_2) - 2 (V + u of K_1) + 180°.
T_2	S_2	0·059 (H of S_2)	(r + w of S_2) + (V of K_1) - 12°.
ν_2	N_2	0·194 (fH of N_2)	(r + w of N_2) - (3V of M_2 - 2V of N_2).
$2N_2$	...	0·133 (fH of N_2)	2 (r + w of N_2) - (r of M_2).
P_1	K_1	0·331 (H of K_1)	(r + w + u of K_1) + 2 (V of K_1).
π_1	K_1	0·019 (H of K_1)	(r + w + u of K_1) + 3 (V of K_1) - 12°.
ψ_1	K_1	0·008 (H of K_1)	(r + w + u of K_1) - (V of K_1) + 12°.
ϕ_1	K_1	0·014 (H of K_1)	(r + w + u of K_1) - 2 (V of K_1) + 180°.

TABLE IX: ADDITIONAL CORRECTIONS TO PRINCIPAL CONSTITUENTS.

Corrections to R cos r of

Multiple of.	μ_2	N_2	M_2	L_2	S_2	$2SM_2$
R cos r: K_2	-0·028	0·039	-0·061	0·137	0·031	0·085
" : T_2	0·015	-0·022	0·035	-0·087	0·017	-0·038
" : ν_2	0·127	0·001	-0·144	0·076	-0·060	-0·043
" : $2N_2$	-1·004	0·127	-0·065	0·045	-0·044	-0·039
" : O_1	-0·002	0·004	0·056	-0·005	0·004	0·026

	Q_1	O_1	M_1	K_1	J_1	OO_1
R cos r: P_1	-0·055	0·077	-0·153	0·015	0·152	-0·143
" : π_1	...	...	...	-1·000	...	...
" : ψ_1	...	...	...	-1·000	...	...
" : ϕ_1	...	...	...	-1·000	...	...

Corrections to R sin r of

Multiple of.	μ_2	N_2	M_2	L_2	S_2	$2SM_2$
R sin r: K_2	-0·029	0·038	-0·065	0·136	0·027	0·081
" : T_2	0·016	-0·022	0·036	-0·087	0·019	-0·036
" : ν_2	0·130	0·000	-0·138	0·074	-0·055	-0·037
" : $2N_2$	-1·004	0·123	-0·059	0·042	-0·038	-0·035
" : O_1	-0·003	0·006	0·055	-0·004	0·002	-0·023

	Q_1	O_1	M_1	K_1	J_1	OO_1
R sin r: P_1	-0·076	0·082	-0·161	0·023	0·145	-0·129
" : π_1	...	...	...	-1·000	...	...
" : ψ_1	...	...	...	-1·000	...	...
" : ϕ_1	...	...	...	-1·000	...	...

Note: If O_1 has been previously allowed for in the analysis by corrections from C_{12} and D_{12} (see table IV) the contributions given above should be ignored.

TABLE 1: HOURLY OBSERVATIONS OF HEIGHT OF TIDE.

Central Day: Feb.21,1951. OGIDIGBE: ESCRAVOS RIVER. Zone: -1.

		00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Feb.	6.	7·1	7·9	8·8	9·4	9·9	10·2	9·7	8·5	7·5	6·7	6·0	5·6	5·7	6·3	7·3	8·3	9·5	10·8	10·5	9·8	9·2	8·3	7·5	6·8
	7.	6·8	7·2	8·0	9·0	9·8	10·6	10·6	9·8	9·3	8·2	7·0	6·2	5·5	5·8	6·5	7·5	8·8	10·0	10·8	10·8	9·8	8·8	7·8	7·2
	8.	6·7	6·8	7·5	8·3	9·1	9·9	10·4	10·5	9·9	8·9	7·5	6·3	5·7	5·6	6·3	7·1	8·4	9·7	10·3	10·6	10·0	9·2	8·3	7·4
	9.	6·8	6·6	7·0	7·7	8·5	9·3	9·7	10·0	9·8	9·2	8·2	6·8	6·2	5·8	6·1	6·8	7·6	8·8	9·8	10·1	10·3	9·8	9·1	8·3
	10.	7·3	6·8	6·8	7·2	7·9	8·6	9·3	9·6	9·8	9·3	8·8	7·9	6·9	6·0	5·9	6·5	7·2	8·2	9·0	9·7	10·1	9·8	9·3	8·7
	11.	7·9	7·1	6·8	6·9	7·4	8·3	8·8	9·2	9·3	9·3	8·8	8·2	7·3	6·7	6·3	6·5	7·2	7·8	8·5	9·2	9·7	9·8	9·5	9·0
	12.	8·3	7·5	6·9	6·8	7·3	7·8	8·3	8·8	9·0	9·2	9·0	8·7	8·3	7·3	6·8	6·7	7·0	7·3	7·7	8·2	8·7	9·3	9·5	9·2
	13.	8·7	8·1	7·5	7·2	7·2	7·5	7·8	8·3	8·6	8·9	9·1	8·8	8·6	8·3	7·8	7·3	7·4	7·8	8·1	8·5	8·8	9·2	9·5	9·2
	14.	8·8	8·5	8·0	7·6	7·3	7·3	7·5	7·8	8·2	8·4	8·6	8·6	8·4	8·3	8·1	7·8	7·6	7·8	8·0	8·3	8·7	9·0	9·3	9·3
	15.	9·0	8·7	8·3	7·9	7·5	7·2	7·3	7·5	7·7	7·9	8·3	8·5	8·6	8·5	8·4	8·3	8·3	8·2	8·2	8·3	8·3	8·5	8·7	8·8
	16.	9·0	8·8	8·7	8·3	8·0	7·7	7·3	7·2	7·3	7·3	7·4	7·8	8·1	8·4	8·8	8·7	8·5	8·3	8·2	7·8	7·9	8·1	8·4	8·8
	17.	9·0	9·2	9·2	8·9	8·6	8·1	7·6	7·3	6·8	6·8	7·0	7·3	7·7	8·0	8·5	8·8	8·8	8·7	8·3	7·9	7·6	7·5	7·8	8·3
	18.	8·7	8·8	9·0	9·3	9·2	8·8	8·3	7·6	7·0	6·7	6·7	7·0	7·3	7·8	8·5	9·2	9·4	9·3	8·8	8·3	7·9	7·7	7·6	7·8
	19.	8·2	8·8	9·2	9·4	9·5	9·1	8·5	7·8	7·1	6·5	6·3	6·5	7·0	7·7	8·3	9·0	9·4	9·3	9·0	8·5	8·2	7·8	7·3	7·3
	20.	7·8	8·3	8·8	9·3	9·6	9·7	9·3	8·4	7·5	6·8	6·2	6·1	6·4	7·3	8·3	9·0	9·6	9·8	9·3	8·7	8·2	7·6	7·1	6·9
	21.	7·2	7·9	8·7	9·3	9·8	9·9	9·5	8·8	7·8	6·8	6·2	5·9	6·3	7·0	8·0	9·0	9·8	10·2	10·3	9·6	8·8	8·0	7·3	6·8
	22.	6·8	7·4	8·3	9·3	10·0	10·4	10·3	9·5	8·5	7·5	6·5	6·0	5·9	6·4	7·4	8·4	9·4	10·3	10·8	10·5	9·8	9·0	8·2	7·3
	23.	6·8	6·9	7·7	8·5	9·3	10·1	10·3	10·1	9·7	8·7	7·7	6·7	6·0	6·1	6·8	7·8	8·8	9·8	10·8	11·0	10·3	9·3	8·3	7·5
	24.	6·8	6·8	7·4	8·3	9·3	10·0	10·3	10·5	10·1	9·4	8·4	7·4	6·4	6·0	6·7	7·7	8·7	9·7	10·7	11·0	10·5	9·8	8·8	7·8
	25.	6·8	6·5	7·1	7·9	8·8	9·4	10·0	10·3	10·4	9·8	9·0	8·0	7·0	6·0	6·1	7·0	8·0	9·0	10·0	10·6	10·5	9·8	9·0	8·2
	26.	7·3	6·8	6·5	7·0	7·8	8·6	9·4	10·0	10·3	10·0	9·3	8·3	7·3	6·5	6·4	6·9	7·8	8·8	9·8	10·3	10·5	9·9	9·3	8·6
	27.	7·9	7·2	6·6	6·8	7·4	8·1	8·8	9·4	9·8	10·1	9·5	8·8	8·0	7·4	7·1	7·0	7·3	7·8	8·8	9·7	10·0	10·2	9·8	9·4
	28.	3·8	8·1	7·4	6·8	7·0	7·3	7·7	8·2	8·8	9·4	9·5	9·2	8·7	8·0	7·3	7·1	7·2	7·8	8·4	9·0	9·3	9·7	10·0	9·7
Mar.	1.	9·3	8·7	8·0	7·4	7·0	7·0	7·3	7·8	8·4	8·9	9·2	9·3	9·1	8·9	8·3	7·8	7·5	7·6	7·9	8·3	8·7	9·0	9·3	9·6
	2.	9·7	9·3	8·5	7·9	7·6	7·3	6·9	6·9	7·3	7·6	8·3	8·9	9·3	9·2	8·8	8·5	8·2	7·8	7·8	8·0	8·3	8·7	9·0	9·3
	3.	9·4	9·4	9·2	8·8	3·3	7·8	7·3	6·8	6·8	7·0	7·4	8·1	8·8	9·1	9·4	9·2	8·8	8·5	8·2	7·8	7·4	7·4	7·8	8·3
	4.	8·8	9·6	9·7	9·1	8·6	8·1	7·6	7·0	6·7	6·6	6·8	7·3	7·9	8·6	9·6	10·2	9·9	9·3	8·6	8·0	7·7	7·4	7·8	8·1
	5.	8·7	9·2	9·5	9·8	10·0	9·5	3·4	7·4	6·8	6·3	6·6	7·3	8·3	9·0	9·5	9·8	10·2	10·2	9·6	8·6	7·6	7·1	7·1	7·4
	6.	7·8	8·6	9·3	9·7	10·0	9·8	9·3	8·4	7·4	6·8	6·5	6·6	6·9	7·6	8·6	9·4	10·3	10·7	10·7	10·2	9·3	8·1	7·3	7·3
	7.	7·9	8·6	9·3	9·7	10·1	10·3	10·1	9·4	8·4	7·2	6·1	5·8	6·3	7·2	8·1	9·1	10·1	10·5	10·3	9·7	8·7	7·9	7·2	6·7
	8.	6·9	7·6	8·3	9·0	9·7	10·0	10·1	9·5	8·5	7·4	6·3	5·8	6·1	6·8	7·7	8·4	9·4	10·2	10·7	10·5	9·8	9·0	8·0	7·1
	9.	6·6	6·8	7·6	8·6	9·4	9·8	10·0	9·8	9·2	8·3	7·3	6·3	6·2											

TABLE 2: ILLUSTRATING USE OF STENCIL FOR X_2

	1	2	1	-2	-4	-2	2	4	2	-2	-4	-2
15.	9·0 9·0	8·3 8·7	7·5 8·0	7·3	7·7	8·3	8·6	8·4	8·3	8·2	8·3	8·7
	1	2	1									

TABLE 3: VALUES OF X_p, Y_p.

OGIDIGBE, ESCRAVOS RIVER.

Central Day: February 21, 1951. Zone: -0100.

Day	X_0	X_1	Y_1	X_2	Y_2	X_3	X_4	Y_4	X_6
Feb 7	250•1	-9•6	-1•3	-30•8	42•4	3•6	0•7	-0•7	-1•8
8	247•7	-7•9	-0•9	-39•1	33•7	3•7	0•8	0•7	-0•2
9	246•6	-8•0	0•4	-43•9	15•7	5•2	-1•0	0•9	-0•9
10	245•6	-7•6	-0•6	-42•9	1•1	-0•9	-1•4	-0•4	1•5
11	245•2	-6•0	0•1	-34•7	-8•9	-1•0	-0•9	-0•4	1•3
12	243•5	-5•3	-1•9	-21•7	-18•7	-0•4	-1•3	-1•1	1•2
13	247•6	-0•1	4•6	-12•4	-18•5	-0•3	0•3	-0•7	-1•9
14	247•0	-2•6	5•9	-4•8	-17•2	0•8	0•2	0•3	-1•3
15	246•5	-1•6	8•3	5•9	-9•9	0•1	0•0	-0•6	0•3
16	245•2	-6•0	7•6	15•4	-2•9	1•4	-0•4	-0•1	-0•7
17	243•5	-8•8	3•4	19•9	10•5	1•1	-0•6	-0•9	0•0
18	246•1	-7•3	4•2	17•0	20•9	0•9	-1•1	0•5	0•5
19	245•0	-9•3	2•8	13•7	28•9	-2•5	0•0	-0•1	1•7
20	244•8	-7•9	0•9	8•0	38•4	0•6	-0•2	0•8	-0•7
21	249•5	-6•8	3•6	-1•0	46•3	-1•5	-0•2	-0•4	-1•5
22	253•4	-7•7	3•8	-19•7	44•6	0•7	-0•4	0•0	-2•8
23	257•0	-6•9	0•4	-35•0	35•8	2•0	-0•1	0•6	-1•5
24	258•8	-4•0	0•5	-41•1	27•8	2•4	-1•6	1•9	-1•1
25	254•2	-3•6	-0•6	-47•3	12•8	2•7	-1•5	0•1	2•7
26	254•5	-0•7	3•4	-46•4	1•4	2•0	-0•4	-0•8	2•1
27	252•5	-0•1	4•8	-34•9	-17•8	2•2	0•7	0•5	0•3
28	252•0	-2•4	6•0	-19•7	-25•0	1•2	-0•2	-1•2	-1•3
Mar 1	250•8	-0•6	6•7	-2•8	-26•6	-0•4	1•4	-0•9	0•8
2	250•8	-3•6	8•2	14•2	-17•8	-3•6	0•0	-0•9	1•6
3	246•7	-3•9	6•6	27•5	-0•3	1•5	1•5	-1•4	-1•6
4	252•6	-4•3	7•2	29•3	16•9	-0•1	-1•6	0•9	-0•7
5	253•7	-1•6	6•2	28•1	30•7	1•9	0•0	-3•2	0•4
6	259•1	-5•7	8•6	7•4	41•6	0•3	1•3	-2•4	-0•5
7	252•4	-8•0	1•5	-0•8	48•0	1•5	2•2	-0•1	0•8

TABLE 4: VALUES OF X , Y , C , D , R cos r, R sin r.

Central Day: February 21, 1951. OGIDIGBE, ESCRAVOS RIVER. Zone: -0100.

	X		X	Y						R cos r	R sin r		R cos r	R sin r
00.	7242·4	1	-147·9	100·4	C10.	-47·5	D10.	248·3	S_0	8·330	...	Q_1	0·004	-0·027
01.	-2·9	11.	3·0	6·0	11.	3·5	11.	-78·7	Mm	-0·004	-0·158	O_1	-0·141	0·057
a.	-134·1	a.	-43·6	-38·1	12.	-245·7	12.	127·1	Msf	0·053	-0·027	M_1	-0·014	-0·012
02.	48·2	12.	-95·1	-56·0	13.	-17·8	13.	-47·6				K_1	-0·052	0·377
b.	-41·6	b.	-3·3	91·3	14.	79·8	14.	4·8				J_1	0·010	0·019
		13.	-6·2	-1·7	D1a.	14·5	C1a.	-84·7				OO_1	-0·009	-0·011
		c.	-31·0	-21·1	1b.	-56·5	1b.	48·9						
		14.	15·9	20·4										
		d.	21·9	-21·6										
32.	24·6	20.	-292·6	333·9	C20.	41·3	D20.	626·5	MO_3	-0·014	-0·012	μ_2	-0·008	0·024
b.	-23·3	21.	7·0	34·4	21.	113·0	21.	17·0	M_3	-0·047	-0·002	N_2	0·068	-0·248
33.	-45·8	a.	30·6	-41·0	22.	2021·5	22.	2188·1	MK_3	0·025	-0·025	M_2	0·963	1·022
c.	1·1	22.	-22·6	1063·7	23.	140·5	23.	-503·9				L_2	0·054	0·009
34.	-12·9	b.	1041·1	60·7	24.	-37·2	24.	3·2				S_2	0·055	0·653
d.	-11·8	23.	184·4	-73·7	D2b.	60·7	C2b.	15·5				$2SM_2$	-0·019	-0·003
		c.	-108·0	-137·8	C12.	-245·7	D12.	127·1						
		24.	-17·0	-32·4										
		d.	15·4	3·2										
62.	-14·8	42.	2·6	8·3	C42.	37·5	D42.	6·3	$2MN_6$	0·004	-0·001	MN_4	-0·013	0·011
b.	-4·7	b.	13·6	-13·0	43.	-10·5	43.	9·1	M_6	0·011	0·023	M_4	0·027	-0·035
64.	-21·9	43.	-4·5	-2·1	44.	32·3	44.	-43·9	MSN_6	-0·012	0·004	SN_4	-0·009	0·009
d.	24·8	c.	1·4	5·3	45.	-18·4	45.	13·4	$2MS_6$	-0·021	0·025	MS_4	0·029	0·003
65.	-10·9	44.	28·8	-5·3					$2SM_6$	-0·014	-0·003			
e.	8·0	d.	-0·5	-9·3										
66.	9·6	45.	-2·7	-3·4										
f.	22·7	e.	0·9	13·2										
67.	5·0													
g.	-2·9													

TABLE 5: COMPUTATION OF H, g.

OGIDIGBE, ESCRAVOS RIVER. 5°34' N, 5°11' E.

Central Day: 21 February, 1951. Zone: -0100.

	S_0	M_m	MSf	Q_1	O_1	M_1	K_1	J_1	OO_1
V	...	250·6	351·3	58·0	308·6	274·4	60·1	310·7	351·6
u	...	...	-0·4	-1·5	-1·5	244·5	1·3	1·8	5·0
	...	10·3	19·3	207·7	216·1	224·6	233·1	241·6	250·2
w	...	...	-17·1	6·2	...	...	-16·9	-6·2	...
r	...	268·7	333·0	278·4	158·0	220·6	97·9	62·4	230·6
f	...	0·873	0·964	1·181	1·181	1·013	1·112	1·163	1·772
1 + W	...	...	1·281	0·846	...	...	0·859	0·846	...
R	8·330	0·158	0·059	0·027	0·152	0·018	0·379	0·021	0·014
H	8·330	0·181	0·048	0·027	0·129	0·018	0·397	0·021	0·008
g	...	169·6	326·1	188·8	321·2	244·1	15·5	250·3	117·4

	MO_3	M_3	MK_3	μ_2	N_2	M_2	L_2	S_2	$2SM_2$
V	317·2	13·0	68·8	17·4	118·1	8·7	79·3	0·0	351·3
u	-1·1	0·6	1·7	0·4	0·4	0·4	-6·9	0·0	-0·4
	353·2	6·9	20·6	73·5	80·8	89·2	97·7	105·0	120·8
w	...	...	-16·9	...	6·2	...	...	-17·1	-34·2
r	220·6	182·4	315·0	*	285·4	46·6	9·5	85·2	189·0
f	1·138	0·946	1·072	0·964	0·964	0·964	1·308	1·000	0·964
1 + W	...	...	0·859	...	0·846	...	...	1·281	1·562
R	0·018	0·047	0·035	*	0·256	1·401	0·055	0·658	0·019
H	0·016	0·050	0·038	*	0·314	1·453	0·042	0·514	0·013
g	169·9	202·9	29·2	*	130·9	144·9	179·6	173·1	266·5

	MN_4	M_4	SN_4	MS_4	$2MN_6$	M_6	MSN_6	$2MS_6$	$2SM_6$
V	126·8	17·4	118·1	8·7	135·4	26·0	126·8	17·4	8·7
u	0·7	0·7	0·4	0·4	1·1	1·1	0·7	0·7	0·4
	80·1	88·5	95·8	104·2	306·1	314·3	321·4	329·5	344·8
w	6·2	...	-10·9	-17·1	6·2	...	-10·9	-17·1	-34·2
r	139·8	307·6	135·0	5·9	346·0	64·5	161·5	130·0	192·1
f	0·929	0·929	0·964	0·964	0·896	0·896	0·929	0·929	0·964
1 + W	0·846	...	1·127	1·281	0·846	...	1·127	1·281	1·562
R	0·017	0·044	0·013	0·029	0·004	0·025	0·012	0·033	0·014
H	0·022	0·047	0·012	0·023	0·005	0·028	0·011	0·028	0·009
g	353·6	54·2	338·4	102·1	74·8	45·9	239·5	100·5	151·8

S_2, MS_4, Msf, $2MS_6$
K_1 V + u = 61°·4
w/f = -13·0
W/f = 0·214
K_2 f = 1·314
w = -17·1
W = 0·281

K_1, MK_3
K_1 2V + u = 121·5
wf = -18·8
Wf = -0·127
f = 1·112
w = -16·9
W = -0·141

N_2, MN_4, $2MN_6$
M_2 3V = 26·1
N_2 2V = 236·2
difference = 149·9
w = 6·2
1 + W = 0·346

$2SM_2$, $2SM_6$
w = 2w(S_2)
W = 2W(S_2)

SN_4, MSN_6
w = w(S_2) + w(N_2)
1+W = W(S_2) + 1+w(N_2)

J_1 as N_2 with w changed in sign.

Q_1 as N_2

* cannot be determined unless a correction is made for $2N_2$; see tables IX and 6.

TABLE 6: CALCULATION OF R cos r AND R sin r FOR SUBSIDIARY CONSTITUENTS.

	K_2	T_2	ν_2	$2N_2$	P_1	π_1	ψ_1	φ_1
ratio	0·272	0·059	0·194	0·133	0·331	0·019	0·008	0·014
f factor	1·314	...	0·964	0·964	...	...	...	...
H factor	0·514	0·514	0·314	0·314	0·397	0·397	0·397	0·397
R = product =	0·183	0·030	0·059	0·040	0·132	0·008	0·003	0·006
term with r	68°·1	68°·1	291°·6	223°·2	82°·3	82°·3	82°·3	82°·3
" " V	-122·8	60·1	-149·9	-46·6	120·2	180·3	-60·1	-120·2
constant	180·0	-12·0	...	...	...	-12·0	12·0	180·0
r = sum	125·3	116·2	141·7	176·6	202·5	250·6	34·2	142·1

	R cos r	R sin r
K_2	-0·106	0·149
T_2	-0·013	0·027
ν_2	-0·046	0·036
$2N_2$	-0·040	0·002
O_1	*	*
P_1	-0·122	-0·050
π_1	-0·003	-0·008
ψ_1	0·003	0·002
φ_1	-0·005	0·004

* already included.

TABLE 7: CORRECTED VALUES OF R AND r FOR SUBSIDIARY CONSTITUENTS.

		μ_2	N_2	M_2	L_2	S_2	$2SM_2$
Corrections to:	R cos r	0·037	-0·009	0·015	-0·019	0·001	-0·005
" "	R sin r	-0·001	0·005	-0·014	0·021	0·002	0·010
Table 4:	R cos r	-0·008	0·068	0·963	0·054	0·055	-0·019
"	R sin r	0·024	-0·248	1·022	0·009	0·653	-0·003
Corrected values:	R cos r	0·029	0·059	0·978	0·035	0·056	-0·024
" "	R sin r	0·023	-0·243	1·008	0·030	0·655	0·007
" "	R	0·037	0·249	1·404	0·046	0·657	0·025
" "	r	38·5	283·7	45·9	40·6	85·1	163·7

		Q_1	O_1	M_1	K_1	J_1	OO_1
Corrections to:	R cos r	0·007	-0·009	0·019	0·003	-0·019	0·017
" "	R sin r	0·004	-0·004	0·008	0·001	-0·007	0·006
Table 4:	R cos r	0·004	-0·141	-0·014	-0·052	0·010	-0·009
"	R sin r	-0·027	0·057	-0·012	0·377	0·019	-0·011
Corrected values:	R cos r	0·011	-0·150	0·005	-0·049	-0·009	0·008
" "	R sin r	-0·023	0·053	-0·004	0·378	0·012	-0·005
" "	R	0·025	0·159	0·006	0·382	0·015	0·009
" "	r	295·5	160·5	321·4	97·4	127·0	328·0

The remainder of the work is similar to that in table 5.

4. *Extension of the Admiralty Method.*

The Admiralty Method of Analysis of Tides for 29 days' observations is similar in principle to the Tidal Institute Method, except that it uses multipliers 1, —1, 0 instead of the multipliers given in tables I and II. The tables of multipliers used in the Admiralty Method and in the extension are given in tables IA and IIA. The original method was devised for use by officers who had not got access to computing machines, and it was found to be convenient to avoid the possibility of products with signs alternating unsystematically, as may happen when the range of any X and Y is small. This was effected by adding temporary datums to X or Y so that only positive quantities were to be treated by the multipliers, and provision was made for the effects of the multipliers on the datums. With the increasing use of computing machines it is not necessary for these datums to be used, and the tables given here make no provision for the use of such datums. (Of course the datum of the observations of hourly heights, relative to the chart datum, or other datum of observations, is not removed from the observations).

The extended tables provide for the computation of third-diurnal and sixth-diurnal constituents, and some additional constituents in other species.

As has already been mentioned, the daily multipliers used in this method do not adequately separate the species of tides so that corrections have to be made for constituents of each species upon constituents of all other species. The formulae to combine the functions obtained after the hourly and daily processes have been completed involve much more calculation than in the original Admiralty Method, for the table of formulae increases in size with the square of the number of constituents sought. The method has been completed with the utmost degree of nominal accuracy, and the multipliers for the corrections are given in tables IIIA and IVA. It should be noted that the Admiralty Method has been followed so that the formulae give PR cos r and PR sin r, where P is given in table VA, together with the angle p, which is the contribution to V (or E) for the time origin of 11.5 hours each day.

As in the Admiralty Method

$$H = PR \text{ divided by } Pf\ (1+W)$$

$$g = r + V + u + p + w$$

where V, u, f, w, W are defined as in the Tidal Institute Method.

Further refinements can be made, as in the Tidal Institute Method, for the effects of K_2, T_2, and other constituents, and the necessary tables for these are given in table VIII (which is common to both methods) and table VIA.

It may be noticed that coefficients for the functions in tables IIIA and IVA differ from those given in the original method. The reason for this is that the new functions contain contributions of the original constituents as well as the constituents for which they have been introduced. The elimination of one of the new constituents introduces additional contributions from the older ones so that the formulae have had to be completely revised. It follows that any attempt to ignore any of the functions may lead to error, if the coefficients of those functions are rather large.

TABLE IA: HOURLY MULTIPLIERS FOR X, Y.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
X_0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
X_1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
Y_1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1
X_2	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1
Y_2	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1
X_3	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1
X4	1	0	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1
Y_4	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1
X_6	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1

TABLE IIA: DAILY MULTIPLIERS.

0	1	2	3	4	5	6	7	a	b	c	d	e	f	g
1	-1	1	-1	1	-1	1	-1	1	0	1	0	1	-1	1
1	-1	1	-1	1	1	-1	1	1	-1	1	-1	1	-1	1
1	-1	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1
1	-1	1	1	-1	1	-1	-1	1	-1	1	-1	-1	1	-1
1	-1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	1
1	-1	-1	1	1	-1	1	1	1	-1	-1	1	-1	-1	1
1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1
1	1	-1	-1	1	1	-1	-1	1	0	-1	0	1	1	-1
1	1	-1	-1	1	1	1	-1	1	1	-1	-1	1	1	1
1	1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	1	1
1	1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1
1	1	1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1
1	1	1	1	-1	-1	-1	-1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1
1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1
1	1	-1	-1	-1	-1	1	1	-1	-1	-1	1	1	1	1
1	1	-1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1
1	1	-1	-1	1	1	1	-1	-1	-1	1	1	-1	-1	-1
1	1	-1	-1	1	1	-1	-1	-1	0	1	0	-1	-1	1
1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1
1	-1	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1
1	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	-1
1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	1	-1	1
1	-1	1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1
1	-1	1	-1	1	1	-1	1	-1	1	-1	1	-1	1	-1
1	-1	1	-1	1	-1	1	-1	-1	0	-1	0	-1	1	-1

TABLE IIIA.

COMBINATIONS TO GIVE 10^3PR cos r.

	S_0	Q_1	O_1	M_1	K_1	J_1	OO_1	μ_2	N_2	M_2	L_2	S_2	$2SM_2$
X00	1000	...	...	...	...	...	...	...	...	...	...	...	...
1	77	-2	13	58	-6	-53	-5	-2	2	-9	-24	2	7
2	28	-14	121	-12	5	-16	-106	2	-2	-47	-2	1	58
C10	5	10	-90	103	1000	-160	35	-9	2	-8	4	-3	...
1	-3	38	23	1000	-14	62	48	14	2	9	39	-3	-7
2	-4	-52	1000	59	-15	48	17	3	1	78	...	1	-31
3	8	1004	27	-73	52	297	-1	...	109	-3	15	-2	-3
4	-7	105	-21	-37	-20	...	-23	144	-8	12	-1	4	8
D1a	-4	-20	55	-130	99	1000	-81	13	-8	9	5	3	2
b	8	-76	-152	171	-66	160	1000	1	4	15	21	-6	-81
C20	...	4	...	...	...	1	...	-41	-30	-40	-33	1000	-36
1	3	...	-5	-22	-5	-25	-2	98	-3	140	1004	-76	-11
2	...	1	-48	-8	4	-8	-37	7	35	1001	-1	15	21
3	-2	-90	-1	17	...	3	-3	6	1000	-64	-15	5	3
4	...	-11	10	1	5	-6	11	1003	-65	80	-23	17	30
D2b	...	7	49	7	-3	6	34	65	-123	-13	-3	15	1000
X32	8	28	-77	168	-77	220	738	-34	26	-113	6	2	-102
3	...	60	10	-275	19	-56	-19	19	-195	7	22	-8	-4
4	-2	-47	-14	...	-4	2	-2	-254	2	-17	...	-5	-4
C42	...	-2	3	-6	3	-9	-33	1	2	33	-1	...	25
3	-1	-12	...	18	-1	5	-1	2	59	-3	-14	2	2
4	-1	3	2	-1	...	...	6	86	-7	3	4	...	-2
5	1	-1	1	-1	1	1	-6	-7	2	7	15	-1	4
X62	-1	-2	6	-8	5	-13	-40	13	-7	-3	8	-3	712
4	...	...	4	-1	4	-5	3	89	-55	22	27	-12	-8
5	1	-3	-10	-24	-2	-26	-16	-81	6	30	27	...	-21
6	-2	-4	33	12	-2	9	71	39	-12	254	25	-20	210
7	-1	-10	14	26	-1	36	19	14	40	-39	-121	16	6

	M_m	MS_f	MO_3	M_3	MK_3	MN_4	M_4	SN_4	MS_4	$2MN_6$	M_6	MSN_6	$2MS_6$	$2SM_6$
X00	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	1000	41	...	...	...	1	...	1	-3	...	...	...	...	...
2	-1	1000	...	-1	-4	2	-1	2	-25	...	...	...	...	1
C10	-8	8	1	...	1	...	-2	1	-2	...	...	...	...	...
1	-25	2	-1	...	1	1	3	1	2	...	...	...	...	...
2	6	-57	-1	...	-6	...	2	-2	24	...	...	...	...	1
3	34	4	-1	-10	2	-3	-1	30	-1	...	...	...	...	...
4	-3	-8	-12	1	-2	8	38	-1	3	...	...	1	1	...
D1a	36	-7	-2	-1	-1	...	4	-3	2	...	...	...	...	...
b	-23	73	-1	5	15	2	1	1	14	...	...	...	...	-3
C20	...	...	-2	-1	...	...	2	2	2	...	...	...	...	...
1	12	3	3	...	2	...	-5	...	-2	...	...	...	...	...
2	...	27	...	3	26	1	-1	...	-35	...	...	...	...	-3
3	-15	...	2	42	-6	1	1	-49	2	...	...	1	...	...
4	4	-4	54	2	-2	5	-61	1	-4	...	...	...	-5	...
D2b	2	-27	4	-7	-30	3	-7	10	-18	...	...	...	...	4
X32	-19	16	75	-4	1080	-6	24	-15	149	...	1	...	2	9
3	30	-5	-31	1042	-172	9	-22	207	-10	1	-2	-2	...	1
4	-5	1	1000	107	36	13	267	4	17	1	-1	2	23	2
C42	...	14	...	-6	-48	11	1	45	1000	...	...	...	...	-14
3	-9	...	-3	-73	9	99	3	1000	-69	-2	1	-1	-2	-1
4	4	-2	-93	-4	3	96	1002	-69	77	...	...	-3	-29	-1
5	7	3	13	-2	-8	1000	-95	-31	-62	...	...	-34	3	-1
X62	1	-1	-4	-5	-62	-7	2	6	38	-61	41	-70	83	1020
4	4	...	-79	-8	...	-15	88	-1	4	-41	-140	-137	1000	71
5	11	6	-2	-4	-6	152	1	...	-4	-113	-50	1050	43	-104
6	2	-24	6	-2	14	13	-11	1	11	-144	1044	32	62	342
7	-14	-7	-9	-5	12	7	9	-4	-11	1008	19	-44	-148	99

TABLE IVA.

COMBINATIONS TO GIVE 10^3 PR sin r.

	Q_1	O_1	M_1	K_1	J_1	OO_1	μ_2	N_2	M_2	L_2	S_2	$2SM_2$
X0a	-4	-13	52	1	59	-19	2	2	5	-30	5	10
b	-3	105	24	-11	23	103	...	...	-50	2	-1	-60
D10	12	-87	98	1005	-167	46	-10	2	-9	9	-7	-8
1	44	29	1009	-20	7	51	12	...	5	38	-2	-6
2	-55	1002	45	-13	35	-88	4	...	83	-7	6	41
3	1000	34	-19	54	297	2	4	110	-9	-16	-1	-3
4	127	-30	-38	-25	3	-28	146	-18	17	-2	9	12
C1a	27	-52	76	-93	-1012	89	-16	4	-8	26	-10	-9
b	48	34	-171	62	-144	-1019	-6	14	41	-29	12	95
D20	4	3	1	...	-1	-3	-41	-30	-41	-34	1000	-34
1	-5	-2	-28	3	26	-1	99	-2	140	1000	-77	-13
2	3	-52	3	-1	3	40	6	35	1000	1	14	-37
3	-90	-5	-16	-2	5	-3	3	1001	-62	8	3	2
4	-20	3	7	2	-7	6	1000	-58	82	-27	19	34
C2b	-16	52	-5	...	-1	-38	-66	123	-21	6	-15	-1000
X3b	-45	63	-89	63	-186	-731	15	-41	-86	-30	11	114
c	102	5	255	-12	20	...	-31	-181	-4	-12	-3	-2
d	74	5	-47	1	6	7	-271	33	-16	3	-5	-9
D42	2	-3	5	-3	8	33	1	2	32	1	-1	-25
3	-15	-1	-18	...	...	1	-1	60	...	17	-1	-1
4	-9	...	2	1	-2	...	83	-1	5	-2	1	2
5	-1	-1	...	...	-2	...	14	-1	5	14	-1	...
X6b	5	-4	6	-4	12	44	1	-3	7	1	-2	-713
d	-35	-2	-1	1	-3	6	48	56	27	-29	15	30
e	-4	-2	-20	2	13	2	86	-15	-8	...	-3	-19
f	1	-46	14	-4	3	73	-68	27	252	-14	16	203
g	5	10	-16	3	24	-13	-4	-38	20	114	-14	13

	M_m	MSf	MO_3	M_3	MK_3	MN_4	M_4	SN_4	MS_4	$2MN_6$	M_6	MSN_6	$2MS_6$	$2SM_6$
X0a	-1083	146	...	...	...	1	2	1	5	...	...	...	...	...
b	-3	-1017	...	-2	-4	-2	2	-4	-26	...	...	...	...	1
D10	-1	-3	1	...	0	-2	-3	...	-4	...	...	...	...	...
1	31	1	-1	...	-1	1	2	-1	-1	...	...	...	...	...
2	...	50	...	...	-5	3	...	3	27	...	...	...	...	1
3	37	-1	-1	-9	...	-2	3	32	-3	...	...	...	...	...
4	3	-3	-13	...	...	18	38	-7	6	...	-1	...	1	...
C1a	-26	1	1	-1	...	-3	-5	...	-4	...	...	...	...	...
b	-11	-53	...	4	12	2	-3	12	24	...	...	...	...	-2
D20	1	2	-2	-1	-2	1	2	1	1	...	...	...	...	...
1	-14	...	4	1	1	-1	-4	...	-3	...	...	...	...	...
2	...	-26	...	4	25	-1	...	-4	-36	...	...	...	...	-3
3	-17	1	3	41	-4	1	-1	-49	3	...	...	1	...	...
4	1	...	54	1	2	-6	-62	5	-1	...	...	...	-4	...
C2b	-2	27	-3	2	-30	-2	7	-10	-16	...	...	..	...	4
X3b	-5	1	-55	265	1000	...	-18	50	147	...	1	-1	-1	11
c	31	-1	152	1000	-117	10	30	193	1	2	-1	-1	1	1
d	-2	...	1059	-74	37	34	285	-39	14	1	-2	2	24	1
D42	...	-14	...	-6	-45	-2	13	23	1002	...	...	...	...	-14
3	-12	1	-6	-71	7	99	-11	1002	-70	-1	-1	-1	1	...
4	...	...	-94	-2	-2	53	1000	-52	84	...	...	-2	-29	-1
5	-8	1	-18	1	-2	1003	-79	-34	-65	...	...	-34	2	1
X6b	1	-1	6	-6	-61	5	-8	-7	36	-15	13	43	-72	1000
d	-3	-1	-89	-5	-1	30	89	2	2	-66	55	301	1042	15
e	-10	...	-1	-2	-4	147	-25	-1	-3	-101	6	1000	-195	-51
f	3	-24	3	-9	-12	-8	10	-1	-13	-58	1000	-200	219	-326
g	-10	5	-6	6	7	4	13	1	-11	1000	-24	-68	-92	71

TABLE VA: VALUES OF P and p.

	P	p		P	p
So	696	...	Mm	453	6·3
Q_1	568	154·1	Msf	438	11·7
O_1	568	160·3	MO_3	284	133·7
M_1	573	166·7	M_3	298	140·0
K_1	448	173·0	MK_3	280	146·3
J_1	550	179·2	MN_4	540	300·4
OO_1	521	185·6	M_4	528	306·6
μ_2	548	321·6	SN_4	555	312·1
N_2	578	327·1	MS_4	540	318·3
M_2	562	333·3	$2MN_6$	280	273·7
L_2	577	339·6	M_6	289	280·0
S_2	447	345·0	MSN_6	309	285·4
$2SM_2$	548	356·7	$2MS_6$	294	291·6
			$2SM_6$	301	303·3

TABLE VIA: ADDITIONAL CORRECTIONS TO PRINCIPAL CONSTITUENTS.

Corrections to 10^3 R cos r of

From R cos r of	S_0	Q_1	O_1	M_1	K_1	J_1	OO_1	μ_2	N_2	M_2	L_2	S_2	$2SM_2$
K_2	-4	-7	7	...	6	-4	6	-8	68	-69	218	41	104
T_2	2	2	-4	1	-3	3	-3	8	-35	40	-134	11	-44
ν_2	3	7	-7	3	-5	7	-5	102	2	-148	43	-41	-31
$2N_2$	3	7	-8	1	-5	6	-8	-1000	124	-78	...	-28	-36
P_1	-10	-91	121	-290	3	199	-131	7	-11	7	-12	6	8

	Mm	MSf	MO_3	M_3	MK_3	MN_4	M_4	SN_4	MS_4
K_2	1	-5	-1	2	-5	...	1	-4	2
T_2	-1	2	...	-1	3	...	...	1	-1
ν_2	-2	3	2	-2	4	-5	-1	1	-2
$2N_2$	-2	3	2	-2	5	-11	-1	1	...
P_1	15	-15	...	...	-3	...	1	-3	1

Corrections to 10^3 R sin r of

From R sin r of	Q_1	O_1	M_1	K_1	J_1	OO_1	μ_2	N_2	M_2	L_2	S_2	$2SM_2$
K_2	-8	...	-10	1	9	-1	-7	68	-65	221	40	110
T_2	2	1	4	...	-3	1	7	-35	33	-134	11	-47
ν_2	2	-1	-2	...	-1	-2	102	2	-150	47	-43	-35
$2N_2$	...	2	-4	...	3	-3	-1000	126	-82	4	-32	-40
P_1	-97	106	-283	-5	215	-145	7	-11	15	-18	15	19

	Mm	MSf	MO_3	M_3	MK_3	MN_4	M_4	SN_4	MS_4
K_2	-3	-1	-1	3	...	...	4	-2	4
T_2	3	...	...	-2	...	-1	1	2	-3
ν_2	27	12	...	-1	...	-3	-2	3	-4
$2N_2$	-4	...	2	...	-1	-11	-1	3	-5
P_1	-6	...	...	1	...	2	2	-1	6

(There are no corrections to sixth-diurnal constituents).

5. *Remarks on methods of analysis.*

When the Tidal Institute Method was first devised, it was realised by the author that the older methods then in use were inadequate to give very accurate results. The analysis of observations for the solar constituents was simple because the observations were in solar time, but the re-reading of the tide gauge to give, say, lunar time for the lunar constituents was prohibitive so that the observation at the nearest solar hour was « assigned » to the lunar hour. Processes such as this are accurate if the effects are computed once for all on exact trigonometrical expressions, but at that time such results were not available, and the accuracy of the analysis depended upon having such a large number of observations that the errors might be treated almost as casual errors. No direct method can be devised so that corrections for one constituent upon another are not necessary. Even if the corrections are not omitted they are rendered extremely complex because of the choice of time origin at the beginning of the first day. The author showed that the calculation of corrections was greatly simplified by the choice of a central time-origin, and he would regard this as an absolute necessity.

For the corrections by those older methods it was assumed that the processes sufficiently separated the species so that one did not affect another, but it is doubtful whether this is true for short lengths of observations. Even in the Tidal Institute Method for 29 days it is impracticable to devise direct formulae which will make M_2 free from the effects of O_1, but the formulae for X and Y are very effective for the isolation of species except in this special case.

The processes used in the Admiralty Method are dependent upon adequate corrections, for the constituents of any species contribute to the constituents of all other species in a small but not negligible degree. As it was first devised, certain smaller constituents like L_2, M_1, J_1 were completely ignored, and the analytical results are not greatly affected by doing so. It was necessary to consider the amount of labour involved, the accuracy of the observations, and the use to be made of the results, so that the original Admiralty Method is a compromise method. Its extension involves much more labour and many other considerations. The author's opinion is that it is better to use the Tidal Institute Method rather than to extend the Admiralty Method, but because a great many analyses have been effected by the latter method it is desirable to place on record what is involved in its extension.

6. *Notes on the effects of casual errors.*

The original methods of harmonic analysis used multipliers which were cosines or sines, and which were usually taken to 3 or more decimals. These are indicated by the mathematical theory which leads to « The Least Square Rule »; the formulae are chosen so as to give the least possible value for the mean of the squares of errors. As an example, for 24 hourly values of a solar diurnal tide the coefficients for one phase will be the cosines of multiples of 15°,

1.000, 0.966, 0.866, 0.707, 0.500, 0.259, 000, ... (a)

with the general divisor 12.00.

The Tidal Institute multipliers are the nearest integers to twice the cosines,

2, 2, 2, 1, 1, 1, 0, ... (b)

with the general divisor 24.52.

The Admiralty Method multipliers are

1, 1, 1, 1, 1, 1, 1, 0, ... (c)

with the general divisor 15.20.

As there is no reason why casual errors should affect any of the observations more than the rest, it follows that on the average the mean value of the square of the errors will be the same (say, e^2) for all data. The signs of the errors will be random, and so the maximum error will occur when it happens that all errors of positive sign occur with positive coefficients in (a), (b), (c), and all errors of negative sign occur with negative coefficients. Hence the maximum errors from (a) are

$$2.000e + (0.966 + 0.866 + \ldots 0.259)\ 4e$$

divided by 12.00. The maximum errors from (b) and (c) are similarly deduced and we obtain results given by

(a) 1.27e; (b) 1.30e; (c) 1.45e;

There is close correspondence between (a) and (b) in these extreme circumstances because the multipliers in the Tidal Institute Method closely conform to the cosine variations, but if a casual error is appreciably large there is not much to be gained by (a) or (b) over (c).

The more usual way of estimating error is to take the square root of the sum of the squares of the products. The sign of the individual error does not enter into consideration, but it must be remembered that the multipliers in (a), (b), (c), are each to be divided by the proper divisor. Thus if a multiplier is m and the divisor is D, and the error is e or —e, then the sum of the squares is

$$\Sigma\ m^2e^2/D^2$$

Thus from (a), (b), (c) we get

$$12e^2/144, \qquad 52e^2/24.52^2, \qquad 22e^2/15.20^2$$

and the square roots give

0.289e; 0.294e, 0.308e.

respectively.

It is evident that the least-square formula (a) has no practical advantage over the integral formulae (b) and (c). Much waste of labour has been incurred and is still being incurred by methods of analysis which use cosines and sines to a number of decimals, as will be revealed by a glance at some of the appalling forms given in textbooks dealing with computation.