STANDARD DEVELOPMENT OF TIDE-GENERATING POTENTIAL

(I.H.B. Circular-Letter No 4-H of 30th April 1954)

Sir,

During the VIth International Hydrographic Conference, May, 1952, the German Hydrographic Institute formulated the following proposal:

« *That the purely harmonic form of the development of the tide-generating potential as first introduced by Dr Doodson in his paper in the Proceedings of the Royal Society (Vol. 100, 1921), London, be generally adopted as a standard form.* »

Moreover, for the *Calculation of the Nodal Variations of Astronomical and Shallow-Water Constituents,* « *that consideration be given to the appointment of a Committee for proposing standard formulae to be generally accepted in the Harmonic analysis of Tides and Tidal Streams for the calculation of the so-called Nodal Variations of the astronomical and shallow-water constituents.* »

The Conference agreed « to form a sub-Committee consisting of Dr. Doodson, Mr. Horn, Mr. Gougenheim and Dr. Marmer to consider these two subjects ».

On 25th September, 1953, Dr A.T. Doodson forwarded to the International Hydrographic Bureau the following information:

« I am now able to give the views of the committee appointed to consider the definitions and accessory tables of the tidal constituents. The documents enclosed commence with a note on the subject by Doodson, followed by comments by other members of the Committee.

« From these it is seen that the committee consider that the time is ripe for the publication of more elaborate tables of the nineteen yearly variations in harmonic constituents, and that these tables will facilitate research and improvements in practice, wherever such is feasible. It will be noted that the agreement of the Committee regarding new versions of f and u implies that they accept the proposal concerning the standard development of the tide-generating potential. »

The Directing Committee of the International Hydrographic Bureau has the honour to communicate the above documents to its States Members as well as to Services responsible for hydrography in other maritime countries, with a view to reaching an agreement on the preparation and publication of these tables in the form a Special Publication by the I.H.B.

Kindly forward the opinion of your Office with comments on the above proposal to the I.H.B.

Yours truly,

Rear Admiral C.L. NICHOLS, U.S.N. (Ret.)
President of the Directing Committee.

COMMITTEE ON TIDAL CONSTITUENTS

At the Conference held by the International Hydrographic Bureau in Monaco in May 1952 the consideration of international agreement on a standard development of the tide-generating potential and on the definitions of the long period factors and phase corrections (denoted at present by f and u) were referred to a committee consisting of

Doodson, Horn, Gougenheim and Marmer,

and this circular to the committee is sent out by Doodson to explain the subject to be considered.

The tide-generating potential was developed in a quasi-harmonic method by Darwin in 1883 and it has been regarded as the standard development. He used the old lunar theory and referred everything to the orbit rather than to the ecliptic; his results were given in algebraic form, arithmetic being used only to decide what terms to omit. The development was quasi-harmonic because he retained factors in the coefficients, and terms in the arguments, (denoted respectively by f and u), which are periodic but which may be considered as constant over fairly long intervals of time, such as a year.

In 1921 Doodson published (in the Proceedings of the Royal Society, A, vol. 100, p. 305) an entirely new development which used more accurate expansions of the longitude and latitude of the moon, as given by Brown. These were referred to the ecliptic and the new development was entirely numerical and truly harmonic. It gave a very much larger number of terms than were given by Darwin and its object was to provide an expansion of such accuracy that research into tidal problems would be greatly facilitated. An elaboration of Darwin's form of development, including many more terms, has since been given in the U.S.A. Manual of Harmonic Analysis and Prediction of Tides (U.S.C. & G.S., Special Publication 98).

When Doodson had completed his development, it was evident that any given tidal constituent was composed of a large number of terms, of which only a part could be represented by f and u. An example of this may be taken for the constituent O_1. The development, following Doodson's notation, is a follows:

Argument-number	Coefficient		Argument in terms of					
			τ	s	h	p	N'	p_1
145.455	12	G'_1	1	— 1	0	— 1	0	0
535	— 218		1	— 1	0	0	— 2	0
545	7105		1	— 1	0	0	— 1	0
555	37689		1	— 1	0	0	0	0
645	16	G'_1	1	— 1	0	1	— 1	0
655	— 108	G'_1	1	— 1	0	1	0	0
665	14	G'_1	1	— 1	0	1	1	0
755	— 243		1	— 1	0	2	0	0
765	— 40		1	— 1	0	2	1	0

There is a datum figure of 5 in each case, except in the first figure, of the argument-number. The variable τ is equal to $15°t - s + h - L$, where L is the west longitude of the place, and the variable N' is equal to —N, (N is the longitude of the lunar node). The other variables have their usual significance. It is not necessary for us to write out the argument in full for each constituent, for the arguments of the terms relative to the argument of the principal

term are alone required. We thus obtain, by re-arrangement, and ignoring temporarily the terms with coefficients G'_1,

Relative argument			Coefficient	Relative coefficient
p	N'	p_1		
0	0	0	37689	1.0000
0	— 1	0	7105	0.1885
0	— 2	0	— 218	— 0.0058
2	0	0	— 243	— 0.0064
2	1	0	— 40	— 0.0011

The first three terms, which are functions only of N', give values of f and u as follows :

$$f \cos u = 1.0000 + 0.1885 \cos N' - 0.0058 \cos 2N'$$
$$f \sin u = - 0.1885 \sin N' + 0.0058 \sin 2N'$$

and it has been verified by Doodson that this and similar formulae for all other constituents give exactly the same values of f and u as were given by Darwin, and tabulated by Baird (of the Survey of India), and as are given in the U.S.C. & G.S. Manual. They agree also with the expansions used by Darwin for f and u direct in harmonic functions of N (see his Scientific Papers, vol. 1, p. 69) but note that his formula for $2\nu''$ should have a negative sign to A_3 (as was discovered recently by comparison with other tables and formulae).

The last two terms are functions of p and of N', and the committee is asked to decide whether the time has arrived for the inclusion of such terms in standard tables of factors and phase-corrections.

Two tables are enclosed, showing the coefficients of all terms in the main constituents of the diurnal and semidiurnal species. Considering firstly the diurnal species it is obvious that M_1 is most affected by these (p, N') terms, and it has been customary to give f and u for M_1 in terms of p and N, though it may be noted that Darwin's values were only half what they should have been. (In actual practice this has not mattered if the same table of factors was used in prediction as well as in analysis). Next in order is O_1 with the term whose coefficient is 243. If O_1 had an amplitude of 3.7689 feet this hitherto neglected term would have an amplitude of only 0.0243 feet. Such high values of H for O_1 are almost non-existent.

All other terms for the diurnal constituents are small, and we may now turn to the semidiurnal constituents. Here the biggest terms (still ignoring the terms with coefficients G') are in L_2. Again it has been customary to include these (p, N') terms in defining f and u for L_2. The next largest perturbations are for M_2, but if the amplitude of M_2 is 9.0812 feet the perturbation is only 0.0053 ft.

It will be evident, therefore, that the inclusion of these (p, N') terms for constituents other than L_2 and M_1 will have little practical significance. If they were included it would mean that instead of a simple table of f and u as functions of N, as is now the case for most of the constituents, it would be necessary to have double-entry tables in p and N as are now required for L_2 and M_1. Even so, the actual tables would not be very bulky.

Considering now the terms involving coefficients G'_1 and G'_2, it should be noted that these coefficients are functions of latitude and are not the same

as the functions of latitude for the main terms, so that the geographical distribution will be different. That is, it is not possible to assume that for any given place the relations between the main terms and these others will follow the relations indicated by the potential. It is not possible, therefore, to prepare for universal use tables which will also include these special terms, and their values can only be obtained by analyses covering a period of 19 years. This matter was examined by Doodson in 1924 (« Perturbations of Harmonic Tidal Constants », Proc. Roy. Soc. A, vol. 106, p. 513). The point to which attention needs to be directed is that several of these special terms (p, N', G') are much greater than any of the neglected (p, N') terms referred to above and it might be argued that it is rather futile to include the neglected (p, N') terms when the others are larger. Similarly we could consider other normal constituents which are usually ignored in tidal predictions though their amplitudes are greater than any of the above (for example, in Doodson's notation, see page 327 of his paper, the terms with numbers 137, 127, 162, 167, 173, 183, 147, 166).

This was the view taken in 1921 by Doodson. He did not at that time feel justified in making changes in established practice, when there was little to be gained in the accuracy of tidal predictions by doing so. The principal reason which could be advanced in favour of a change now is that tidal research is not moribund. The principal purpose of Doodson's development was to provide a sound basis for a critical examination of the tidal constituents found by analysis, and research is now taking place with a view to the inclusion of the neglected (p, N', G') terms which are of sufficient importance in many places to justify inclusion (see the paper by Doodson on harmonic perturbations, with special reference to Bombay and St. John). This research work, however, would be vitiated unless the other group of neglected terms in (p, N') were included.

It is therefore suggested for the consideration of the committee that perhaps the time has arrived when the tables of f and u should be supplemented by others including the (p, N' terms) which would be available for research workers in the future. The new factors and phase-corrections might well be denoted by new symbols (say j and v) to avoid confusion. The comprehensive tables should give f, u as well as j and v so that no compulsion is exercised on computers who are satisfied with f and u.

The Committee might take into consideration whether it is necessary to prepare tables for all the constituents, or whether a compromise is possible, so that only the more important constituents are included in the new tables. Obviously M_2, N_2, ν_2, are the only terms among the semidiurnal constituents which would require special tables, and as the terms are small they might be expressed as factors to be applied to f and u, so that the double entry tables could be at large intervals in p and N. It should be noted that S_2 and P_1 have small nodal variations which have been neglected hitherto. The (p, N') terms are not such that several constituents could use the same factors to f and u, and the factor method would not be a good one for O_1 because of the large variations of f and u.

Harmonic Terms of Diurnal Constituents.

Relative phase			165·555	145·555	135·655	155·555	175·455	127·555	185·555	137·455	125·755	163·555
p	N	p_1	K_1	O_1	Q_1	M_1	J_1	σ_1	OO_1	ρ_1	$2Q_1$	P_1
0	-2	0	...	-218	-42	...	...	...	...	...	...	14
0	-1	0	1050	7105	1360	...	87	218	...	258	180	-199
0	0	0	-53050	37689	7216	...	-2964	1153	-1623	1371	955	17584
0	1	0	-7182	...	...	...	-587	...	-1039	...	...	...
0	2	0	154	...	...	...	13	...	-218	...	...	...
0	3	0	...	...	...	...	...	...	-14	...	...	...
0	0	2	...	...	...	...	...	...	...	...	...	-11
-2	-2	0	...	...	-28	...	...	...	...	...	...	...
-2	0	0	...	...	...	...	...	...	-240	...	...	...
-2	1	0	...	...	...	...	...	...	-48	...	...	...
-1	-2	0	...	...	...	17	...	...	...	...	...	...
-1	-1	0	...	...	...	-197	...	...	...	...	...	...
-1	0	0	...	...	...	-1065	...	...	...	...	...	...
1	-1	0	...	...	...	85	...	...	...	...	...	...
1	0	0	...	...	...	-2964	...	...	...	...	...	...
1	1	0	...	...	...	-594	...	...	...	...	...	...
1	2	0	...	...	...	17	...	...	...	...	...	...
2	0	0	...	-243	-19	...	46	...	...	-78	...	-26
2	1	0	...	-40	...	...	29	...	...	24	...	...
2	2	0	...	...	...	...	17	...	...	...	...	...
Coeffs. of G'_1												
-1	-1	0	...	...	-84	...	...	...	...	...	-23	...
-1	0	0	-36	12	-211	...	...	-11	-40	...	-58	...
-1	1	0	...	...	...	...	...	...	-16	...	...	...
0	-1	0	...	...	...	98	...	...	...	...	...	...
0	0	0	...	...	...	-661	...	...	...	...	...	...
0	1	0	...	...	...	86	...	...	...	...	...	...
1	-1	0	...	16	...	...	...	...	...	...	...	...
1	0	0	-13	-108	-13	...	-241	...	...	-18	...	...
1	1	0	...	14	...	...	...	...	...	...	...	...

The first group of terms gives normal values of (f,u)
The second " " " include all those which it is suggested should now be taken into account.

Harmonic Terms of Semi-diurnal Constituents.

Relative phase			255·555	245·655	247·455	265·455	263·655	237·555	235·755	273·555	275·555	272·556
p	N	p_1	M_2	N_2	ν_2	L_2	λ_2	μ_2	$2N_2$	S_2	K_2	T_2
0	-2	0	47	...	...	...	...	...	...	...	...	...
0	-1	0	-3386	-648	-123	95	24	-104	-86	94	-147	...
0	0	0	90812	17387	3303	-2567	-670	2777	2301	42358	11506	2479
0	1	0	...	...	...	...	...	...	...	...	3423	...
0	2	0	...	...	...	...	...	...	...	...	372	...
-2	-2	0	...	-63	...	...	...	...	-14	...	...	...
-1	0	1	...	14	...	...	...	...	...	...	...	...
2	-1	0	...	...	...	-12	...	...	...	...	...	...
2	0	0	53	...	17	643	...	...	...	...	...	...
2	1	0	19	...	-12	283	...	...	...	...	...	...
2	2	0	...	...	...	40	...	...	...	...	...	...
Coeffs. of G_2'												
-1	-1	0	...	-97	...	...	...	...	-27	...	...	...
-1	0	0	32	-569	...	...	...	-29	-156	...	29	...
1	-1	0	...	...	...	-31	...	...	...	...	...	...
1	0	0	86	11	15	525	...	...	...	...	...	...
1	1	0	16	...	...	99	...	...	...	...	...	...

The first group of terms gives normal values of (f, u)
The second " " " include all those which it is suggested should now be taken into account.

COMMENTS BY W. HORN

The following comments by Horn, in a letter to Doodson, should also be considered by the Committee. The Committee will see that he has already carried out most of the work that would be involved.

« Harmonic development of the tide-generating potential. My reasons for advocating the purely harmonic form as standard form are:

a. that the most complete numerical calculation which now exists, viz. that given in your paper of 1921, is of that form;

b. that this form closely corresponds to that of the modern lunar theory, by Brown, after which the current lunar ephemeris is computed;

c. that mathematically correct theory of the representation of the tides in deep or shallow water, to the degree of including all astronomical variations which deserve consideration, can be built up on the base of the purely harmonic development of the potential. (The set of orthogonal functions necessary and sufficient for the representation in expansion form of the lunar potential = that for the solar potential = that for the tides in deep water = that for the tides in shallow water = all six-dimensional Fourier terms, which is not self-evident. Vide my paper of 1948.) I know of no other solution to the problem.

Nodal variations. The corrections are introduced in order to compensate for a temporary reduction in the dimension number of the expansion. If you put it thus one is led very naturally to what you propose to call (j, v), and a special explanation is required of why one should content oneself with applying (f, u) only.

However, I do not intend to persuade other people to use (j, v). I only wish to be free to use (j, v) when I prefer to do so, and I have thought the question ripe for being discussed at Monaco after it had become evident that you and we had been proceeding on different lines without being aware of it.

The ideal of an harmonic analysis would be the purely empirical determination of all constants, including (j, v) or the harmonic equivalent, from at least 19 year observations. That would enable one to also settle, for the individual port, the problems of the terms which include geodetic functions other than those of the principal constituents, and of what may still be uncertain in the adopted formulae for the nodal variations in shallow water. In my analysis of high and low waters I have expressed everything in harmonic terms and determined their values empirically.

We have calculated tables of the annual values of (j, v) for 65 constituents up to the year 1999, and daily values for the 10 principal constituents up to 1959. I shall be glad to excerpt for you whichever of the values you may think suitable to serve as examples in the report. »

COMMENTS BY A. GOUGENHEIM

1. A. Gougenheim is in favour of the adoption of the propositions put forward by Dr. A.T. Doodson in his note of 19th January, 1953, regarding the consideration of the terms in (p, N') in the harmonic formula of the tides.

2. As indicated by Dr Doodson, these terms would only be taken into account in the case of M_1 among the diurnal constituents, of $M_2L_2N_2\nu_2$ among the semi-diurnal constituents, as they are far too small in the case of other constituents.

3. The customary method of determination of f and u for the M_1 and L_2 constituents would remain unchanged. Complementary tables for the M_2, N_2, ν_2 constituents would give the elements required for proceeding from the values of f and u to those of j and v. The calculations already carried out by W. Horn could be used advantageously in the drawing up of these tables.

4. It is desirable that the attention of users be drawn to the fact that the terms thus introduced in (p, N') have only a purely theoretical interest and can be ignored in all regular calculations as, on the one hand, they are inferior to the uncertainty produced in tidal observations by measuring devices and by various causes of disturbance, and on the other hand, their consideration adds nothing to the accuracy of the harmonic formula, as other terms of equal importance in (p, N', G') are disregarded.

COMMENTS BY H.A. MARMER

Obviously it will be desirable to agree on a standard development of the tide-generating potential and on the definitions of the long-period factors and phase corrections. But as you put it at one time « the translation of these proposals into practical methods, however, is a matter for careful consideration ».

I am of the opinion that it would be desirable for research purposes to have the tables for f and u supplemented by others arising from the fuller development of the potential, leaving it for the various authorities engaged in tide predictions to adopt the new tables as may be most convenient after careful consideration.