INTERCONTINENTAL CONNECTION OF GEODETIC SYSTEMS

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SUMMARY

In order to obtain the World Geodetic System we need five quantities: the geodetic latitude φ_0, longitude λ_0, and azimuth A_0 of the initial point I of this system as well as the equatorial radius a and flattening α of the reference ellipsoid. φ_0, λ_0 and A_0 will be obtained from the formulae

$$\varphi = \varphi' - \xi_g ; \qquad \lambda_0 = \lambda' - \eta_g \sec \varphi ; \qquad A_0 = A' - \eta_g \operatorname{tg} \varphi \qquad (1)$$

where φ', λ' and A' are the astronomical coordinates and ξ_g and η_g the deflection of the vertical components computed gravimetrically. The gravity formula supplies the flattening α. As to the equatorial radius a, this is supplied by the existing triangulations when we locate them exactly on the ellipsoid by correcting the astronomical coordinates of the end points of the triangulation chain, using the formulae similar to formula (1). To get for the different triangulations exactly the same scale, we must measure in various parts of the world standard base lines by the (Väisälä) interference comparator.

For the intercontinental connection we have three methods: direct, semi-direct and indirect methods. In the direct method (Shoran, Hiran) we need to know only the velocity of light and relatively short triangulation chains for calibrating the equipment which measures the long distances. The lines up to 880 km. have been measured so far with the relative accuracy of 1 : 120 000 (Aslakson). The semi-direct consists of three celestial methods: the solar eclipse method, the occultation method and the moon camera method. In all these methods the moon is one triangulation point. Consequently its distance from the observation points at the time of photographing the moon must be known with a high degree of accuracy. For that purpose we photograph the moon from at least two points of the same continent which are connected with one another by triangulation.

In the indirect, or gravimetric method we determine astro-gravimetrically the geodetic latitude, longitude and azimuth as well as gravimetrically the undulations N and deflection of the vertical components ξ_g, η_g of the initial points reference ellipsoid, and compute the distances between the initial points along the reference ellipsoid. N, ξ_g, η_g give the tilt and distance between the reference ellip-

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soid and geoid. Thus we can reduce the geodetic base lines from geoid to ellipsoid. The gravimetric method supplies, of course, the quantities N, ξ_g and η_g at any needed points in the neighbourhood of which sufficient gravity surveys exist, regardless of the continents on which these points are located.

The scientific mission of geodesy is to determine the size and shape of the earth and also to study with other sciences the structures of the earth's interior. Its practical task is to carry out the measurements and computations which are needed for making good and reliable maps for different purposes. The most important practical objective of geodesy is the determination of the exact co-ordinates of the control points on the earth's surface. Without these control points no adequate mapping work would be possible. The more accurate the co-ordinates of the control points, the more reliable the maps.

In the history of geodesy we find three epochs: the spherical era, ellipsoidal era and geoidal era. During the spherical era, which lasted from the time of Eratostenes (276-194 B.C.) to the early part of the eighteenth century, the earth was assumed to be a sphere. So the objective of geodesy was only to measure the radius of this sphere. This epoch brought to geodesy the telescope, logarithm tables, triangulation, relatively accurate geodetic base lines and the important arc measurement of J. Picard.

The ellipsoidal era began in 1738, when P.L.M. Maupertuis published his important book « La figure de la terre » in which he was able to prove on the basis of the arc measurements carried out in France and Lapland that the earth is in fact a flattened ellipsoid of revolution, as Newton and Huygens had already claimed earlier. During this period, which lasted to the turn of the twentieth century, several geodesists computed reasonable values for the dimensions of the earth's ellipsoid, i.e., the equatorial radius a and the flattening of the meridian α. In addition, the turn of the nineteenth century brought to geodesy, through the genius of the 18-year-old K.F. Gauss and A.M. Legendre, the important adjustment computation as well as the definition and preliminary length of the meter measured by the French scientists. It is known that the meter does not correspond exactly to its definition. The length of the Paris meridian is not exactly 10 000 000 m., but according to the International ellipsoid slightly longer, i.e. 10 002 286 m. The nineteenth century introduced the methodical basic triangulation works of Gauss and F.W. Bessel, the mathematical basis (Stokes' formula) (1) of physical geodesy, as well as the international cooperation required for the development of geodesy.

The short but important period of this century is the geoidal era, during which it has been possible to undertake the determination of the detailed form of the geoid. This is particularly important, since it is no longer sufficient to know the reference ellipsoid along broad lines, as a knowledge of its accurate dimensions as well as the detailed shape of the geoid are also required. This is a difficult task which can be mastered only by the new methods. This geoidal era has devised new instruments for geodetic and gravimetric measurements, adopted the International ellipsoid and International gravity formula to be used in geodetic and gravimetric studies, carried out extensive isostatic studies which are of basic significance to physical geodesy, made several long range triangulations and applied the electronic, celestial and gravimetric methods to geodesy.

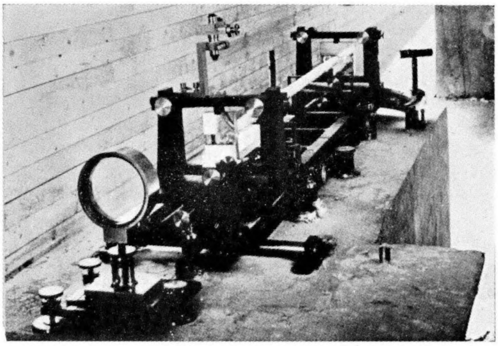

Fig. 1
Sketch of Väisälä interference comparator of the Finnish Geodetic Institute.

During the idyllic period of the nineteenth century, when every country was satisfied provided it had control points of its own, the problems of geodesy were not yet very complicated. If the country in question was not too large, any reasonable reference ellipsoid could be used, since the effect of possible errors in its dimensions could be very small and would do no harm to practical mapping. The reliability of the co-ordinates of the control points mainly depended on the accuracy of the astronomic, triangulation and traverse measurements. Relatively simple adjustment computation procedures eliminated any inner discrepancy of the geodetic system in question. It did not matter how much the geodetic systems of different countries disagreed with one another. As late as 1948, for instance, the differences in the co-ordinates of a same control point, according to the lecture of Admiral Nares of the International Hydrographic Bureau of Monaco, delivered at the General Assembly of the International Union of Geodesy and Geophysics in Oslo, amounted in latitude, longitude and metres, between the Danish and Swedish systems to 0″.4, 5″.5 and 95 m., between the Danish and German systems to 6″.3, 8″.9 and 250 m., between the Danish and Norwegian systems to 5″.5, 1″.0 and 171 m., and between the English and French systems to 5″.43, 4″.03 and 191 m. No one knows the extent of differences between the geodetic systems of the various continents.

We know that this mess cannot last much longer. We must get rid of the too large number of existing geodetic systems. Owing to hydrographic surveying and aviation, it is of basic significance that the co-ordinates of the different countries and continents belong to the same system. The existing geodetic systems, even across the oceans, must therefore be connected as accurately as possible to the world geodetic system.

It does not matter where the initial point of the world geodetic datum is located, whether in England, France, Germany or in the United States of America. The important point is that we have a single world geodetic system.

For that purpose we must have:

(1) A common scale throughout the world;

(2) New arc measurements along the continents and across the oceans;

(3) Improved dimensions of the reference ellipsoid;

(4) The detailed shape of the geoid,

(5) Very accurate values for the undulation N of the geoid, and of the deflections of the vertical components ξ and η at the initial points of the different geodetic systems, or at some other astronomical point of the systems in question; and,

(6) Supercontrol points for long-range triangulation.

The significance of *a common scale* is obvious. If the length unit obtained by the base line measurements is different in various countries and on different continents, the computed long geodetic lines to be used in the world-wide computations will not be comparable with one another and will cause systematic errors, which no one knows and which consequently can not be considered.

Because of this fact we have to measure, in as many countries and continents as possible, accurate standard base lines on which to calibrate the wires to be used in the measurements of the usual field base lines. The light interference method invented three decades ago by the Finnish geodesist and astronomer Y. Väisälä and put in practice by the Finnish Geodetic Institute, supplies the needed standard base lines. Until the present time such standard base lines have been measured in Nummela (Finland), in 1948 and in 1955, and in Buenos Aires (Argentina) in 1953. The length and relative accuracy of these base lines are: in Nummela 864 m. and 1/17 000 000 (3), in Buenos Aires 480 m. and 1/9 000 000 (4).

Fig. 1 shows a part of Väisälä's interference comparator with the collimator lens and the mirrors at both ends of the quartz gauge.

As this method is now in actual use, I shall try to explain with the help of Fig. 1 the principle involved: (5)

« White light from a pointlike source L is made parallel by a collimator lens C and divided by means of the screen S into two parts. While one part is reflected several time by the mirrors M_1 and M_2, the other travels back and forth the distance between the mirrors M_1 and M_3 once only. The beams of light meet in the focal plane of the observation telescope T where they bring a diffraction patch of light. If the lengths travelled by the beams are equal, the lines of diffraction interference I cover the image. In white light these lines will, as a result of the different diffractions of various wave lengths, disappear when the difference in the lengths of the light beams is more than 1.3 μ. Still higher accuracy is attainable if the interference lines are directed symmetrically into the centre of the patch of light. To obviate the difficult and time-wasting adjustment of the mirrors to positions in which interference is observable, a compensator consisting of two plane-parallel glass plates P is interposed in the paths of the beams immediately in front of the objective of the telescope. By turning either of these glass plates of the compensator the optical distance of the beam of light passing through it can be extended, and the interference lines thus found and directed into the center of the image. The difference in length can be computed from the reading of the angle of rotation. Thus, the comparator renders it possible to multiply a known length (the distance between the mirror surfaces) by an integral number. The first distance is determined by means of a one meter end-gauge of fused silica (Fig. 1), while the short distance between the mirror surface and the slightly rounded end of the end-gauge is measured by Newton-fringes. »

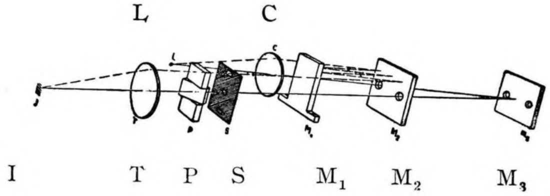

Fig. 2

A part of the interference comparator with the collimator lens and the mirrors at both ends of the quartz gauge meter.

The significance of this method has been realized by the highest authorities of international geodesy. The International Union of Geodesy and Geophysics at its Rome in September 1954 consequently passed the following resolution:

« The International Union of Geodesy and Geophysics resolves that these member-countries adopt the following program in so far as possible:

« ......

« (3) Establish a standard base-line in each country using the Väisälä method (or similar apparatus) for assuring a uniform scale in all networks and for calibrating invar tapes and geodimeters.

« ...... »

Similarly the U.N. Regional Cartographic Conference for Asia and the Far East in Mussoorie in February 1955 adopted the following resolution:

« The United Nations Regional Cartographic Conference considers that in view of the remarkably accurate results that are obtainable with the Väisälä Comparator of the Finnish Geodetic Institute, which has been calibrated against the international prototype of the International Bureau of Weights and Measures, and in view of the difficulties and uncertainties associated with the physical standards of length for calibration purposes, recommends to the Governments of the Asian countries that a few standard base-lines in this region should be established by the Väisälä method for assuring a uniform scale in all networks and for calibrating invar tapes and other equipment. »

For measuring the exact dimensions of the reference ellipsoid we need geodetic *yardsticks*, i.e. accurately measured arcs in different parts of the world, and the more of them we have and the longer they are the better. These yardsticks have been measured up to the present time exclusively by classic triangulation methods. On the basis of astronomical observations of latitude and longitude at or near the end points of these yardsticks we can locate them at approximately the right places on the earth's surface. This astro-geodetic method can, of course, be used on the continents, as for instance the important triangulation of the American Army Map Service (6), carried out in recent years under the supervision of M. Floyd Hough, shows. Thus the African arc measurement from Cairo to Cape Town, started 75 years ago, was finished in 1954.

This classic method is, however, rather slow, and in the oceans it fails completely. Furthermore, it is not possible to locate the end points of the triangulation chains with sufficient accuracy, as this method cannot supply the correct deflections of the vertical at these points.

To *connect the continents* with one another we need the air-borne electronic (Shoran, Hiran), celestial and gravimetric methods, made available to science a few decades ago.

Shoran, and its recent improvement Hiran, have been dealt with in this periodical. A few words concerning it will therefore be sufficient.

For *Shoran* measurements we need two ground stations A and B and one (or more) air-borne stations, which carry the transmitting equipment for broadcasting the electromagnetic impulses. The airplane flies in loops approximately over the middle of line AB. The Shoran readings give the time which elapses when the impulses travel from the airplane to the ground station and back. From this time the distance A-airplane can be computed, as well as in a similar way the distance B-airplane. The sum of these distances is different depending on how far the airplane is from the line AB. Every reading gives a single value for the distance A-airplane-B. These values will be reduced to the minimum distance, which supplies the distance between the ground stations.

The principles of Shoran were discovered in America by Seeley in 1938. After the war this system was developed particularly by Captain Aslakson of the U.S. Coast and Geodetic Survey. According to his report to the Rome assembly of the International Association of Geodesy last September, Shoran and Hiran were used in the Greater Antilles in 1950-1952, as well as in the Lesser Antilles in 1954; moreover, they were used to connect Norway and Scotland in 1953, Crete and Africa in 1953, and Scotland and Iceland in 1954. The advantage of this method is that very long distances—even up to 880 km.—can be measured and the observation error will not increase with the distance as it does in triangulation methods. The relative accuracy of 1 : 120 000 has already been reached so that this method can replace triangulation particularly in areas where the classic triangulation is difficult to carry out.

According to Aslakson (7) the Hiran method makes intercontinental connections possible.

There are three different celestial methods: the solar eclipse method, the occultation method and the moon camera method, which all use the moon as one triangulation point. The *solar eclipse method* was developed for measuring distances across the oceans by Ilmari Bonsdorff, the late Director of the Finnish Geodetic Institute (8), a decade ago. When we measure by the sound film technique the exact moment when the totality of the eclipse begins and ends at two stations, each on different continents, we are able to compute with a relatively high degree of accuracy the distance across the ocean between these continents. For intercontinental ties this method was used in 1945, 1947, 1948 and 1954. The eclipses were unfortunately so capricious—cloudy skies largely hindered observations—that only once was it possible to compute by this method the distance across the Atlantic. This was done by T.J. Kukkamäki (9) on the basis of the observations of the Finnish expedition during the total solar eclipse of 1947 in Brazil and Africa. The accuracy is of the order of 100 m.

In the *occultation method* the moments when a star disappears behind and emerges from the limb of the moon are observed. When the distance of the moon is known, the distance between the observation points can be computed as in the solar eclipse method. The U.S. Army Map Service has used this method, although only a few results have so far been published.

In the *moon camera method* of Wm. Markovitz (10) (U.S. Naval Observatory), we have to photograph from the observatories of different continents the moon and neighbouring stars. The moon camera is so constructed that the star and the moon's limb are stationary during the observation and their image distinct. When we measure the small angular distances of the distinct points of the moon's limb from the neigbouring stars, the direction from the observation point to the moon, as well as the distance between these points, can be relatively easily computed. This method supplies the geocentric radius of the earth at the observation point, according to Markovitz, with an accuracy of about 40 m. The variation of these geocentric radii with the latitude and longitude of the observation points can give the general shape of the earth with the accuracy stated if we have a sufficient amount of observation points. International cooperation between about 20 observatories on different continents is to be established.

The *equatorial radius* values computed by Clarke (1866) and A. Bonsdorff during the last century, and by Helmert, Hayford, Heiskanen, Krassowsky and Jeffreys in this century (6378 km. + 206 m.; 444 m.; 200 m., 388 m., 397 m., 245 m. and 099 m.), disagree with one another so much that the largest deviation is 345 m. Although the mean error of the a-value computed from these determinations is only ±48 m., it is in any case larger than the accuracy of observations and computation methods would allow.

These differences are obviously caused by the fact that the *geoid* is an *irregular surface:* in the area of some arc measurements, under the reference ellipsoid to be computed; in the area of some other arc measurements, above it. Depending on the arcs that have been used and on the way the observations have been reduced, we must therefore get different results. Fig. 3 explains this (11).

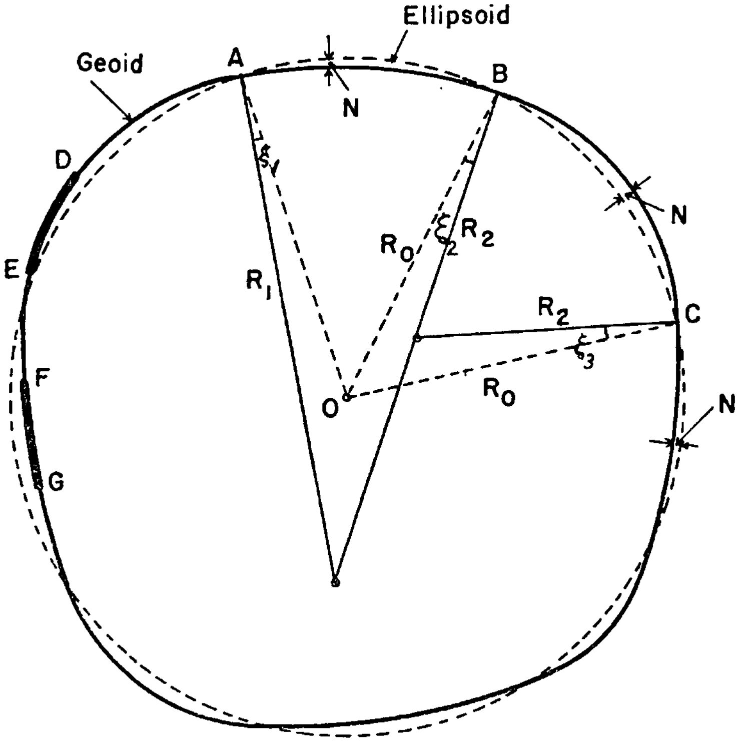

Fig. 3

Because of the undulations N and the deflections of the vertical ξ_1, ξ_2, ξ_3 we easily get wrong dimensions for the earth ellipsoid. Arc AB gives too large a radius R_1, arc BC too small a radius R_2, while the correct value is R_0, Similary the arc FG gives too large, and arc ED too small a value for R. Through isostatic reduction we can « smooth » the undulations N and the deflections of the vertical ξ and get better results. The effect of ξ_1, ξ_2, and ξ_3, however, can be eliminated completely only if we know them. The gravimetric method supplies all these quantities ξ.

Between points A and B, the geoid is under the ellipsoid; between points B and C, above it. The measured arc AB gives therefore for the earth's radius too large a radius R_1, while the arc BC gives too small a value R_2; the correct value would be R_0. In the figure the undulations N of the geoid and the deflections of the vertical ξ_1, ξ_2 and ξ_3 have been drawn, both exaggerated nearly half a million times. In order to get better results we have to « smooth » the undulations N. The isostatic reduction, which Hayford used nearly half a century ago, and Bowie, Heiskanen, Vening Meinesz, Lejay and others later on, does this. It cannot however, eliminate the whole effect of the undulations N; this can be done only by the gravimetric method, which yields the absolute deflections of the vertical.

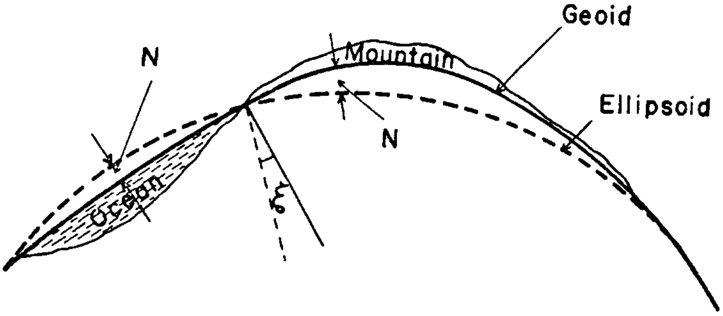

Fig. 4

Mass surplus of the mountains and mass deficiency of the oceans cause undulations N of the geoid and deflections of the vertical ξ. In the figure these quantites have been exaggerated about 5 000 times.

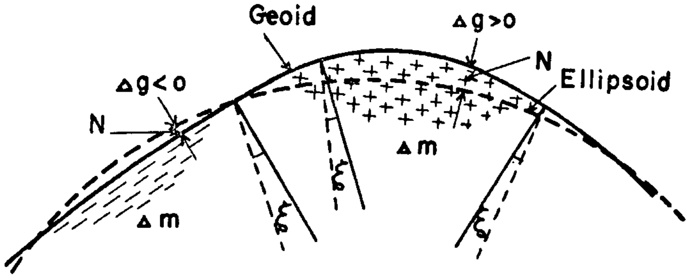

Fig. 5

The invisible mass anomalies Δ_m (surplus or deficiencies) bring about the gravity anomalies Δ_g, undulations N of the geoid and deflections of the vertical ξ. Δ_g can be observed, Nand ξ can be computed. This is the fundamental idea of the world-wide gravity program of the Columbus Group.

Our figures 4 and 5 show why the geoid is irregular and why the deflections of the vertical and the undulation of the geoid exist. There must be some reason. This reason is the irregular mass anomalies, visible or invisible. They bring about the gravity anomalies Δ_g, undulations N of the geoid and deflections of the vertical components ξ and η. Δ_g can be observed, N, ξ and η can be computed.

It is easy to understand that the gravity value is too small (gravity anomaly negative) if the density of the material under the station is too small, while the gravity value is too large, if too heavy a mass exists close to the station.

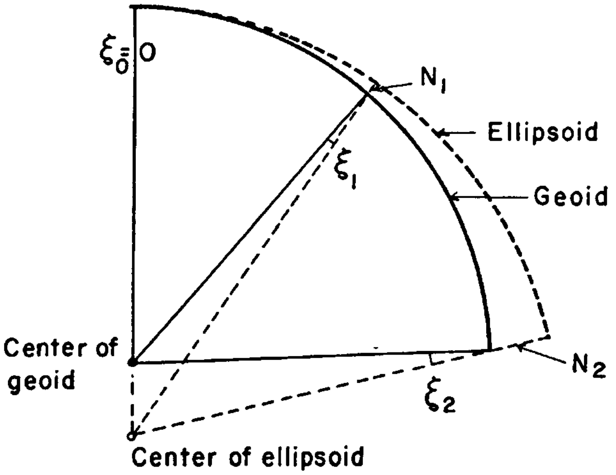

Fig. 6

Limitations of the arc measuring method. A wrong reference ellipsoid (in this case too large) causes systematic undulations N_1, N_2 of the geoid and systematic deflections of the vertical ξ_1 and ξ_2 which are not real.

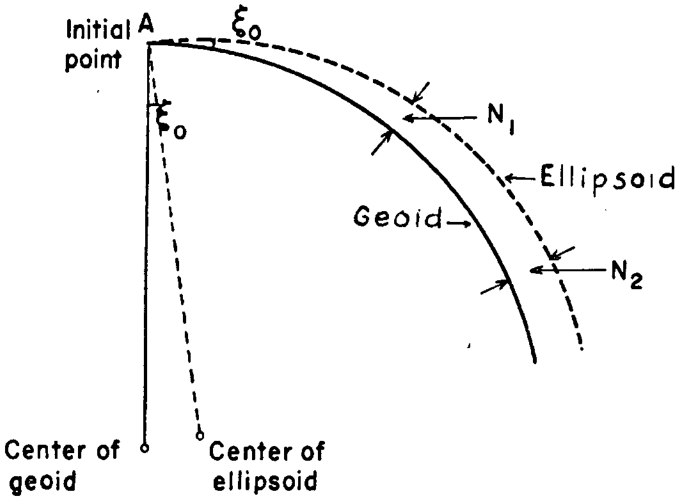

Fig. 7

The unknown deflection of the vertical ξ_0 at the initial point A of the geodetic system causes systematic undulations N of the geoid which in fact do not exist.

Fig. 6 and 7 explain the limitations of the arc measurement method. The former figure shows that we get fictitious undulations of the geoid if we do not know the exact dimensions of the reference ellipsoid. Similarly (Fig. 7) we get fictitious undulations of the geoid which are caused by the wrong deflection of the vertical used at the initial point A of the geodetic system. Therefore it is of basic significance to know the real deflections at the initial point of all geodetic systems. Without them we cannot convert these systems to a single system.

When using the measured arcs for checking the dimensions of the reference ellipsoid we must correctly locate these yardsticks on this ellipsoid. In other words we must determine accurately the central angle of the ellipsoid corresponding to the measured arcs. Astronomical observations are not sufficient. We must know the absolute deflections of the vertical at the end point of the measured arc in order to convert this arc to the ellipsoid. If we know, for instance, the deflections of the vertical ξ_1, ξ_2 and ξ_3 at point A, B and C of our figure 3, we get from the measured arcs AB and BC the correct radius of curvature R_0 instead of the incorrect values R_1 and R_2.

When no better method has been available, geodesists, in checking the dimensions of the reference ellipsoid on the basis of arc measurements, have so far used the criterium $\Sigma\xi^2 = \text{min}$, or $\Sigma\eta^2 = \text{min}$, or $\Sigma(\xi^2 + \eta^2) = \text{min}$, depending on whether the arc has been measured in the direction of the meridian, or perpendicular to it, or in some other direction. This method is quite good if the measured arcs are very long, for instance of the order of the length of the earth's radius. However, if they are shorter, or if we have covered by astro-geodetic points an area where the real deflections of the vertical are systematically either negative or positive, we may obtain an entirely incorrect result. For instance in Europe the criterium $\Sigma(\xi^2 + \eta^2) = \text{min}$ will not do because the average value of ξ in large area is of the order of $+5''$ and the average η of the order of $+2''$ to $+3''$.

The gravimetrical method helps us to increase substantially the accuracy of the dimensions of the reference ellipsoid (12). We need only make very accurate astronomical observations at or in the neighbourhood of the end points of the measured arc. In addition we carry out good local gravity surveys around these points. If we then have available the general gravity survey at least up to 1000-2000 km. from these points, and some knowledge of the gravity field still farther from them, it is easy to compute gravimetrically the deflections of the vertical components ξ_g and η_g at both end points of the arc. On the other hand we compute the astro-geodetic deflection of the vertical components ξ_a and η_a at the other end point of the arc, having chosen some arbitrary values —for instance zero—at the initial point. The quantities ξ_a, η_a depend on the reference ellipsoid used, while ξ_g, η_g are nearly independent of it. If now the differences $(\xi_g - \xi_a)$ and $(\eta_g - \eta_a)$ at both end points of the arc are nearly equal,. the reference ellipsoid is good. If there is difference, say, of $10''$, we have to correct the a-value of the reference ellipsoid so that this difference disappears.

Needless to say, every measured arc, at the end points of which astronomical observations and a good local gravity survey have been carried out, can be used for checking the dimensions of the earth. By the least-square method the different measured arcs of this kind will together supply the best possible correction to the equator value of the reference ellipsoid used.

This method of computing the correction to the a-value might be simplest, since it requires no astronomical observations except at or near the end points of the measured arcs. It is of course desirable to have astronomical observations also at a few points between the end points of the measured arc.

In the case of large areas, such as those in the United States of America, in Europe and so on, that have been covered by astro-geodetic points instead of by measured arcs, it is best to compute the gravimetrical N, ξ and η only at the initial point and then at several points in different directions from it and close to the boundary of the surveyed area, as Vening Meinesz suggests. If again the differences of the gravimetric and astronomical deflections of the vertical are at all these points approximately equal, the reference ellipsoid used does not need any correction. If, however, systematic differences appear, we have to correct the dimensions so that these systematic differences disappear.

The gravimetric method is superior also in that it is not necessary to use the astronomical triangulation points themselves as the computation points (13) because they are generally located on hill tops or in rugged mountain areas where the reduction of the astronomical and gravimetrical deflection of the vertical is rather complicated. Instead we choose any point of the lowland where the topography and gravity fields are smooth. At such point, we make very accurate astronomical observations, and in the neighbourhood of it we make a good local gravity survey, if such does not already exist, and join this point to the close triangulation point either by local triangulation or by traverse. For instance, the end point of the base lines would be well adapted to this purpose.

To obtain the gravimetric undulations of the geoid and deflections of the vertical, we must:

(1) Connect the gravity base stations of different countries and ocean islands as accurately as possible to the same world gravimetric system. Whether this system is based on the absolute gravity at Potsdam, Paris, Teddington or Washington, does not matter. What we must have is a single system. For that purpose the International Geodetic Association at its Meeting in Rome in September 1954 organized international cooperation for the measurement of accurate gravity base stations of the first order and gravimetric calibration lines for calibrating the gravimeters. Mr. Martin (14) from France, and Mr. Morelli (15) from Italy, have made connections between the European and African gravity base stations. Of the utmost importance, however, is the gravity survey of the Wisconsin University Group under the leadership of G.P. Woollard (16) which has connected to the same system more than 1500 gravity base stations of different continents and ocean islands.

(2) Carry out new gravity measurements at sea. Thanks to the Vening Meinesz pendulum (17) apparatus and to the different types of underwater gravimeters it is also possible to measure gravity in the oceans and in such a way as to cover 70 per cent of the earth's surface by gravity measurements. Vening Meinesz himself, as well as institutions in France, Italy, Japan and Great Britain, but particularly the Columbia University Group under the leadership of Maurice Ewing and Lamar Worzel, have together measured, using submarines, about 4000 gravity points in the oceans.

Underwater gravimeters are suitable for the gravity survey of shallow waters up to depths of 200 m. They have already been used by the Gulf Oil Co. for more than 10 years (18). Several institutions in various countries are now interested in these measurements. We have full reason to hope that during the Geophysical Year 1957-1958 international cooperation can be established for the gravity survey of shallow waters and of the oceans. The difficulty of having submarines available for ocean measurements might however hinder the gravity survey of the open oceans.

One point still requires to be mentioned. There has been some doubt as to the speed, accuracy and reliability of gravity measurements at sea. It has been stated that only about 10 points can be occupied during one month, that the ocean measurements have systematic errors of many milligals, and cannot therefore be used for any geodetic purposes.

The experience of Vening Meinesz and of the Columbia University Group however shows that over 100 points can be occupied in one month, and that the average speed is 3-5 points a day. On the other hand the comparison of the continent measurements and ocean measurements close to the coast lines indicates (Worzel) (19) that no systematic difference between the land and ocean observations exist. When the same points were occupied after an interval of several years, the measurements agreed to within an accuracy of a few milligals. Moreover comparison of the astro-geodetic and gravimetric deflections of the vertical in the Mediterranean and around it, made for the Army Map Service by Miss Irene Fisher (20), showed that both methods lead to similar results and therefore indicate that gravity measurements in the Mediterranean are reliable.

(3) Make gravity surveys on continents where large areas are still lacking in gravity station nets. The modern gravimeters, which allow gravity readings to be obtained in three minutes with 20 to 50 times more accuracy than with pendulum equipment in two days, will enable the gravity survey of enormous areas to be carried out within a few years. The elevations of the observation points might possibly be obtained by radio-altimeters and statoscope readings of the elevation of the airplane when taking photogrammetric pictures in regions where levelling lines are not yet available. On the other hand gravity surveys along railways and precisely levelled highways supplies in many countries a reasonable gravity station net. The usual altimeters are able to supply the elevation of gravity points not very far from the levelling points, lakes or ocean coasts. This method has been used for instance in Finland with great success.

(4) Finally reduce the observed gravity values to sea level. The isostatic reduction method according to the Airy-Heiskanen system (normal thickness T of the earth's crust = 30 km., density difference between the underlayer and the earth's crust = 0.6) seems to fit best. The isostatic method supplies gravity anomalies which do not largely depend on the elevation of the topography or on ocean depths. In other words, the isostatic gravity anomalies are more representative than any other types of anomalies. A drawback of the isostatic method is the tedious work the isostatic reduction of the enormous gravity material requires, as well as the fact that we have to consider also the indirect effect of the reduction, first in obtaining the gravity values valid at the co-geoid and then in converting the co-geoid to the actual geoid.

Of basic significance is the error of representation studied particularly by J. de Graaff Hunter (21) (1934) and, on the basis of much more extensive material, by R.A. Hirvonen (22) (1953). The result of these investigations is the following: If one gravity station has to represent a square or more properly a spherical trapezoid with the side lengths 5', 10', 20', 30', 40', 1°, 2°, 3°, the probable error of 2.0, 3.0, 4.5, 6.0, 7.5, 10.5, 13.2 and 14.7 milligals will appear. According to Hirvonen the last-mentioned error of representation, 14.7 milligals, may be close to the maximum. The average gravity anomaly of still larger areas, decreases with the increase in area. For instance, the gravity anomaly of a hemisphere is very close to zero, if the gravity formula used has been derived on the basis of a sufficient number of gravity stations in the different parts of the other hemisphere.

Consequently it is wise to use one gravity station as representative of the areas not exceeding $3^\circ \times 3^\circ$. Beyond this boundary it is best to use, in the unsurveyed areas, the isostatic gravity anomaly zero.

In world-wide studies it is best to use the reference ellipsoid and gravity formula which are in harmony with one another. The International ellipsoid: $a = 6378388$ m., $\alpha = 1/297.0$ and the International gravity formula

$$\gamma = 987\,0490\ [1 + 0.00522884 \sin^2 \varphi - 0.0000059 \sin^2 2\varphi]\ \mathrm{cm/sec^2}$$

satisfy this condition. In fact, the gravity formula is referred to the ellipsoid and not to the spheroid, and the α-value, 1/297.0 of the International ellipsoid can be derived from the coefficient $\beta = 0052884$ of the International gravity formula with the help of Clairaut's formula and vice versa.

This is an important statement. If we change one of these crucial parameters, α or β, we also have to change the other. Therefore we must be conservative in changing any of these parameters.

The first gravimetrical computations of the continental undulations of the geoid on the basis of Stokes' formula were made at my suggestion by my assistant and colleague R.A. Hirvonen (23) in Finland in 1934, and by my student, the late Dr. L. Tanni (24), in 1948 on the basis of much better gravity material. Consequently his geoid is more accurate. In fact, the error of his N-values is frequently smaller than 5 m., and in general less than 10 m.

The first gravimetric deflections of the vertical were computed, using the Vening Meinesz formulae (25), by the Russian I. Kasansky (26) in 1935, in his investigation of the Moscow area.

These pioneer works show that we can already use the gravimetric method successfully, particularly as we now have four times as many gravity measurements at sea as a decade ago, when Tanni made his computations. In addition, such countries as Japan, New Zealand, Australia, South Africa, Madagascar, and considerable areas of Arabia, Tunisia and Morocco have now been covered by a rather good gravity station net, while 10 years ago they only had scattered gravity stations. In Argentina and Canada gravity survey has been very active and successful.

Later the gravimetrical deflections of the vertical were computed by Honkasalo (13) in Finland, by Wideland (13) in Sweden, by Baglietto (13) in Argentina, by Miss Irene Fisher (20) in the area of Mediterranean and by D. Rice (27) of the U.S. Coast and Geodetic Survey for the middle part of the U.S.A., as well as in the West Indies area (28). As to the gravimetric undulations of geoid the study of Rice in the West Indies is most important. In all these studies only the relative values of N, ξ and η, have been considered as the effect of the gravity field of the earth beyond the distance of 1000 km. from the computation points has been, owing to the lack of gravity material, either neglected or estimated only roughly.

Theoretical studies concerning the gravimetric method, or physical geodesy, as we now call it, has mainly been carried out at the Isostatic Institute in Helsinki, at the International Gravity Center in Paris, in England, in the Netherlands, and in recent years particularly in the Mapping and Charting Research Laboratory of the Ohio State University.

As to the supercontrol points (12), these are necessary to check the accuracy of long-range triangulation, particularly if carried out under difficult conditions, such as in the heart of Africa, or in the high mountains of the North and South American Cordilleras, where accuracy cannot be too high. These control points can be obtained easily by the astro-gravimetric method. The astronomical observations supply the direction of the plumbline, while gravimetric deflections of the vertical relate to the effect of the tilting between the geoid and the reference ellipsoid. As the formulae

$$\begin{aligned} \varphi &= \varphi' - \xi_g \\ \lambda &- \lambda' - \eta_g \sec \varphi \\ A &= A' - \eta_g \operatorname{tg} \varphi \end{aligned}$$

show, the gravimetric deflections of the vertical convert the astronomical latitude, longitude and azimuth (φ', λ', A') from the geoid to the ellipsoid, and give co-ordinates (φ, λ, A) which are equivalent to the geodetic co-ordinates. As the accuracy of the gravimetric deflections of the vertical is of the order of $1''$, and of the astronomical observations of the order of $0''.3$, the error of the astro-gravimetric supercontrol point is of the order of 30 m., which is less than the error of long-range triangulation.

The equations mentioned above also supply, of course, the control points (without triangulation) for mapping work where 30 m do not correspond to more than the drafting error of the map, i.e., for maps on the scale of 1 : 100 000 and less.

The writer is particularly glad to have had the opportunity (29) to establish in 1950, and to conduct in the Mapping and Charting Research Laboratory of Ohio State University, the world-wide gravity program sponsored by the Cambridge Research Center of the U.S. Army Air Force. The main objectives of this program were given at the Brussels Assembly of the IUGG (30). The writer's work « On the World Geodetic System » (31), published in 1951 in Finland, in 1952 in Columbus, Ohio, and in 1953 in this periodical, is a comprehensive study of the gravimetric method. No less than thirty countries, most of the leading oil companies and many prominent geodesists from all over the world are cooperating with us in solving the principal geodetic problems of the new era described above. This close cooperation is the best guarantee that the World Geodetic System will be established in a few years.

I take this opportunity to express my best gratitude to the scientists and organizations of different countries for their willing cooperation, as well as to the Cambridge Research Center of the American Air Force for its support, which made this world-wide gravity study possible and to the Research Foundation and the Mapping and Charting Research Laboratory of the Ohio State University.

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