NEW METHODS FOR COMPUTING HOMOFOCAL HYPERBOLIC GRIDS (FRENCH DECCA CHAIN) ON CHARTS IN CONFORMAL PROJECTION

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(From *Navigation*, French Institute of Navigation, Paris, Vol. II, No. 8, October 1954).

Introduction.

It is well-known that experiment with the Decca medium--range radio-navigational system has shown that it is possible to consider the position-line curves as meeting the standard definition of hyperbolae, i.e. the loci of points whose differences in distance from the two foci are constant, with reference in this connection, of course, to *geodetic* distances on the earth's surface.

Since the shape of the earth, according to results supplied by geodesy, may be assimilated to that of an ellipsoid of revolution whose parameters have now been ascertained with a satisfactory amount of accuracy, the problem is theoretically determined and may be treated by mathematics as soon as all the necessary numerical data have been established. The hyperbolae in the system may accordingly be plotted on the charts.

The actual testing of each Decca chain is then expected to supply the overall confirmation of these assumptions and of the plots, and to indicate residual corrections, (i.e. discrepancies between the theoretical and actual location of the position lines) which may appear locally following anomalies in wave propagation.

The plotting of homofocal hyperbolic systems on charts is a mathematical problem that is difficult to solve with absolute precision, and various methods have been proposed to this end. However, certain approximations may be considered that are compatible with the problem's physical features and may be advantageously used to simplify computations. Thus, in the case of very-short-range hyperbolic systems (<100 km.), it is possible to reduce the problem to the *plane*, and the plot of the hyperbolae may then be obtained by the following convenient standard parametric formulae :

$$X=(\cos x \text{ ch } y).\, f \qquad \text{with} \qquad \cos x=\frac{\rho - \rho'}{2f}$$

$$Y=(\sin x \text{ sh } y).\, f \qquad \text{with} \qquad \text{ch } y=\frac{\rho + \rho'}{2f}$$

ρ and ρ' : vector radii

$2f$: distance inter foci.

In the general case of systems used in air navigation, with ranges of several hundred (Decca) or several thousand (Loran) kilometres, the *indirect method* has mainly been used, in which the hyperbolic value ($\rho - \rho'$) for points on the geographic grid is computed exactly (i.e. round latitude and longitude values), the latter being taken in sufficient numbers to enable interpolation over the meridians and parallels of the intersecting hyperbolae having a round parametric value.

The computations relating to the points on the grid can be handled with absolute precision by using one of the many methods for computing long geodetic lines. The legitimacy of later interpolations may be evaluated on the basis of standard techniques in the use of tables by interpolation; if need be, additional points on the grid may be computed. Properly developed, the method is satisfactory from the practical aspect, as it is adapted to standard types of computation. It is extremely efficient when groups of operators fully conversant with the method are available (it should be stressed in passing that such computations involve series of largely varied operations which are difficult to transpose into the realm of automatic calculations by card-programmed electronic computers).

When first faced with this problem, before the French Decca Chain went into operation (1952-1953), the Technical Bureau of the Institute's Geodetic Section was fairly well acquainted with the performance of card-programmed electronic computers, and as a result of personal investigation, we moreover knew it as possible to obtain the rectangular coordinates of hyperbolae on conformal projection charts directly through the use of formulae of a standard type. We thus decided the proper time had arrived to test a new method, and that useful progress would result owing to the increasing substitution of automatic computing by card-programmed machines for manual computing.

We therefore propose to submit herewith, if not a complete technical report of this work, which would be of only slight interest to readers of this publication, at any rate a description of the basic features of the method used, and of the main experimental results observed during operations.

General Description

The main feature of the method is that the *plane rectangular coordinates* are determined directly, by projection, of any points on the given hyperbolae, therefore involving a direct plot of the hyperbolae on the conformal charts.

An additional distinctive feature of the method from the practical aspect is the use of *algebraic* working formulae, which are well adapted to automatic card-programmed computers. This refinement is made possible since the rectangular coordinates of a plane conformal projection are determined for the computed points instead of latitude and longitude.

In order that the subject may be accurately defined, it should first be made clear that an *auxiliary sphere* is involved, closely resembling the ellipsoid of revolution for the area concerned: between the sphere and a limited area of the ellipsoid (in the case of Decca), there is a quasi-isometric relationship (in which length measurements are almost exactly retained), so that the hyperbolae on the sphere may be regarded as reflecting the hyperbolae on the ellipsoid. The slight discrepancy that exists may be evaluated in precise terms by means of the scale coefficient, and will be found to be negligible in so far as Decca is concerned.

Having thus reduced the problem to the sphere, we can henceforth rely on consideration of *isothermic nets* of coordinate curves. The properties of such systems are well-known: they consist of sets of orthogonal curves so combined as to create a pattern of infinitesimal squares by intersection. By establishing a relationship between such an isothermic net and lines drawn on a plane with x and y

as their Cartesian coordinates, we obtain a conformal representation of the surface (in this case a sphere) on the plane, and vice versa.

Let us then take two separate orthogonal isothermic nets plotted on an identical surface, and let x, y, Y, X be the coordinates of an identical point in the two systems. The two charts obtained in the two planes xy, XY are both plane conformal representations of an identical object, and the relationship $xy \leftrightarrows XY$ is thus a conformal representation of the plane on itself, so that an analytical relationship $U = f(z)$ necessarily exist between the complex variables $z = x + iy$ and $U = X + iY$ respectively. The latter may be represented by a series development of the type:

$$U = U_0 + \alpha z + \beta\ z^2 + \gamma\ z^3 + \ldots,$$

which may be directly applied to a card-programmed electronic computer, even if of large degree. The operations required to form successive terms such as $x^2 - y^2$, $2\ xy$, $x^3 - 3\ xy^2$, $3\ x^2y - y^3$, correspond to a constant algorism, i.e. the multiplication of two complex numbers, and the machinery is so designed that it can repeat algorisms a great number of times within a very brief interval.

To sum up, the method used by us consisted in taking on the one hand the isothermic pattern based on the spherical hyperbolae, and the isothermic pattern corresponding to the Lambert rectangular coordinates on the other; in determining the coefficients of the analytical relationship $U = f\ (z)$ between the coordinates of an identical point in the two systems; and in making use of this latter relationship, taking advantage of its algebraic character and of the possibilities offered in this connection by card-programmed electronic computers.

An initial step will therefore consist in determining the isothermic system based on the homofocal hyperbolae, and in making the transition from the ordinary geometrical coordinates (the sum and difference of vector radii) to the isothermic coordinates. A standard formula supplies the answer, which actually only exists for the sphere (and not for the ellipsoid in the general case).

A second stage consists in deriving the relationship between such coordinates and the chart coordinates, and this may be accomplished empirically by taking an adequate number of coordinate pairs $(x_i y_i X_i Y_i)$ and by determining the coefficients of the relationship $U = f\ (z)$ by the system of linear equations corresponding to these control points.

We shall now examine the particular features of these various partial problems.

Use of Intermediate Sphere

An extremely adequate spherical representation, described as the Gauss spherical projection, transposes the ellipsoidal figures to the sphere of mean curvature at the midpoint of the projection. This representation possesses the following basic properties:

— An extremely low scale coefficient (under $0.2.10^{-5}$, within a 600-km radius around the midpoint);

— Simple formulae in the transition from the sphere to the ellipsoid. These are of the following type:

$$\Delta\varphi = f\ (\Delta\varphi') \qquad [\text{defined by } \Delta\mathcal{L}, = k\Delta\,.\mathcal{L}']$$

$$\Delta\lambda = k\Delta\lambda',$$

where φ, φ'; λ, λ' designate latitudes and longitudes on the sphere and ellipsoid respectively, $\mathcal{L}$ and $\mathcal{L}'$ meridional parts on these surfaces, k a parameter approaching unity $[1+e'^2 \cos^4 \varphi_0]^{\frac{1}{2}}$, and the symbol Δ represents a discrepancy in coordinates with reference to the midpoint (it should be noted that for position φ_0, λ_0, the corresponding position on the sphere is φ'_0 ($\neq \varphi_0$), λ_0).

The scale discrepancy of this spherical projection in the area of Decca pattern coverage is so slight that it may be disregarded altogether in view of the inherent inaccuracies of the Decca system. ± 0.02-lane is the estimated amount of stability for any given point in the most favourable areas of the Decca pattern, which corresponds to an evaluation of the difference (ρ — ρ') to within 7 m (Purple pattern), and therefore to a definitely higher order of magnitude than scale corrections (1).

For practical purposes, therefore, the problem of plotting hyperbolae on the Gaussian sphere may be approached by making use of the distances defining the points just as they stand. (There are three distances: the focal distance $2f$; ρ, and ρ', which are the distances to either focus). After the spherical coordinates of the points have been obtained, these are transformed on the reference ellipsoid, and thence on the plane of the chart projection used, the conformal nature of the representation being retained throughout.

The possibility of making use of the properties of Gauss' representation in order to reduce the problem to the sphere has been recognized and applied earlier by other authorities (S. Ballarin, Ref. [6] and K. Ansorge, Ref. [7]).

Isothermic System of Curvilinear Coordinates on the Sphere, including Hyperbolae as a Family of Coordinate Lines

Reduced to the sphere, the problem of plotting the hyperbolae may, as we mentioned above, be dealt with by ordinary spherical trigonometry (2).

In the different method selected by us, the isothermic and orthogonal network based on the hyperbolae of the homofocal pattern is needed. The orthogonal curves to such hyperbolae are of course the spherical ellipses having as their foci F and F' and which may be located by their parameter $2u = \rho + \rho'$. After ascertainning and defining the orthogonal curvilinear net we require, we must then define thereon a system of coordinates possessing the required isothermic property, i.e. one possessing the same structure as a plane Cartesian grid.

A general approach to the problem of isothermic curvilinear coordinates on a surface leads to certain required adequate conditions of accomplishment, which are capable of fulfilment as regards hyperbolic networks on the sphere, but generally not as regards the ellipsoid of revolution. We are compelled as a result here to use the sphere as the intermediate surface.

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(1) A different line of reasoning may be followed and the scale discrepancy in the Gauss ellipsoid/sphere projection be compared with the inherent accuracy with regard to knowledge of the speed of Decca wave propagation, governing the transformation of phase-differences into distance-differences. The amount of uncertainty with respect to this speed is of a definitely higher order than the scale discrepancy.

(2) There is an interesting article on the direct plotting of spherical homofocal hyperbolae by P. Hugon, entitled: *Note on the rectilinear representation of spherical hyperbolae* (Service Central Hydrographique, Paris, 1951).

The particular investigation with regard to the net of isothermic homofocal coordinates on the sphere was made the subject of intensive research by two standard authorities, Peirce and Guyou, with the object of constructing charts allowing of such a coordinate system as a Cartesian reference system, resulting in the remarkable planispheres bearing their name supplying a doubly periodic representation of the earth's surface. The work of Peirce and Guyou is concerned with special homofocal systems (so-called « equilateral » systems) in which the focal length is equal to one-fourth of the terrestrial meridian; however, the same principle may be applied to any type of focal system, upon which special maps of the sphere might also be constructed.

Our method in plotting the hyperbolae on a conformal projection essentially consist in using the equations of the conformal relationship existing between the Peirce-Guyou type of chart in the Decca homofocal system and the proposed general Lambert chart.

The Peirce-Guyou formulae as set forth and proved in the *Traité des Projections* by Driencourt and Laborde therefore solve the problem arising with regard to the change in variables. They enable the transition to be made from the geometric parameters $u = 1/2\ (\rho + \rho')$ and $v = 1/2\ (\rho - \rho')$ to the isothermic parameters $p = p\ (u)$, $q = q\ (v)$ which are most conveniently related to the rectangular coordinates of the conformal projection.

Expressing and Computing the Homofocal Isothermic Coordinates p, q.

The Peirce and Guyou formulae are:

$$p = \int \frac{du}{\sqrt{\cos^2 f - \cos^2 u}}, \qquad q = \int \frac{dv}{\sqrt{\sin^2 f - \sin^2 v}}$$

supplying the following simple form with regard to the length factor of the sphere:

$$dS^2 = (dp^2 + dq^2) \sin \rho \sin \rho'.$$

For computation purposes, there is advantage in introducing an additional set of parameters L_E and M_E :

$$u \rightarrow L_E \rightarrow p, \qquad v \rightarrow M_E \rightarrow q,$$

which are of concrete geometric significance on the sphere (see Figure 1).

The transformation equations thus take on the following form:

$u \rightarrow p$:

$$\sin L_E = \frac{\cos f}{\cos u},$$

$$p = \int \frac{du}{\sqrt{\cos^2 f - \cos^2 u}} = \int \frac{dL_E}{\sqrt{1 - \cos^2 f \sin^2 L_E}}$$

$v \rightarrow q$:

$$\sin M_E = \frac{\sin v}{\sin f},$$

$$q = \int \frac{dv}{\sqrt{\sin^2 f - \sin^2 v}} = \int \frac{dM_E}{\sqrt{1 - \sin^2 f \sin^2 M_E}}$$

which is Legendre's standard form (1).

The transition from L_E to p and from M_E to q constitutes, from the numerical aspect, one of the main difficulties in the present method, and may even be described as a key operation of the process.

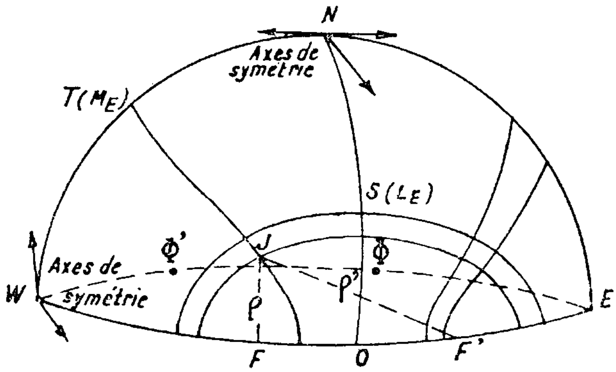

Fig. 1

Diagram of Spherical Focal System.

L_E represents the arc distance between apices of the ellipse and the focal axis.

Similarly, M_E represents the arc distance between the focal axis and apex T of the hyperbola (which is also an ellipse with its foci at F and φ).

The following terms should also be noted:

$\rho + \rho' = 2\,u, \quad \rho - \rho' = 2\,v.$

Two separate methods are followed, owing to the difference in the orders of magnitude $\sin^2 f \neq \cos^2 f$:

(a) In the case of the integral q (M_E), the smallness of the « modulus » (approximately 0.01 for the French Decca chains) suggests a convergent series development of the type (by putting $\sin^2 f = k^2$):

$$q = M_E + \frac{1}{2}k^2 \int \sin^2 M_E \, dM_E + \frac{1}{2} \cdot \frac{3}{4} k^4 \int \sin^4 M_E \, dM_E ,$$

where the development may be limited to the first corrective term (in the case of Decca) and use may be made of Wallis' Integral Tables as computed at the National Geographic Institute (2). Computation is of a standard simple type, and it may be pointed out that the maximum value of the corrective term, where $M_E = 100^G$, is 50 centesimal minutes for patterns in the French Chain.

(1) Legendre put :

$$F(k, \varphi) = \int \frac{d\varphi}{\sqrt{1 - k^2 \sin^2 \varphi}} \quad \text{where } k^2 = m^2 : \text{« modulus » of elliptic integral.}$$

(2) *Note sur le calcul des Grandes Géodésiques, avec XI Tables annexes,* by J.J. Levallois and M. Dupuy, National Geographic Institute, 1952.

(b) On the other hand, the integral $p\,(L_E)$ has a modulus in the immediate neighbourhood of one, and we are therefore faced with the problem of calculating the most difficult type of elliptic integral. Legendre's Tables cannot be used, since interpolation is impossible in the region concerned (i.e. moduli close to unity). Recourse is had to the ingenious transformation originated by Landen, i.e. a special change of variable simultaneously modifying the upper limit L_E and modulus m of the elliptic integral (see formulae in Appendix). The transformation can be so dealt with as to raise the modulus to the value of one, in which case the elliptic integral becomes identified with the standard « meridional part »: $\mathcal{L}$

$$\mathcal{L} = \int^{\theta} \frac{d\theta}{\sqrt{1 - \sin^2\theta}} = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$$

which is easy to compute directly.

In the case of patterns in the French Chain, two successive Landen transformations enable the transition to be made from the entry L_E to the entry argument L_2 for the meridional part.

To obtain p corresponding to a given L_E by this method requires about fifteen simple numerical operations.

Use of Previously Obtained p, q Coordinates. Obtaining and Use of Working Formulae.

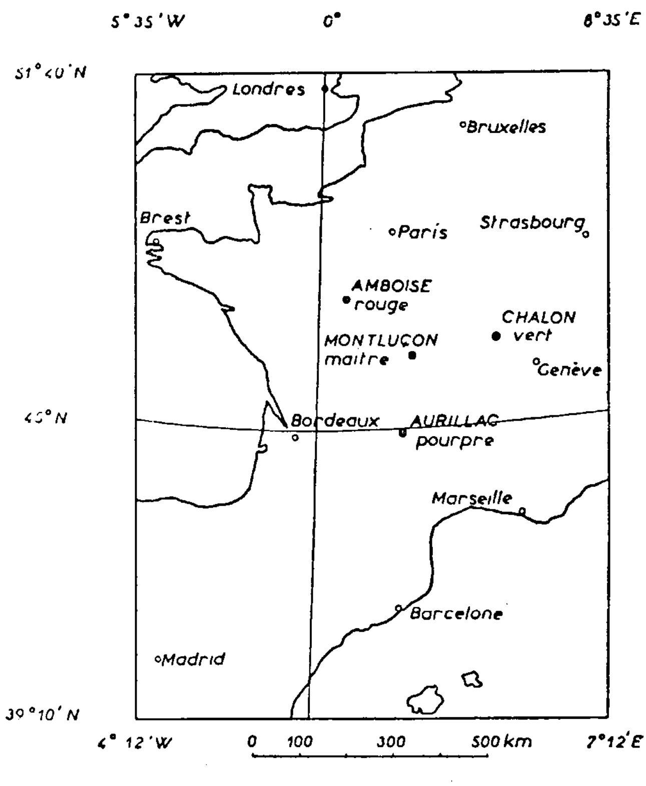

Fig. 2
French Decca Chain Array on 1:10⁶ Lambert Map of National Geographic Institute

We have now reached a stage where, with reference to a point known by its distances ρ, ρ' to the foci (or by the equivalent parameters u, v), it is possible to relate new parameters p and q included within a system presenting a known type of analytical relationship with the plane rectangular coordinates of the conformal chart.

The coefficients of this analytical relationship may be determined as soon as the corresponding coordinates X, Y and p, q are known for a sufficient number of points.

The desired indications as to the number of points required and their arrangement in the plane are supplied by theory (1). The preceding indications have moreover shown how operations should be carried out in practice for each of the selected points. Such points may be taken on fixed hyperbolae (hence be defined by u, v; whence p and q), their positions be calculated by spherical trigonometry, and their X, Y chart rectangular coordinates derived. Conversely, points may be taken as referred to X and Y, and ρ, ρ', be derived, followed by u and v, and finally p, q.

Thus the pair $(p_i q_i \rightarrow X_i Y_i)$ which will determine the coefficients of the « working formula » are obtained.

At the stage where the working formula is to be applied, a point-by-point plot of certain hyperbolae known by their parameter v is desired. It will accordingly be necessary first to compute the values of q corresponding to such values of v. Values of p will moreover so be selected as to ensure the appropriate distribution of control points on each individual hyperbola (see Fig. 3).

A network of values p and q corresponding to points located on the required hyperbolae is thus obtained, where the latter intersect with certain arbitrarily selected ellipses. Applying the working formulae to this set of values, we get the X and Y rectangular coordinates of these points on the chart.

Checks

As the working formula is determined from a certain number of control points, it has the approximate character of interpolation formulae, and the accuracy of the positions obtained should be verified by careful checking. Such checks are made by computing the exact distances to the foci according to the rectangular coordinates of the points on the chart, whence the *actual* distance-differences $2v = \rho - \rho'$ are obtained, which should be compared with the *theoretical* $2v$ value of the hyperbola. The XY position is considered to be acceptable if Δv (the theoretical v minus the actual v) is below a certain limit (defined as 10 m.), consistent with the known physical accuracy of the Decca system.

Practical Application of Method in Plotting French Decca Chain on 1:1 000 000-Scale Map on the Lambert Projection of National Geographic Institute

Having described the general characteristics of the method adopted, we shall now supply various specific data with respect to the French Decca Chain and related computations effected in 1953 at the National Geographic Institute's Technical Bureau.

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(1) The theoretical study of this relationship as applying to the general problem of the spherical hyperbolic net would not come within the scope of this article. An approximate idea thereof may be had, however, by considering the *plane* homofocal hyperbolic net, which, using the same symbols, is written :

$$(X + iY) = \sin(p + iq)$$

(rect. plane coord.) (homofocal isothermic coord.)

The French Chain (see Fig. 2) consists of three patterns, defined by four transmitters, as follows:

Common Master	Patterns	Focal distance (in km)
Montluçon	Green pattern	179.5
	Red pattern	174.1
	Purple pattern	157.2

The effective range of the system is evaluated at approximately 500 km., thus supplying positions that can be used for maritime navigation purposes near the coast. Inherent position-fixing accuracy under favourable conditions, on the basis of cumulative tests, is reported as 0.02 (one-fiftieth) of a mean lane, or approximately 10 m.

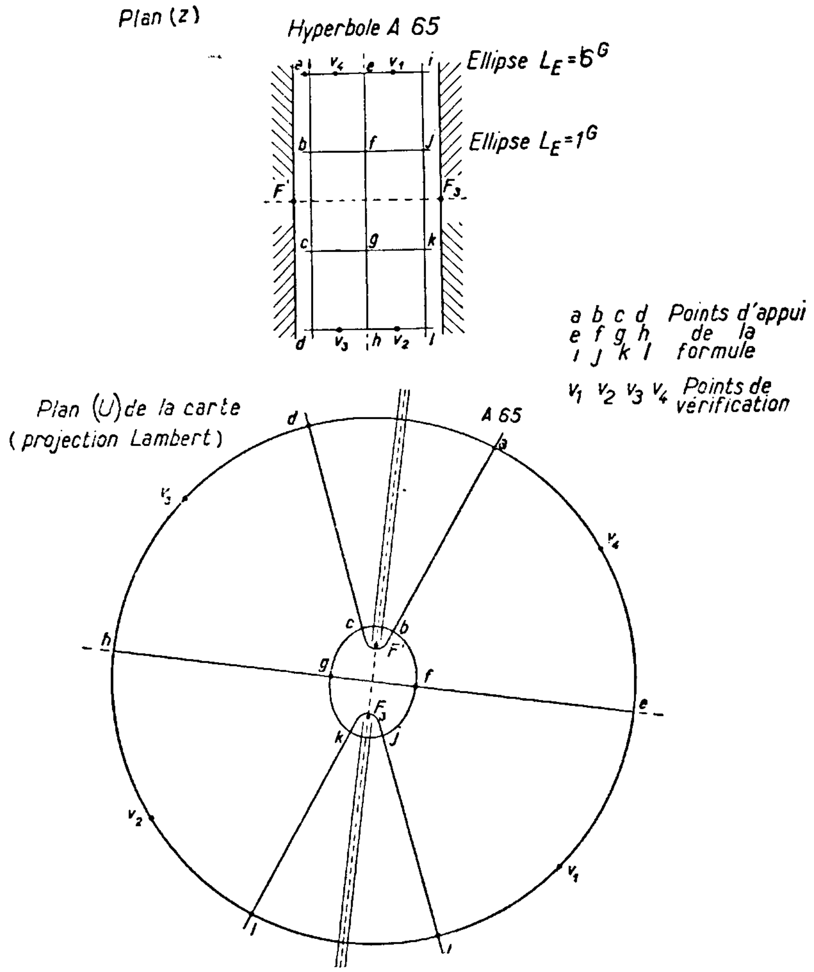

Fig. 3
Arrangement of Control Points for Purple Pattern Working Formula.
(It will be noted that Hyperbola A 65 is here considered as the useful limit in the pattern.)

The aeronautical chart to be completed consists of a special mosaic deriving from the National Geographic Institute's general map of Europe on the scale of 1 : 1 000 000 in Lambert projection: a conformal projection of the International Ellipsoid on the plane. The area to be covered extended northwards as far as London, and southwards to the Azores, thus requiring that curves be plotted over an 800 to 1,000-km stretch from the central area, and therefore over a far longer distance than the optimum effective range of the system.

On this chart the control points along the hyperbolae could be taken at 100-mm intervals (appr. 100 km. over the ground) in the outer areas, but in the central zones of the patterns it appeared necessary to increase the frequency of the control points by two or even by four. The block diagram in Fig. 4 shows the standard arrangement adopted. In the transverse sense, preliminary investigation showed that at the scale of 1 :1 000 000 it was possible to draw all the inserted curves by graphic interpolation, using only the *sector limit* curves (DEF...) as a basis, except in the marginal areas of each pattern, where a narrower interval must be taken in the direction of the base line region. In the first sector adjacent to the base line (from A_0 to B_0, and the symmetrical sector), each hyperbola should enven be computed *separately,* although no such attempt was made here for reasons of economy, and the hyperbolae were plotted only approximately as far as the two-thirds mark of the first sector. About seven hundred control points in each pattern thus had to be computed.

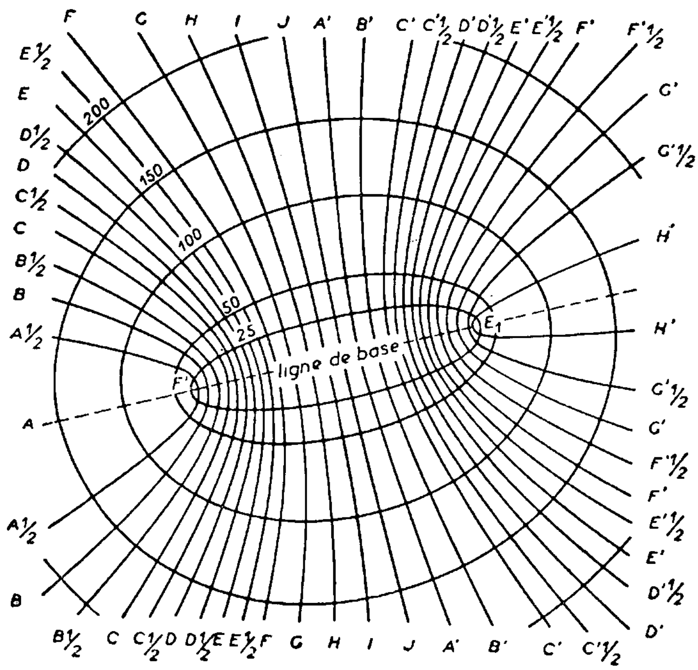

Fig. 4

Model of Control Points for Plotting of Hyperbolae
(Central Area)

Note. The control ellipses are spaced 100 km apart beyond the figure (300 and 400 km control ellipses).

The general method described below was more or less standard for all three patterns, the principal difference consisting in the control point arrangement used for the working formulae.

For purposes of readier numerical calculation, a set of thirteen points in the *Red* and *Green* patterns has been selected only along the axes of symmetry in the system; firstly, because the points could readily be positioned, owing to the fact that they were located on hyperbolae reduced to great circle arcs, and secondly, since the coefficients of the working formula could be obtained by means of simple combinations carried out with the coordinates of these basic points.

When put to the test this arrangement proved to be of insufficient scope with respect to the area to be covered, resulting in a decrease in the quality of checks in the inner and outer regions of the pattern. (In every case where the discrepancy Δv between the assumed value of $\rho - \rho'$ and the actual value derived from the coordinates exceeded 6 m., the position as obtained from the electronic computations was suitably corrected).

The lessons thus learned were duly applied in the case of the *Purple* pattern, in which the set of control points was more adequately adapted to the area involved. The arrangement in this case (see Fig. 3) is such that the entire sector between the « quasi-degenerate » A 65 hyperbola and its image, and the ellipse with an L_E value of 600 km., is included within the control polygon. (The zone thus delimited includes the area covered by the French Geodetic Survey). Complex interpolation theory indicates that an evaluation of the maximum position error *within* this figure is supplied by checks obtained at the midpoints of the outer sides of the control polygon (points V_1 V_2, V_3, V_4).

By proceeding on the same basis as heretofore (by checking $\rho - \rho'$ and $\rho + \rho'$ derived from the rectangular coordinates), a maximum position error of 31 m. was found for these four points.

This shows that the process enables close delineation of the net of ellipses and homofocal spherical hyperbolae to be obtained. This result may be attributed to the high degree of the working formula, which we shall now briefly describe.

The sets of control points consisting of thirteen or twelve basic points were used in determining the numerical coefficients of 11th-degree complex algebraic formulae designed for a card-programmed electronic computer: it is of course obvious that such high-degree formulae hardly invite calculation by hand (1).

The formulae have the following aspect:

$U = X + iY = f(z) = f(p + iq) = U_0 + \alpha z + \beta z^2 + \gamma z^3 + \ldots + \mu z^{11}$,

where U_0, α, β, γ,... as are U and z themselves, are complex numbers with two coordinates. The components of successive powers of z are derived from one another in the constant sequence:

$$z^{p+1} \ldots (\text{which is written } x_{p+1} + iy_{p+1}) = (x + iy)^{p+1} = z^p . z$$

$$= (x_p + iy_p)(x + iy) = (x_p x - y_p y) + i(x_p y + y_p x)$$

(1) The decision as regards the degree of the working formula is governed by the accuracy required at long distances. The decrease in the terms of the series development (the « convergency » of the development) can be predetermined by theoretical means, and can likewise be ascertained empirically by the working formulae. In the working formulae for the *Red* and *Green* patterns, the figure supplied by the 11th-degree term, 450 km from the centre of the pattern, is of the order of 100 m.; for the following term, it would be a few decametres and therefore negligible.

This algorism of complex number multiplication can be introduced once for all time into the computer programme. With the I.B.M. equipment available for making the Decca computations, the « card-programmed » procedure was followed, each programme-card operation (at the rate of one every 0.6-second) initiating a new complex multiplication. The total amount of computation per point required slightly less than a minute. In other words, it was possible to compute the entire set of 600 or 700 control points in each pattern within the space of a few machine-hours. There are grounds for believing that certain technical refinements in programme design may result in a further appreciable decrease in the time element, using a similar type of commercial electronic equipment, and in its thus being brought down to a few seconds per point (1). Final calculations (supplying the desired coordinates of the control points) consequently take up a practically negligible amount of time as opposed to the time spent in preparing the working formulae — a characteristic and abiding trait in electronic computation.

Practical Conclusions

We believe that the method described above may be recommended owing to the following main advantages:

— The coordinates of control points on hyperbolae with a round parametric value can be obtained directly;

— The coordinates obtained are the direct *chart* coordinates of points in the conformal projection used. If the latter is the Mercator projection, the coordinates of longitude and the meridional parts are obtained, whence ordinary latitude may be derived directly;

— Since electronic computation is involved, as many control coordinates as desired may be obtained, with no necessity for transverse interpolation (i.e. the control coordinates of interpolated hyperbolae may be obtained directly);

— In particular, the difficulties involved in interpolation in the marginal sectors (near the base line extensions) no longer prevail: the working formulae remain valid for these sectors. It should be pointed out, however, that in order to plot the hyperbolae in this section, the working formula should be drawn up for a system extending to the outer margins of the pattern, which was not done systematically in the case of the French Decca Chain.

Opposing such advantages, there is the question of the rather delicate preparation of the control points and obtaining the working formulae. The problem here is one of organization and the rationalization of computations, and it may well be that new experiments will vastly improve the still largely satisfactory results that have been obtained in the present computing of the first French Decca Chain.

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(1) These results were actually obtained by Ing. Géographe H.M. DUFOUR, of the Technical Bureau of the National Geographic Institute's Geodetic Section, to whom important work on the purely analytical determination of coefficients of the transition formulae is also due.