NOTE ON THE GIVRY CORRECTION

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In most textbooks on navigation, and hence in the opinion of most people dealing with problems in Mercator projection, the difference in loxodromic and orthodromic bearings is computed according to the formula $\frac{\sin \varphi_m}{2}$, where φ_m is the mean latitude of the observer and observed point.

It is the aim of the present note to show that there is no real reason for the use of the mean latitude in this expression instead of using simply the observer's latitude φ_1. The fact is that the latter as a whole gives an approximation just as good as φ_m.

The formula seems at first to have been established by the French Hydrographic engineer Givry, and his expression was in fact $\frac{\sin \varphi_1}{2}$, where φ_1 is the observer's latitude. But then it became the habit in most textbooks on the subject to deduce the correction from what has been called the « convergence of meridians », and to this we must ascribe the introduction of φ_m. In an article in *I. H. Review*, Vol. XVIII, No. 2, Nov. 1941, page 9, Gougenheim shows how the method works, and how we may obtain a third order approximation from it in using the following formulae, originally given by Germain:

$$\left.\begin{aligned} \tan \frac{\gamma}{2} &= \tan \frac{K_1 + K_2}{2} = \tan \frac{\lambda}{2} \cdot \frac{\sin \frac{\varphi_2 + \varphi_1}{2}}{\cos \frac{\varphi_2 + \varphi_1}{2}} \\ K_1 &= \frac{\gamma}{2} - \frac{1}{12} \cdot \frac{(\varphi_2 - \varphi_1) \cdot \lambda}{\cos \frac{\varphi_2 + \varphi_1}{2}} \end{aligned}\right\} \quad (1)$$

where K is the correction sought for, λ the difference in longitude, φ the latitude, γ the convergence of meridians and the indices 1 and 2 refer to the observer and observed point respectively. It is a remarkable refinement that the correcting term of second order: $\frac{1}{12} \frac{(\varphi_2 - \varphi_1)}{\cos \varphi_m} \lambda$ leads to a quantity where third order terms are included. The tables in handbooks of radio signals are based on the formula, and Gougenheim shows how it can be put in a still shorter way is if an auxiliary latitude ψ is introduced instead of φ , a procedure which has been used in the « Tables des Radiosignaux à l'usage de la Navigation ».

However, if for special purposes, for instance in connection with surveying, we need more figures in the corrected angle than can be obtained from the given tables$\left(\frac{1^\circ}{10}\right)$, (1) is transferred to the corresponding first order formula: $\frac{\gamma}{2} = \frac{\sin\varphi_m}{2} \cdot \lambda$. Only in rare cases, near the poles or over great distances, need we take care of the higher terms, and therefore have to resort to the whole system (1), which at once gives a third order approximation and does not distinguish between the second the third order.

In the Danish Hydrographic Office we have not used the convergence of meridians but proceeded in a more straightforward way to obtain the Givry correction. The method is given in a textbook on hydrography written by the former hydrographer, Commodore Ravn (who however gives the correction to the first order), but we may of course follow different other ways (the method of Givry is not known to the writer).

The result to the third order will be:

$$\left.\begin{aligned}
K_1 &= \frac{\sin\varphi_1}{2} \cdot \lambda + \frac{2\text{-}3\sin^2\varphi_1}{12\cos\varphi_1} \cdot p\lambda + \frac{\sin\varphi_1}{24}\left(\cos^2\varphi_1 \cdot \lambda^3 - \frac{p^2\lambda}{\cos^2\varphi_1}\right)\\
&= \frac{\sin\varphi_1}{2}\lambda + \frac{2\text{-}3\sin^2\varphi_1}{12}\cot\alpha_1 \cdot \lambda^2 - \sin\varphi_1 \frac{1\text{-}2\sin^2\varphi_1}{24}\lambda^3\\
&\qquad - \sin\varphi_1 \frac{5\text{-}6\sin^2\varphi_1}{24}\cot^2\alpha_1 \cdot \lambda^3\\
&= \frac{\sin\varphi_1}{2}\eta + \frac{2\text{-}3\sin^2\varphi_1}{12}\xi\eta + \sin\varphi_1\cos^2\varphi_1 \quad \frac{\eta^3}{24} - \frac{\xi^2\eta}{8}\\
&= \frac{\sin\varphi_1}{2}\sin\alpha_1 \cdot \rho + \frac{\sin\alpha_1\cos\alpha_1}{6}\rho^2 + \frac{\sin\varphi_1}{48}\\
&\qquad (\sin^2\varphi_1\sin^2\alpha_1 + 2\,[\cos^2\alpha_1 - \sin^2\alpha_1])\sin\alpha_1 \cdot \rho^3
\end{aligned}\right\} \quad (2)$$

where K, φ and λ are the same as above and: γ is $\varphi_2 - \varphi_1$, ξ and η the differences $x_2 - x_1$ and $y_2 - y_1$ of the x and y co-ordinates on the Mercator projection but in units of the radius of the earth (x northward, y eastward).

α the bearing of the orthodrome

$\rho = \sqrt{\xi^2 + \eta^2}$

If we want to control the influence of the ellipsoidal shape of the earth the first term containing the eccentricity ε is:

$$+ \frac{\sin^2\varphi_1 \cdot \cos\varphi_1}{2} \gamma \cdot \lambda \cdot \varepsilon^2 \text{ and the like.}$$

We are now going to compare systems (1) and (2).

In cases where third order approximation is claimed, I think (1) should be preferred from a computing point of view (the approximation is of course the same).

In case of a second order or first order approximation I think a possible advantage as to computation of one of the systems over the other is hardly worth mentioning. But if in the case of first order approximation, which is by far the most important, we take the relative accuracy of the two expressions:

$$K_m = \frac{\sin \varphi_m}{2} . \lambda, \qquad K_1 = \frac{\sin \varphi_1}{2} . \lambda$$

we find that the neglected second order terms are:

$$\text{for } K_m \left(: \frac{\sin \varphi_1}{2} \lambda + \frac{2\text{-}3 \sin^2 \varphi_1}{12 \cos \varphi_1} p \lambda \right) - \frac{\sin\left(\varphi_1 + \frac{\rho}{2}\right)}{2} \lambda = \frac{1}{12 \cos \varphi_1} . p \lambda$$

$$\text{for } K_1 : \qquad + \frac{2\text{-}3 \sin^2 \varphi_1}{12 \cos \varphi_1} . p \lambda$$

This shows that in cases where third order terms are completely negligible but the second orders are about critical, we may use the following table to compare the accuracy of the two expressions:

φ	$-\dfrac{1}{12 \cos \varphi_1}$	$\dfrac{2\text{-}3 \sin^2 \varphi_1}{12 \cos \varphi_1}$
0°	— . 083	+ . 167
10°	— . 085	+ . 162
20°	— . 089	+ . 146
30°	— . 096	+ . 120
40°	— . 109	+ . 083
50°	— . 130	+ . 031
60°	— . 167	— . 042
70°	— . 244	— . 158
80°	— . 480	— . 436

From this it follows that in the interval 0° to 30° K has some advantage while from 40° and upwards the reverse is true; in a smaller region between 50° and 60° even to such an extent, that K_1 here in fact gives a second order approximation.

Aside from the fact that the recently devised U.T.M. system may make geodesists prefer K_m to K_1 (the φ and λ values involved acquiring quite another meaning of course) it must be justified to conclude, that the Givry correction should equally well be computed from $\frac{\sin \varphi_1}{2}$ as from $\frac{\sin \varphi_m}{2}$ and in fact within a certain region with a significantly better result.