THE RANA RADIONAVIGATOR

CHARACTERISTICS, TESTING AND OPERATION OF A RADIONAVIGATIONAL CHAIN

by Henri LACOMBE, *Professeur of Physical Oceanography, Chief Hydrographic Engineer of the French Navy*
(detached for special service)

(Translated from *La Revue Maritime*, Paris, September 1955.)

The French Navy Hydrographic Office, early in 1954, made the acquisition of a RANA Radionavigational Chain which was tested in April 1954, and then used for routine hydrographic investigations in September by the French and North African Coastal Survey, which operates off the coast of Morocco during the spring and summer each year. As head of the Survey, we gave close attention to these operations from the surveying vessel *Amiral Mouchez*; the board of editors of *La Revue Maritime* appears to believe that an account of the work carried out with this device might be of interest to its readers.

I. — PRINCIPLE AND COMPOSITION OF A RANA CHAIN

The RANA process (from RAdio NAvigation), designed by Messrs. Honoré and Torcheux, radio engineers, covers a large number of combinations of continuous electro-magnetic waves, and may be applied to fixing the position of a mobile by measuring the phase-differences of radio oscillations received by the mobile. The Hydrographic Office's chain, built by the *Compagnie des Compteurs de Montrouge*, consisted of a chain supplying hyperbolic position lines. It will be readily apparent that reception by a mobile of waves radiated by two aerials of known geodetic position enables determination of the hyperbola on which the mobile is located by measurement on this mobile of the phase-difference between such waves, provided the waves are originally locked in phase. Such waves cannot, however, consist of ordinary continuous waves on an identical frequency, as then the receiver aerial would cause superimposition of the waves and no system of separation would enable identification of the signals from each aerial or the consequent measurement of their phase-difference.

This requires, at various stages, combinations of waves that can be separated by the receiver, but that are nevertheless such that the ultimate directly perceptible result will be a difference in phase as between currents which should, obviously, be of identical frequency, and such that the relative phase difference will only depend on the position of the mobile in relation to the aerials and not on time considerations.

Various existing instruments, such as the British Decca Navigator, make use of varying wave combinations. The original feature of RANA as compared with Decca is that use is made of frequency subtractions (beats), technically a simpler and more reliable method than the multiplied and divided frequencies used in DECCA.

Each pair of aerials supplies a position line of the mobile. In the RANA chain used, there are three such pairs: each pair is absolutely independent, and supplies a position line. There are thus three position lines, ensuring greater reliability; any anomaly causing the displacement of one of the position lines is revealed through the appearance of an appreciably large « cocked hat » (1), i.e. the three position lines form a triangle of uncertainty of appreciable dimensions.

A diagram of a pair of transmitters, or of the « pattern » supplying the position line, is shown in Figure 1. This « basic installation » consists of the following:

A) A « free » (master) transmitter L continuously emits two waves of frequency N_1 and N_3 (of wave-length λ_1 and λ_3), e.g. : $N_1 = 1620850$ c/s. and $N_3 = 1623050$ c/s., differing by 3 kilocycles and stabilized by means of thermo-controlled crystals.

B) A « slave » transmitter A, located about 10 kilometres away from L, likewise emits two continuous waves of frequency N_2 and N_4 (of wave-length λ_2 and λ_4), the difference being 3 kilocycles also, and such that:

$$N_2 = N_1 + 38 \text{ c/s.}$$
$$N_4 = N_3 - 38 \text{ c/s.}$$

The 38-cycle beats resulting from the superimposing of N_1 on N_2, and of N_3 on N_4, as received at a servo-receiver R adjacent to transmitter A, are sent to the slave transmitter by a coaxial line about 500 metres long in which a zero-phase relationship is maintained. Upon reaching the slave transmitter, an appropriate locking device maintains them at an identical frequency and phase with the 38-cycle oscillations of a low-frequency oscillator which is included in the slave transmitter and in a way « sets the beat ».

With the phases of the beats thus locked, what frequency combinations are obtainable and with what result ?

Let d_L and d_A (Fig. 1) be the distances of servo-receiver R to stations L and A. Let f_1, f_2, f_3, f_4 be the phases of similarly-indexed oscillations at time $t = O$.

At point R, the phases of waves N_1, N_2, N_3, N_4 are as follows, designating the speed of electro-magnetic wave propagation by V:

$$1 : 2\pi N_1 t - 2\pi N_1 \frac{d_L}{V} + f_1$$

$$2 : 2\pi N_2 t - 2\pi N_2 \frac{d_A}{V} + f_2$$

$$3 : 2\pi N_3 t - 2\pi N_3 \frac{d_L}{V} + f_3$$

$$4 : 2\pi N_4 t - 2\pi N_4 \frac{d_A}{V} + f_4$$

(1) « Chapeau » : designates the figure formed by plotting the three position lines observed in the area of the fix desired.

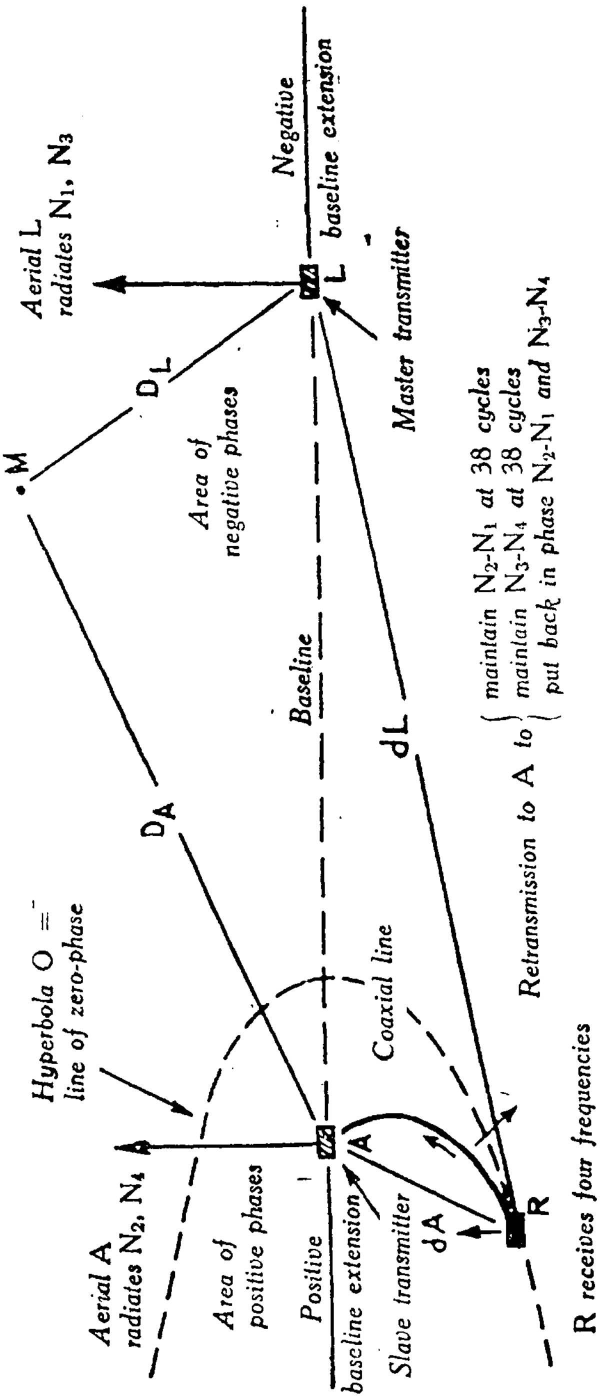


Figure 1. — Basic RANA Installation.

The beat of waves 2-1 is of phase:

$$2\pi(N_2-N_1)t - \frac{2\pi}{V}(N_2d_A - N_1d_L) + f_2 - f_1$$

The beat of waves 3-4 is of phase:

$$2\pi(N_3-N_4)t - \frac{2\pi}{V}(N_3d_L - N_4d_A) + f_3 - f_4$$

Since $N_2-N_1 = N_3-N_4$ in accordance with the frequencies selected, the phase difference of beats 2-1 and 3-4 are not dependent on time, but on distances d_L and d_A and, of course, on the phases proper, f_1, f_2, f_3, f_4. If at R such phase difference is maintained at zero, we get:

$$O = \frac{2\pi}{V}\left((N_3 + N_1)d_L - (N_2 + N_4)d_A\right) + f_2 - f_1 - f_3 + f_4$$

$$= \frac{2\pi}{V} . 2N(d_L-d_A) + f_2 - f_1 - f_3 + f_4 = \frac{2\pi}{2}(d_L - d_A) + f_2 - f_1 - f_3 + f_4$$

by putting:

$$2N = N_3 + N_1 = N_2 + N_4 \text{ and } \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda_3} + \frac{1}{\lambda_4};$$

N and λ being the mean frequency and mean wave length transmitted by each aerial, N = 1622350 c/s.

The locus of points where the phase relationship of the constituent beats is zero is the hyperbola branch whose foci are located at A and L and which passes through R.

This being established, if the same line of reasoning be adopted as regards any point M located at distances D_L and D_A respectively from L and A, the phase-difference of the beats is derived as:

$$\Delta\Phi = \frac{2\pi}{\lambda/2}(D_L-D_A) + f_2 - f_1 - f_3 + f_4 = \left(\frac{2\pi}{\lambda/2} D_L - D_A - (d_L - d_A)\right)$$

The locus of points where $\Delta\Phi$ is constant is a hyperbola of foci A and L, the distance between, 2c, being supplied by geodesy.

The number of phase-difference revolutions of the beats is $\frac{\Delta\Phi}{2\pi} = n$, whence the basic formula derived by Chief Hydrographic Engineer Grousson is obtained:

$$D_L - D_A = n\frac{\lambda}{2} + (d_L - d_A)$$

D_L-D_A being the length of axis 2a of the hyperbola in the pattern, characterized by n and on which M is located.

C) A shipborne receiver contains a phasemeter for each of the patterns, enabling the decimal fraction of n, and such fraction only, to be read off. The hyperbola on which the mobile is located can only be defined, therefore, by ascertaining the integral value of n, i.e. an adequately approximate position of the mobile.

Owing to the value of the RANA wavelengths, an extra revolution of the phasemeter corresponds to a mere 90-metre variation in D_L -D_A. In hydrographic operations, where D.R. is invariably closely held to, adjustment of the phasemeter at the inshore ends of the sounding lines enables the integral value of n at that point to be known, and a revolution-counter mechanically locked with the phasemeter needle supplies a continual count of the revolutions measuring the phase-difference, provided transmission and reception are not interrupted. But if such adjustment has not been made or if the mobile comes from seaward, the ambiguity may not be resolved until after a considerable time, and may not take place until a satisfactory fix is obtained with (within) in sight of land.

In order to mitigate this drawback, each RANA pattern is provided with a system for resolving ambiguities radioelectrically and instantly, independently, as it were, of the previous « history » of phasemeter readings, such as when reliance is placed on the mechanical interlocking of the phasemeter needle and a revolution-counter. Each of the L and A aerials emits an additional frequency, say N_5 = 1783085 c/s. at L and $N_6 = N_5 + 38$ c/s. at A. Such frequencies are then combined with N_1 and N_2, following which the difference 2-1-(6-5) is taken. Since $N_6\text{-}N_2 = N_5\text{-}N_1 = \dfrac{N_3 + N_1}{20}$ the basic formula of this additional pattern sensibility, called the « coarse » pattern as opposed to the first or « fine » pattern, is written:

$$\frac{\Delta\ \Phi}{2\,\pi}\ \text{coarse} = \text{n coarse} = \frac{2\,\pi}{20\,\lambda/2}\ D_L\text{-}D_A\text{-}(d_L\text{-}d_A),$$

which means that the « coarse » phasemeter makes one revolution while the « fine » phasemeter makes 20. The frequency combination designed for the « coarse » phasemeter therefore has the same result as a similar operation of the « fine » phasemeter on a wavelength 20 times longer. The ambiguity with regard to the integral value of the phase-difference therefore affects 20 lanes instead of one, and is much more readily resolved.

This represents only one instance of the remarkable flexibility of the frequency combinations that may be applied with regard to RANA. It would be possible to devise yet another degree of pattern sensibility.

It should be noted that the direction of rotation selected in the design of the phasemeters is such that they rotate in the direction of increasing graduations (clockwise); as the mobile moves from the master towards the slave transmitter, a phase of 0.00 is read off on the hyperbola branch passing through the slave transmitter. On one side of the branch, the one containing A, the phases are positive; on the other side, containing L, they are negative. In order to avoid negative phases, and above all the necessity of reading the phasemeters counter-clockwise, a round number of revolutions (500) over and above the actual number of lanes in the pattern is conventionally added to the numbers, i.e. to the actual phase-differences. The figure obtained is preceded by an additional cipher corresponding to the number of the pattern. Thus hyperbola 0.00 in Pattern I becomes hyperbola 1500; in Pattern II, 2500, and so on. Hyperbola 1383.42 actually corresponds to phase 1383.42 — 1500 = — 116.58.

The number of lanes in a pattern may readily be obtained from λ and from the length 2c between transmitters A and L. On the baseline, between such points, D_L -D_A varies by $\dfrac{\lambda}{2}$ as the mobile shifts by $\dfrac{\lambda}{4}$, for as D_A increases by such amount on the line, D_A decreases by a corresponding amount and the difference increases by $\frac{\lambda}{2}$. The number of lanes in a pattern is therefore $\frac{AL}{\lambda/4} = \frac{2c}{\lambda/4}$; where AL = 10 kilometres, we get approximately 220 lanes, since $\frac{\lambda}{4}$ = approximately 45 metres. The lanewidth on the baseline is 45 metres, and increases away from this axis. It may readily be shown that the lanewidth is equivalent to $\frac{\lambda}{4 \sin u/2}$ adjacent to a point M where segment AL subtends angle u. The factor $\frac{1}{\sin u/2}$ denotes the relative spread of the hyperbolae in a pattern. The shorter the wavelengths used, the greater becomes the linear accuracy for a given accuracy of phase. If propagation may be assumed as being isotropic about the centre of the pattern, the speed of propagation V that should be assumed may readily be derived from the distance AL determined by geodetic means, the frequency N, and the difference between the n_1 and n_2 readings obtained on the extensions of the focal axis, or baseline extensions (1), which are the branches of the degenerate hyperbola in the pattern. We get:

$$V = \frac{4.AL.N}{n_1 - n_2}$$

Such isotropic properties do not generally occur in hydrographic operations, owing to the usual proximity of the shore and the AL line, and to the difference in the speed of propagation over land and at sea. The pattern must be calibrated by other methods, which will be discussed later.

To summarize, the chain used by the Survey consists of:

— 3 pairs of ground transmitting stations, such as the one shown in Figure 1 and housed in sectional huts;

— 1 shipborne receiver, including 3 « fine » phasemeters — one per pattern — and 1 « coarse » phasemeter enabling the coarse readings for the three patterns to be obtained by switching;

— 1 monitor receiver at a fixed point, either ashore or on a vessel at anchor in the harbour, the receiver indications with regard to the three coarse and three fine patterns enabling the detection of any phase-jumping in a pattern, i.e. the control of the overall stability of the chain.

II. — QUALITIES OF A RADIONAVIGATIONAL CHAIN. CONDITIONS FOR EVALUATION.

The qualities that may be expected in a radionavigational chain are various, and tests for their evaluation were made with respect to the following points, in accordance with Hydrographic Office specifications:

1. Delay in stabilization when first warming up or after a brief interruption.

2. Stability and reliability showing the ability of the chain to supply identical readings when the mobile proceeds through a given geometric point at various times

(1) Called « lignes d'ombre » (shadow-lines) in the original French text.

and from different directions. If by chance, due in particular to variations in electromagnetic propagation, certain readings should show differences at various times for a fixed point, steps should be taken to verify that all the hyperbolae in the pattern involved have shifted by an identical number of hundredths of a revolution.

Consistency of the chain denotes its ability, following the best possible determination of the correction values for the calibration, to produce in certain areas three concurrent position lines, i.e. a « cocked hat » of extremely small size.

Subject to adequate accuracy of the geodetic coordinates of the transmitting stations, two terms should normally be met with in calibration corrections: a term varying linearly with the number of the hyperbola and arising from an incorrect value of the speed of propagation of radio waves: from this term the exact value of speed over water may be derived; and a constant term whose origin must be traced to the propagation of the signals ensuring synchronization: thus the paths LR and AR followed by such signals do not occur over the sea but in a medium where the speed of propagation is different. A variation in this speed which especially affects the path AR... may also be inferred in the immediate vicinity of the aerials.

Such propagation effects, known as anomalies, can of course but be endured, since it is not yet known how they work. But radionavigational chains may possibly provide an excellent means of investigating such phenomena.

3. The accuracy of a chain denotes its ability to provide radio fixes coinciding with geodetic fixes. It is often impossible to compare the two types of fix far to seaward and away from the stations. It is true that the area selected in the Seine Estuary tests was such that comparisons were possible at a remote distance from the stations, but owing to propagation anomalies, comparisons were conclusive in the case of a single pattern.

Moreover, the datum provided by a geodetic position has no absolute value higher than the accuracy of the datum. A geodetic position at sea can only be obtained with sufficient accuracy by means of simultaneous sights of the masts or mast lights of the moving vessel, taken with three suitably sited land theodolites. Aside from the totally inadequate accuracy of the compass bearing, the standard subtense method used in hydrography supplies inaccurracies amounting to as much as ten metres. And in the central area of a RANA pattern, near the bisector of the line between the aerials at a distance from shore of about 6 miles, where under favourable visibility conditions angles between coastal marks may adequately be measured, 10-metre uncertainties with regard to the geodetic position involve uncertainties in the RANA chain of 10 hundredths of a phasemeter revolution. In the areas of greater lanewidth, however, at distances remote from the bisector and near the baseline extensions, such a linear uncertainty causes a phase uncertainty of a mere one-hundredth of a revolution. A scattering of the parameter values must therefore be expected leading to determination of greater calibration corrections in the central areas of the patterns than at the extremities thereof. We shall encounter this fact later on.

We shall moreover learn that the calibration methods actually available determine both consistency and accuracy jointly in certain areas; outside these areas, it is often only possible to check consistency, even though conclusions may be drawn as regards accuracy, which is of course the essential criterion of a chain since it implies practically every other desirable quality.

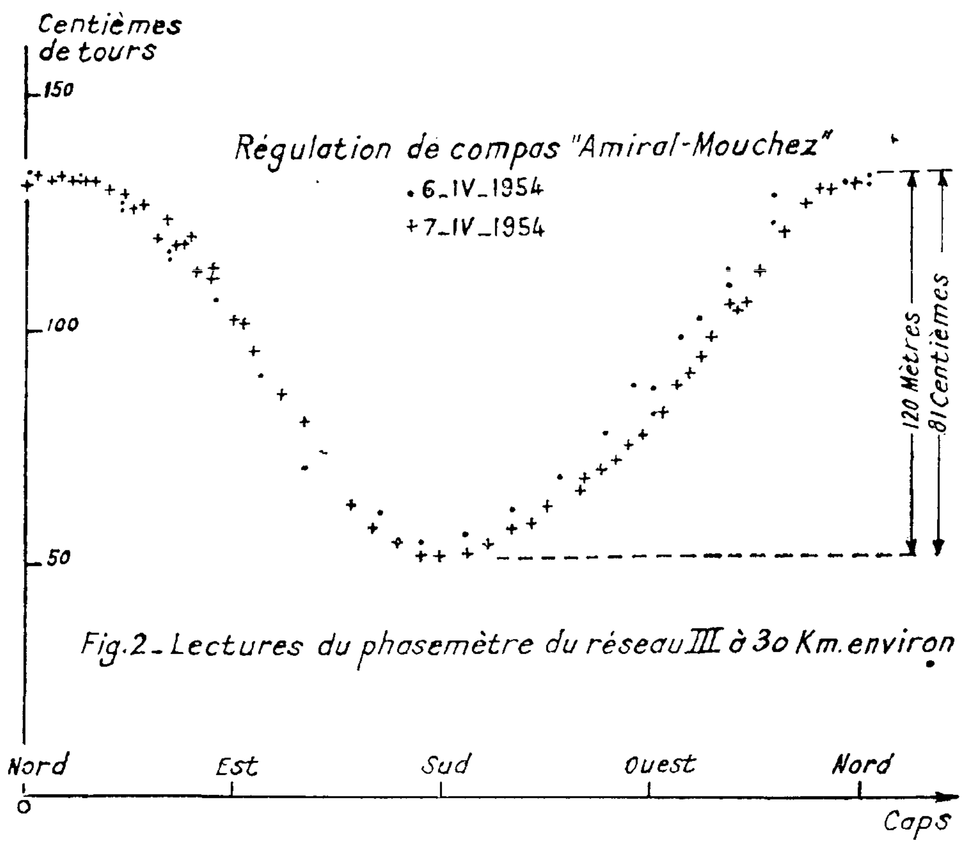
4. The last quality to be considered is range, by day and by night.

III. — RESULTS OBTAINED WITH RANA CHAIN IN SEINE ESTUARY (APRIL 1954)

The qualities discussed in the preceding section were all subjected to investigation during the April 1954 trials, but many of them were re-examined and dealt with more thoroughly and with greater accuracy during the offshore survey of the Moroccan coast in September 1954. In the Seine Estuary operations, one of the patterns was sited in the Trouville-Houlgate area, the second pattern in the Longues-Ver area, and the third in the Saint-Vaast-Barfleur sector.

1. *Stabilization after starting the equipment.* Phasemeter readings become stabilized one half-hour after warming up. After a few minutes interruption, stabilization is rapidly recovered.

2. *Stability and reliability.* The monitor receiver shows maximum oscillations of the order of 4 centimetres, with much more frequent average values. The average discrepancy for a few dozen consecutive observations may reach 1 to 1.5 hundredth of a revolution.



Hundredths of a revolution.

« Amiral Mouchez » Compass Adjustment.

Pattern III Phasemeter Readings at approximately 30 km.

North East South West North

Headings

During compass adjustment operations in the inner roads off Cherbourg on 6th and 7th April, while determining the various headings of the *Amiral Mouchez,* which was anchored to a mooring-block and hauled off by tug at the stern, we observed the geodetic positions of the bridge by hydrographic circle and took simultaneous readings of the « fine » phasemeters for the three patterns. Accounting for the distance between the RANA receiving aerial and the point of observation, we were able to check that the dimensions, in all three patterns, deducted from the extreme values as shown by the phasemeters during a complete swing of the ship, coincided to within a few metres with the length of the path described by the aerial. The most sensitive pattern (1 hundredth represents 1.40 metre), at different headings occupied during several swings on two separate days, showed the series of positions illustrated in Figure 2.

Crossing one of the baseline extensions from appreciably different directions supplied values showing a maximum discrepancy of 4 hundredths; the continuity and uniformity of values obtained in the vicinity of this line when crossed by the mobile in an appreciably normal direction indicate the stability over a short period of the readings.

The absence of any effect of the supporting mobile — which must be included as a reliability factor of the shipborne receiver unit — was checked by setting the ship's course towards the four cardinal and intercardinal points in the vicinity of a buoy. Reducing the value read off to the value at the buoy was done by measurement or evaluation of the distance to the buoy as it lay on the ship's beam. These operations were carried out in Pattern I at a distance such that the angle subtended by baseline AL of the transmitters amounted to approximately 60°. This is a useful specification, since where two stations are located in an approximately identical direction in relation to the mobile, the ultimate absence of effect might result from identity of effect on two waves coming from this same direction. The absence of heading effect was noted as accurately as assumption regarding the buoy's stability would allow. Maximum phase-difference values amounted to 13 hundredths, corresponding in the trial area to less than a 12-metre discrepancy; an amount that can be plausibly assigned to the drift of the buoy.

Interference due to ship radio broadcasts was also investigated: it does not prevent the use of RANA.

3. *Consistency and accuracy — Calibration.* Consistency tests were carried out on two separate occasions: first on a small (1 :100,000) and later on a larger (1 :20,000) scale. On the scale of 1 :100,000, consistency was immediately verified owing to the practically absolute concurrence of the three hyperbolic arcs constructed on the 1 :100,000 projection and interpolated in a hyperbolic pattern plotted for every 20 revolutions. The almost identical correspondence of the geodetic fix with the radio fix, both of which were graphically plotted, was verified at this scale.

Although these were already favourable results, and sufficient in practice for offshore sounding purposes, it was imperative that a more thorough check be made of consistency and accuracy. This might be done at two additional levels:

a) By means of a graphic plot at scale 1 :20,000 of the radio fix and geodetic fix, the latter being obtained by hydrographic circle observations, and by evaluating the size of the radio « cocked hat » and the discrepancy of its centre in relation to the centre of the cocked hat formed by the measurements obtained by the subtense method from shore signals. The « theoretical » values of the phase-differences may then be computed with respect to the visual fixes plotted on the projection, and the discrepancy noted as between the theoretical value and the value actually read off.

b) Accuracy may be carried still further by computing simultaneously the visual fix and RANA fix, by evaluating consistency from the size of the cocked hat formed by the computed RANA position lines and by determining the discrepancies between the theoretical values of the phase-differences — from the most likely position of the optimum visual fix selected in the three patterns.

On 16th and 21st April 1954, the *Amiral Mouchez,* while cruising off the cliffs between Cap de la Hève and Cap Antifer, fixed its RANA position and visual position every four minutes.

It was then discovered that the three hyperbolic arcs plotted on the scale of 1 :20,000 from the rough readings formed a cocked hat of approximately 60 metres invariably showing a similar arrangement, and that the RANA fix was always located in the same direction away from the visual fix. The cocked hats were of systematic character.

This led both Chief Hydrographic Engineer Grousson and ourselves to the belief that such discrepancies might be due to the fact that synchroization over the three patterns occurred following travel of the waves over land, with the result that the useful speed of propagation was approximately one seven-hundredth less than over water (299250 km/s. as opposed to 299680 km/s.). It was therefore necessary to add, to the rough readings, corrections depending on the d_L -d_A values of the patterns and amounting to 22.20 and 24 hundredths of a revolution respectively. The application of these corrections notably decreased the radio cocked hats, i.e. increased consistency of the chain, and brought the RANA fixes appreciably closer to the visual fixes, the average discrepancy being reduced to 30 metres or so.

But the fact that the rough values were unacceptable involved the necessity of actual calibration of the chain. For this purpose, comparisons were continued over as wide as possible a range of phase-difference values within the three patterns, although lack of time prevented comparison by computational means. Coordinates of the visual fixes were carefully plotted on the scale of 1 :50,000 and the theoretical phase differences were derived by computation; discrepancies as between values read off and theoretical values were evaluated, and plotted on three graphs in terms of the hyperbola numbers in each pattern.

In Pattern II (Longues-Ver), a satisfactory grouping was obtained, but a slight discrepancy — independent from the hyperbola number — remained in relation to zero-value. It was therefore necessary to add a residual correction to the correction relating to the error in speed over the phasing path. The independence of the value of the correction with regard to the hyperbola number implied that the rate of propagation over water was the right one (299680 km/s.).

Similar investigations over the other two patterns led to comparable conclusions, but revealed anomalies in areas which, although fairly restricted in size, in Pattern I included the La Hève-Antifer zone, and thus reduced the significance of the previous comparisons in this locality. Beyond the anomalous areas, discrepancies between theoretical and read-off values varied between 6 and 8 hundredths of a revolution around the average value, especially in the centre of the patterns. A large portion of this scatter could be attributed to errors in the visual fixes, which had been graphically plotted on the scale of 1 :50,000, but, through lack of time, not computed. More intensive investigations were carried out during the surveying operations off Morocco. But the tests so far made were of sufficient thoroughness to warrant use of the chain for routine hydrographic purposes during the summer of 1954.

For the reader's information, it may be pointed out that the anomalies seemed to occur mainly in areas where the radio beams were tangent to the cliffs.

IV. — RESULTS OBTAINED OFF THE COAST OF MOROCCO IN SEPTEMBER 1954

The operation of the RANA chain in standard hydrography presented additional problems. The Survey was charged with the selection of transmitter sites, the computation of the hyperbolic patterns and practical range tests. Moreover, improvements in the investigation of consistency and accuracy were introduced, by developing methods for computing the radio fix.

The Survey was faced with the above number of problems within a period of a few weeks, and was compelled to solve them on a day-to-day basis. Owing to a series of circumstances, of which a major one consisted in the rapid transition from the trial stage to the operational stage, the Survey was faced with the necessity of relying on its own available personnel, and especially engineering personnel, which was limited to Hydrographic Engineers Le Fur and Terrasson, to Lt. V. Sarindu of the Thai Navy and ourselves.

The 1954 surveying programme off the Moroccan coast (Fig. 3) consisted in part of soundings in an area bounded by perpendiculars drawn to the coast at Fedala and Port-Lyautey and extending 75 miles in the offing, thus aligning itself on the edge of an area sounded in 1952 with a DECCA Navigator Chain which had been made available to the Survey during that year, under Chief Engineer Brémond. Inshore near the 100-metre depth contour the 1954 sounding area was adjacent to the limits of the area sounded in 1953 by conventional methods and located from 8 to 12 miles offshore.

1) *Choice and Determination of Transmitting Station Sites* (Fig. 3).

The choice of station sites depends on several factors: some are of a purely practical nature, others geometrical, and the remainder of a radio-electric character.

a) *Practical factors.* Such factors essentially consist in accessibility, and local housing and supply facilities for the personnel assigned to the operation and maintenance of the stations. In principle, three men taking the watch in shifts (the instrument and radio watch) suffice for each station, but a radio repairman (an engineer of the *Compagnie des Compteurs*) and an additional repairman for the generating units must be available at one of the centre-pattern stations. In the 1954 survey area, the choice was relatively wide from this practical aspect owing to the density of population and of lines of communication. The shore area north of the mouth of Oued Sebou as far as Moulay-bou-Selham nonetheless had to be rejected owing to the lack of roads, and the latter-named town was too remote for adequate quality of the intersections (see b) below).

b) *Geometrical factors.* The sites selected for the three patterns had to be such that the hyperbolae would intersect one another at sufficiently wide angles over the entire survey area. For off-shore surveying purposes at a distance of 80 miles it would have been convenient to separate the end stations as much as possible, but not for inshore operations, in this case located at the limit of the 1953 soundings,

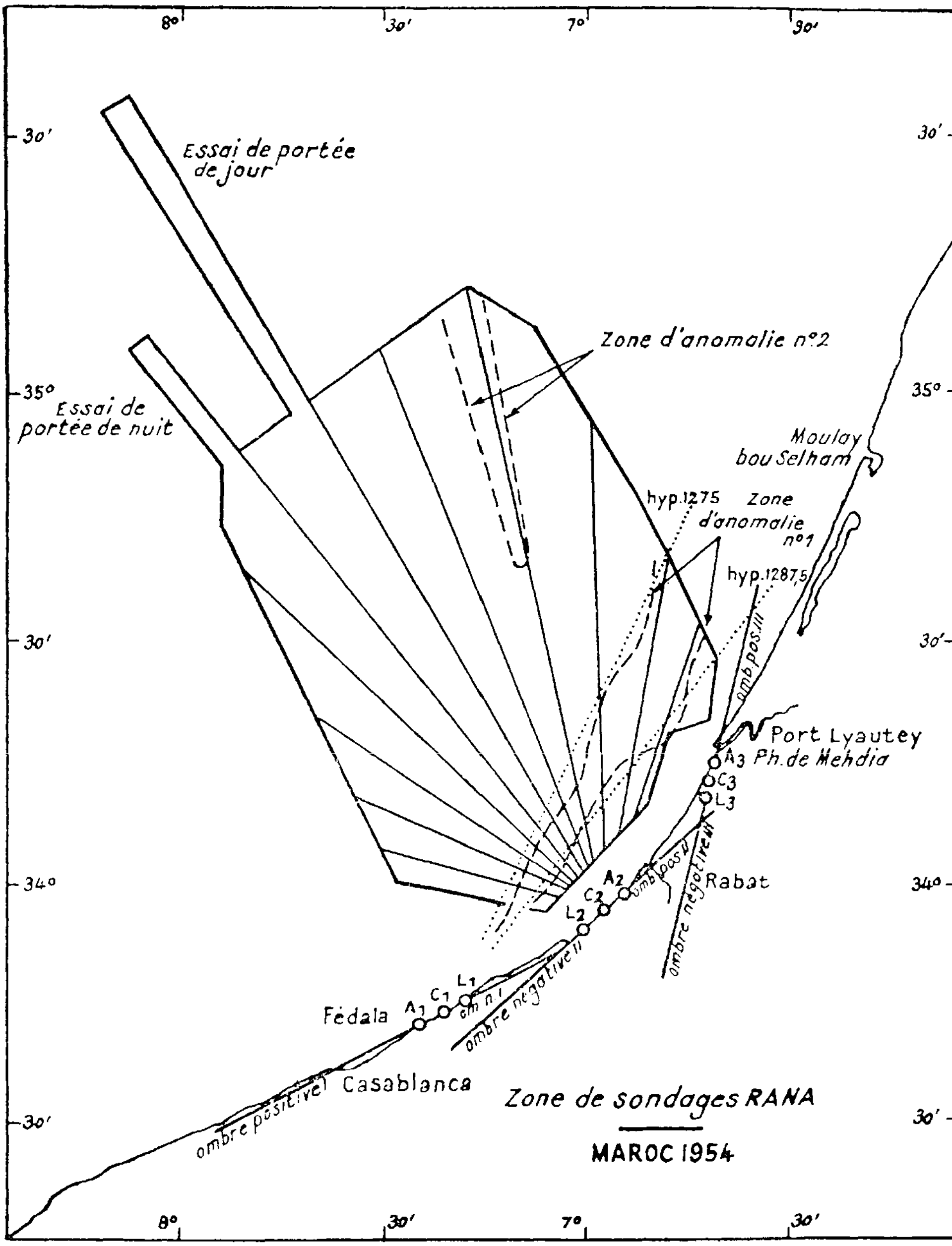


Fig.3

Day range test

Anomalous zone No. 2

Night range test

Anomalous zone No. 1

Negative baseline extention

Positive baseline extension

Rana Sounding Area

from 6 to 12 miles seaward. An adequate angle of cut in the latter area could only be obtained by siting the pattern centres at a maximum interval of 40 kilometres.

c) *Radio-electric factors.* Installation of the aerials requires the addition of a counterweight consisting of a horizontal set of cables of 25-metre radius centred on the foot of the aerial. The line of vision seaward from the aerial must be unobstructed and located over gently sloping land. Stations must be sited at least 200 metres away from power or telephone lines and from groups of trees, in order that retransmission from such conductors and phase disturbances may be avoided.

The final choice is shown in Figure 3. Specifications listed under a), b) and c) were generally met with, except as regards station A_1, which was sited in the vicinity of oil-tanks in the port of Fedala, and to which an anomaly was later assigned. The site was retained, however, in view of other advantages. In the following account, the subindices added to A, L and R refer to the numbers of the patterns to which the stations belong. The monitor receiver (RC) was installed on the *Estafette*, a tender operated by the Survey, which remained anchored in Casablanca harbour. Readings of the 3 « fine » and 3 « coarse » patterns were obtained thereon through use of a single switched phasemeter.

Fixing the mobile's position by phase-difference readings naturally requires that the foci of the hyperbolae and that the slave receiver in each pattern be located within a well-defined coordinate system. On the coast of Morocco, an excellent Lambert geodetic network readily enabled determination of the aerial coordinates by ordinary geodetic methods. The Lambert system involves a conventional shrinkage in scale in order to bring the mean scale of the portion of the spheroid covered by the Lambert net to unity, as well as an inherent increase in scale away from the mean parallel of the system. The true distance between two points accordingly slightly differs from the distance measured from the Lambert coordinates, but in practice over large areas it is possible to adopt a mean value for the total contraction or expansion in the system. This result may likewise be accomplished by contracting or expanding the radio wavelengths in the same ratio, which was the procedure adopted: the wavelengths were « reduced » to the Lambert projection by means of a 23 one-hundred-thousandths reduction in the values derived from the frequencies and speed of propagation of 299,680 km/s. adopted as the correct value during the Seine estuary tests.

The next step thereupon consists in computing the hyperbolic patterns with the reduced wavelengths on the Lambert plane.

2) *Computation of Equiphase Hyperbolae in the Three Patterns.*

The time allotted for computation of the hyperbolae was extremely short, especially as regards Pattern III, whose slave receiver was not located in Lambert coordinates until the day before soundings were to begin. We accordingly believed that we should first attempt rapid approximate calculations that would enable soundings to be carried out at the scale prescribed. More accurate computations would be made as necessary when drawing up the fair sheets. We decided that the most expeditious method would be the computation of several points along the asymptotes of the hyperbolae, followed by that of the distance from the hyperbolic point to the corresponding asymptote. This method had been followed by Chief Engineer Grousson in the approximate calculation of the hyperbolic coverage of the Seine estuary.

If 2a denotes the double major axis of the hyperbola that requires to be computed, i.e. the difference in the distances of one of its points to foci A and L ($2a = D_L - D_A$), then:

— 2c denotes the distance between foci, which is a pattern constant;
— θ the angle between the asymptote and the focal axis;
— D, the distance from a point on the asymptote to the centre of the hyperbolae;;
— d, the distance of this latter point to the hyperbola, and which is appreciably equivalent to:

$$d = \frac{c^2 \sin 2\theta}{4D}$$

The error involved when such a value for d is taken is less than:

$$\frac{c}{32} \quad \frac{c^3 \sin 4\theta}{D(D^2 - c^2 \cos 2\theta)} \qquad \text{if } \theta < 45°,$$

and less than

$$\frac{c}{32} \quad \frac{c^3}{D^3} \quad \sin 4\theta \qquad \text{if } \theta > 45°,$$

according to computations carried out by Hydrographic Engineer Terrasson.

As over most of the survey area these expressions are extremely small owing to the low values of c (maximum value 6,000 metres) in relation to the D values, the hyperbolae were found to be sufficiently accurate upon their construction by the foregoing method on the prescribed scale of 1:100,000.

In actual practice, after computing θ by $\cos \theta = \frac{a}{c}$, and then 2θ, it was easy to obtain the coordinates of points located at various round distances of D, then the d distances which were plotted graphically at the required scale. A hyperbolic branch could thus be computed in 8 minutes. This procedure was followed for round hyperbolic values every 5 revolutions, or, in certain places, every 2.5 revolutions.

It was later discovered that the equation of the hyperbolae, when referred to the asymptotes, $xy = \frac{c^2}{4}$, supplied fully as rapid and precise a computation method, but that this procedure was inconvenient in those cases — which in actual practice occurred frequently — where the angle between the asymptotes differed too remotely from a right angle.

In any case, such methods of calculation are faster than those involving hyperbolic curves, i.e. reference of the hyperbola to its axes.

3) *Pattern Calibration.*

Calibration of the patterns was carried out in order to determine the necessary corrections to the phasemeter readings and thus ensure maximum consistency and accuracy of the chain. Values obtained on the baseline extensions and from the monitor receiver, and the systematic comparison of geodetic and radio fixes within sight of land, supply the data needed to correct the phasemeter readings in the three patterns, in such a way as to make the RANA and geodetic fixes coincide as closely as possible throughout the area where such data can be obtained, and to ensure pattern consistency.

a) *Intersection of baseline extensions :* Depending on the manner of siting the ground stations, such intersections may be carried out ashore or at sea. The following table lists the results obtained, the land values being supplied by the monitor receiver installed on a truck, and the sea values by the shipborne receiver. A discrepancy of 5 hundredths occurred in the monitor receiver readings during the trials, and was discovered and determined by comparing the readings with those of the shipborne receiver, equipped with a zero-setting device; but the date the monitor receiver went out of order — probably as a result of its being shaken up by the truck — could not accurately be determined. Discrepancy values (as between the theoretical and actual values of the phasemeter readings) are shown for the six baseline extensions; the land values, possibly subject to an error of 5 hundredths due to poor adjustment of the monitor receiver, are the most likely values. The extreme value of 8 followed by a question mark is without doubt the first value obtained after the mishap. It should possibly be reduced to 3, which would align it with other values.

	S.W. Baseline Extension	N.E. Baseline Extension
Pattern I......	Sea: 12, 9, 9, 9, 9. (Value 12 was obtained prior to final adjustment.) Land: 10, 9, 10. (After partial travel over sea.)	Sea : None. Land: 2, 0, 1, -1.
Pattern II......	Sea : None. Land: 3, 4, 2, 3, 4.	Sea: None. Land: 4, 0, 1, 1, 8 (?)
Pattern III......	Sea : None. Land : 3.	Sea: None. Land: None.

« None » indicates that the corresponding intersection was impracticable or difficult. All these values were obtained during daylight.

Figure 4 shows the curve of variations in phase-difference readings as plotted against time for a course normal to the positive (NE) baseline extension of Pattern III.

b) *Monitor receiver readings:* The monitor receiver was set up on the *Estafette*, and readings taken every fifteen minutes showed the stability of the information supplied by the phasemeters of Patterns I and III throughout sounding operations. The rate of scatter, while low at night (1 hundredth), occasionally reached 2 or 3 hundredths during the warm hours in the middle of the day. In Pattern II, two stability rates were noted : one at night extending from 1900 to 0830; and a daytime rate (when readings increased by approximately 20 hundredths of a revolution) which became stabilized after a transition period lasting about an hour. At 1730 or thereabouts, readings decreased and settled at the night rate after an hour and a half or an hour and three-quarters. The twofold rate occurred with regularity during the eleven days of sounding operations, except for two days.

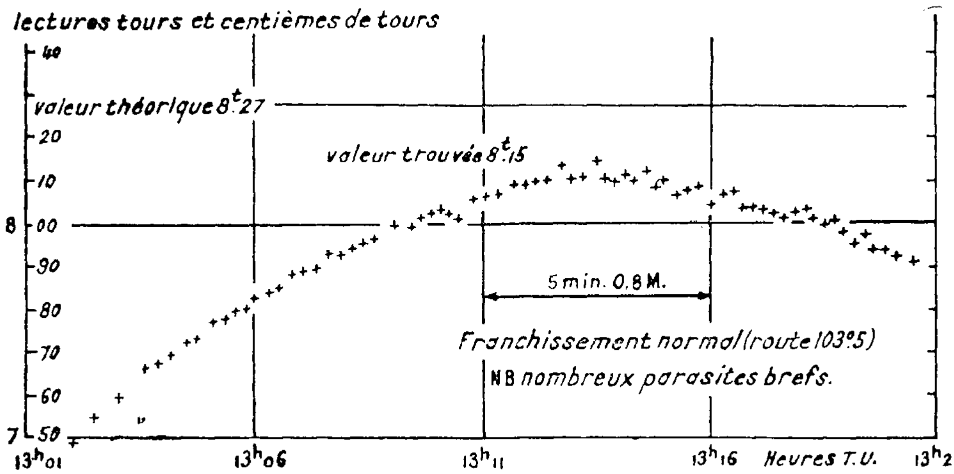


Fig. 4

Readings (revolutions and hundredths of a revolution).

Theoretical value 8.27 *revs.*

Actual value 8.15 *revs.*

Normal crossing (bearing 103.5°).

N.B. — Frequent interference of brief duration.

The question arose as to whether or not the change in rate affected the whole pattern. In anticipation of the results discussed later herein, it may be stated that the entire pattern moves in the same amount as at the monitor receiver. Computation of the RANA fixes, after applying the corrections mentioned hereafter, shows the existence during the transition periods, of « cocked hats » formed by the three hyperbolic curves. If the fix supplied by the intersection of the hyperbolae in Patterns I and III is assumed to be correct, and the correction to Pattern II that will eliminate the « cocked hat » is determined, it will be found that the correction is the same, to within 2 hundredths, as the monitor receiver correction required to obtain the theoretical value of the fix. Consistency of the chain is therefore high.

c) *Comparison of visual and RANA fixes:* At the end of operations, numerous visual fixes, obtained by measuring angles between landmarks, were computed, and the theoretical values that should have been obtained were compared with the actual phase-difference readings in the three patterns. The differences between the theoretical and actual values were then plotted against the number of the hyperbola in each pattern (Fig. 5). A small vertical segment, centred on the point corresponding to the most likely visual fix, represents the uncertainty, in hundredths of a revolution, relating to the visual fix uncertainty value, considered as equivalent to the linear dimensions of the cocked hat formed by the three visual position lines, in a normal direction to the hyperbola in each pattern. (See the paragraph on accuracy, under II above.)

The diagram shown, which relates to Pattern III, illustrates the fact that the discrepancy obtained is independent of the number of the hyperbola (i.e. the speed of propagation selected is therefore correct), and that it has a constant value throughout the pattern. Similar results, although somewhat less consistent and complete, were provided for the other two patterns. The significant outcome of this is that, throughout the entire range of the hyperbolae in the three patterns, and for the entire area in which comparisons were effected as between the visual and

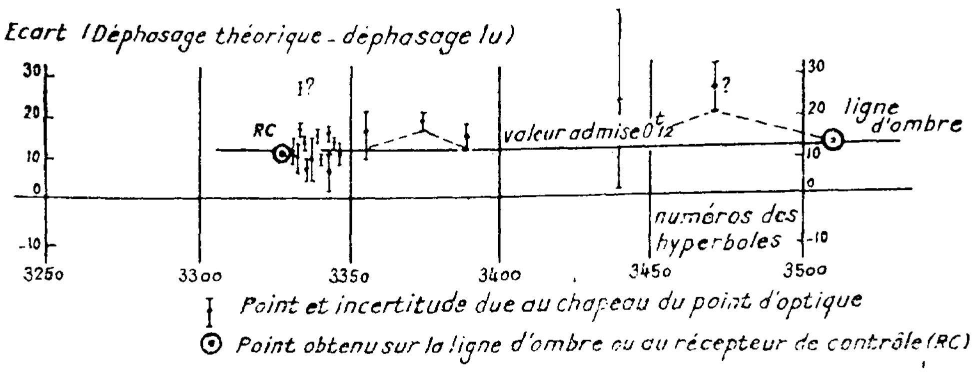


Fig.5

Difference between theoretical and actual values.

Accepted value. *Baseline extension.*

Hyperbola numbers.

Fix and uncertainty due to cocked hat of visual fix.
Fix obtained on baseline extension or at Monitor receiver.

RANA fixes, a constant calibration correction for each pattern ensures both consistency and accuracy of the chain. The method used in discovering the correction for each of the patterns involves an absence of cocked hats (i.e. a state of consistency) and a coincidence of the geodetic fix with the RANA fix (i.e. a high degree of accuracy).

Finally, corrections to the values read off were applied as follows:

Pattern I :	+ 0.08 revolution.
Pattern II :	— 0.02 revolution by day.
	+ 0.18 revolution by night.
Pattern III :	+ 0.12 revolution.

The question might be asked whether these corrections can be attributed to the difference in the speed of propagation over land and over sea during the synchronization process ? But in this connection, it must be noted that the path of the waves occurs over land in Pattern III and almost entirely over water in Pattern I; yet the correction values are largely the same. As regards Pattern II, can a day-rate be assumed during which synchronization might occur over the sea, and a night-rate when it might occur over land ? Here the answer is that the extent of the discrepancy between rates appreciably corresponds to the difference in propagation speed over land and sea. The problem of explaining calibration corrections is therefore not solved.

4) *Checking Pattern Consistency after Adoption of Calibration Corrections.*

A check was still required as to whether consistency verified for the inshore area was retained farther out at sea. But accurate investigation of this question could only be made by computing the RANA position lines, as in any case the cocked hats could not be evaluated — except in anomalous areas — on the 1 : 100,000 scale at which the hyperbolae were plotted.

How can the RANA fix be computed, or rather, how can the three hyperbolic elements supplied by the corrected RANA readings be plotted on a large-scale graph ? The intersection of hyperbolae involves the solving of equations of the fourth degree, but the use of methods for obtaining an approximate fix brings the degree down to unity, provided the approximate position obtained by plotting on the 1 : 100,000 scale be sufficiently close to the actual position.

Two sets of methods may be used for computation purposes. One set of processes includes those in which, first of all, the direction of the asymptote is computed, then one of its points, then the distance from the latter to a hyperbolic arc. The next set consists in comparison of the corrected phasemeter readings with the phase-differences computed for the approximate fix. Various methods of computation may be used in either computation group.

Operations carried out in Morocco made use of the first group, which was a transposition of the hyperbolic plot method. θ is computed, as previously shown; then by a standard geodetic process, the coordinates of the point of intersection of the asymptote and one of the parallels to the coordinate axis passing through the approximate fix are derived. Distance D of the point of intersection to the centre of the hyperbolae is thus easily obtained. The use of the formula $d = \frac{c^2 \sin 2\theta}{4D}$ gives the distance between the asymptote and a point on the hyperbola; plotting the parallel to the asymptote through such point supplies a hyperbolic arc element of usually adequate accuracy. If the location is less than 4c away from the centre of the pattern, either a simple corrective term may be computed, or equation $xy = \frac{c^2}{4}$ of the hyperbola referred to its asymptote may be used. x is identical to the distance D computed above, and y is derived and plotted parallel to the other asymptote. On a large-scale graph (1 :5,000, i.e. 1 mm. per 5 metres), a segment of asymptote, followed by a small hyperbolic arc, are plotted in both cases. The same computation is made with regard to the remaining two patterns. The entire computation of a RANA fix takes about 25 minutes.

A series of RANA fixes, distributed at hourly intervals over the lines used on 8th and 9th September, 1954, for daylight range tests, has been computed. The following table shows, as against the distance of the computed fix to the centre of the middle Pattern II, the smallest-sized computed triangular cocked hat, expressed in metres and in hundredths of a revolution in Pattern II (or I).

Distance of mobile from centre of Pattern II in km.	Size of cocked hats in metres	Size of cocked hats in hundredths of a revolution Pattern II (or I)
11	10 m.	10 /100 (1/100 in I)
29	20 m.	9 /100 (3/100 in I)
48	< 5 m.	< 1.3/100
68	< 5 m.	< 1 /100
87	35 m.	< 5.5/100
107	< 5 m.	< 0.6/100
129	22 m.	2.3/100
146	< 5 m.	< 0.5/100
164	9 m.	0.7/100
181	25 m.	1.8/100
198	30 m.	2 /100
214	30 m.	1.9/100

Other RANA fixes were computed in various areas at approximately regular intervals, but outside the anomalous zones, which are discussed in an ensuing section. The following table shows the statistical distribution of sizes of cocked hats for a total of 70 computed RANA fixes.

Size of cocked hats (in hundredths of a revolution)	Number of cases encountered
From 0 to 0.5 hundredth	21
From 0.6 to 1.5 hundredth	18
From 1.6 to 2.5 hundredths	21
From 2.6 to 3.5 hundredths	5
From 3.6 to 4.5 hundredths	3
Above 4.6 hundredths................	2
	Total 70

The average value for a cocked hat is 1.4 hundredths.

By means therefore of calibration operations carried out within sight of land and leading to the selection of corrections best ensuring pattern accuracy and consistency in the area involved, consistency is likewise obtained for offshore areas to within an average of two-hundredths and better, apart from the anomalous zones. It may also be claimed that accuracy of the pattern is of the order of 2 hundredths of a revolution. Since maximum consistency and accuracy have been obtained as regards inshore areas, and since such coexist due to the selection of calibration corrections, there is no reason why these factors should not likewise both be present offshore; and by checking one, i.e. consistency, the other naturally follows. A discrepancy of two hundredths in the phase-readings corresponds, in such a chain arrangement as the one adopted in Morocco, to linear uncertainties of the order of one metre on the baseline, 10 to 20 metres at 140 kilometres, and 20 to 40 metres at 200 kilometres distance of the mobile to the stations.

5) *Anomalous Zones.*

The systematic investigation of consistency in the various survey sectors by computation of the RANA fixes showed the existence of two anomalous zones, numbered 1 and 2 (Fig. 3), in which the cocked hats have a systematic appearance and which are bounded by the dotted lines in the figure.

One of the zones (No. 1) is partly within sight of the land, and the numerous visual position lines computed from angular measurements enabled the RANA cocked hat, which was likewise computed, to be definitely attributed to an anomaly — a very gradual one — in Pattern I. The intersecting of the hyperbolae of Patterns II and III systematically supplied a fix which was generally 10 to 15 metres (1) away from the visual position line, while the linear discrepancy of the hyperbola in Pattern I reached 100 metres and more. This anomalous zone is clearly limited to two hyperbolae in Pattern I and covers a sector of approximately 13° aperture. The maximum anomaly reaches 11 hundredths. The existence of the zone may be attributed to the presence of oil-tanks in the vicinity of Fedala harbour. Station A_1 was located 1,000 metres to the southwest of a large number of tanks; others were less than 500 metres from A_1, and much nearer R_1.

An additional zone (No. 2) which shows anomalies of unknown origin, but which, unlike Zone No. 1, could not be delimited and analyzed, owing to its presence out of sight of land, was discovered on 13th September, 1954, at about mid-day, at ranges between the mobile and the land exceeding 105 kilometres. The maximum anomaly was 8 hundredths and may have been temporary, as no anomaly was apparent on either side of Zone 2 along hyperbolae covered at less than a 24-hour interval.

6) *Range Tests.*

Tests for range were carried out both by day and by night. At long ranges the phasemeter needles seemed to become steadily more sluggish and highly sensitive to interference. During the daytime, a steady decrease in the gain setting was noticed, then, beyond 200 km., it was apparent that this setting must be due to interference. No such clear-cut variation was apparent at night, and the instant selected for the ship to come about was dictated by a rather subjective impression that the phasemeter needle was relatively insensitive to signals. The distances between the mobile and the stations at the instant of coming about seem to constitute the effective range limits. We obtained:

By night : 164 km., i.e. 88 miles;
By day : 214 km., i.e. 116 miles.

These values are worthy of note, when it is considered that the power radiated by the aerials on each frequency was approximately 1 watt.

7) *Practical Utilization. Uncertainties Experienced.*

The sounding lines followed were the hyperbolae in Pattern II, generally 5 revolutions apart. The regular rate of spread of the hyperbolae ensured proper distribution of the sounding lines, which were spaced at intervals varying from 500 metres at a distance of 8-10 miles offshore to 5,000-6,000 metres at 75 miles. A RANA fix was obtained every five minutes.

---

(1) The distance between the point of observation and the aerial may enter into the discrepancy.

The regular spacing of the soundings at the scale of the survey (1 :100,000) did not necessarily mean that the sounding vessel must strictly hold to the hyperbola involved : a shift of 5 hundredths of a revolution at 100-kilometre range resulted in a linear discrepancy of approximately 35 metres, or less than 4 tenths of a millimetre at 1 :100,000. A shift in course was ordered whenever the linear discrepancy with regard to the course exceeded a maximum of 100 metres. But it is believed that a maximum discrepancy of 20 metres in either direction (i.e. twice the beamwidth of the ship) may be maintained with sufficient ease, as long as the lanewidth amounts to no more than 1,000 metres (1 hundredth-revolution is equivalent to 10 metres). A shift from one line to another was readily accomplished by the accurate translation of the ship's turning radius into one-hundredths of a revolution, an operation which was necessary in order to pick the right time to put the helm over so that the ship would end up on the prescribed hyperbola with the required heading.

A few uncertainties arose, particularly inshore, in the areas of greater sensitivity of a pattern. It has already been mentioned that the coarse readings were obtained by successive switching in the three patterns; in areas where the hyperbolae are closely adjacent, the delay involved in the prior « fine » phasemeter reading may result in an uncertainty of one revolution in the « coarse » reading. This uncertainty often may only be resolved by referring to the sequence of the readings and, due allowance being made for the ship's speed and course, by estimating the effect of the delay in their recording, especially where a more or less well-known or delimited anomalous zone is involved. This disadvantage may be somewhat remedied by causing the « coarse » reading of the most sensitive pattern to be taken by an additional observer. Improved design of the phasemeters and substitution of a 1 :10 ratio for the 1 :20 ratio between the coarse and fine patterns would at least partially eliminate such uncertainties.

V. — CONCLUSION

The prototype of the RANA chain was tried out in the Seine estuary in April, 1954, and within a few months it was possible to use it in routine hydrographic operations, a result which is ample proof of the excellent qualities of the equipment. Only few of us indeed, on the eve of the April trials, believed in the immediacy of adequate overall performance, and fewer still were the optimists who dared to hope that the equipment might be used during the 1954 surveying expedition.

The equipment reached Morocco during the early part of August, 1954, after having suffered considerable damage owing to poor packing, but also due to its excessive fragility. Additional difficulties arose in connection with its installation on shore owing to the above-mentioned circumstances and the lack of personnel, yet by 1st September the equipment was ready for use in sounding operations. With the exception of two engineers and a technician of the *Compagnie des Compteurs,* the survey was compelled to rely almost solely on its own staff.

Soundings were carried out over a fifteen-day period lasting from 1st to 16th September, 1954, during ten days and eleven nights, and a distance of 2,500 nautical miles was covered in an area measuring 10,000 square kilometres. None of the sounding operations had to be suspended as a result of a breakdown.

The various characteristics of the chain were studied during operations, and results show that, allowance being made for all corrections and for the anomalous zones, which were easily detected owing to the presence of 3 position lines, the chain supplied phase-difference readings with an accuracy of the order of 2 hundredths of a revolution. For the type of chain set up in Morocco, this figure corresponds to a linear accuracy of a few ten-thousandths of the distance of the mobile to the pattern centres. This order of accuracy is amply sufficient for hydrographic purposes.

The soundings revealed an extremely unusual type of submarine relief that has been described elsewhere (1); but the most interesting feature of the survey was the establishment of the excellent hydrographic adaptability of an instrument of French design and manufacture. The equipment stood the test brilliantly, even though revision of various parts is required to reduce its weight and increase its mobility and robustness for transportation purposes.

(1) « Relief du talus continental marocain au large de Rabat » (Relief of Moroccan Continental Slope off Rabat), *Bulletin du Comité d'Océanographie et d'Etude des Côtes*, Paris, May 1955.